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OAK RIDGE National Laboratory

Sum-of-Fractions Method

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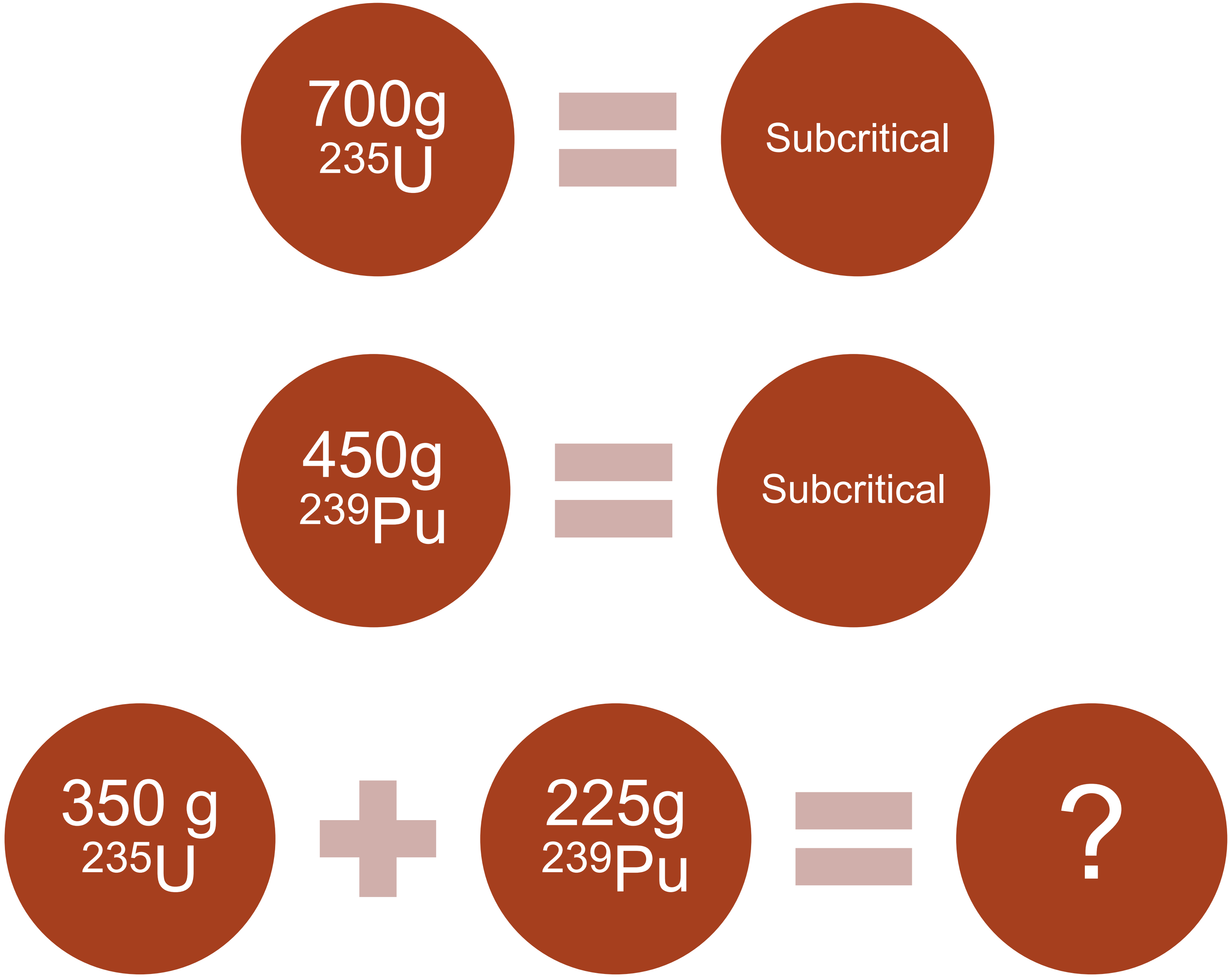
Purpose

- Addressing the presence of multiple fissionable nuclides in a moderated system
- Validation with benchmarking gaps
  - Calculational margin
  - Margin of subcriticality



Background

- ANSI/ANS-8.15-1981
  - "… the sum of ratios of the mass of each fissile nuclide to its limit does not exceed unity…"
  - Only for water moderated and reflected systems
- Three methods of note were derived from the ANSI/ANS-8.15-1981
  - Rule-of-Fractions
    - ✓ Direct application of ANSI/ANS-8.15-1981
  - Fissile Gram Equivalent
    - ✓ Rule-of-Fractions restricted to a specific moderated and reflected system
  - Sum-of-Fractions (SoF)
    - ✓ Generalization of the Rule-of-Fractions that extends use beyond water moderated and reflected systems

Background (cont.)

- Revision of ANSI/ANS-8.15-2014 removed Rule-of-Fractions
- ANSI/ANS-8.15-2014
  - If mixture is of "…different isotopes of the same material… the subcritical limit for the limiting nuclide should be used…"
  - Or "… the user may calculate their own subcritical limits for mixtures of material ..."
- Work presented here provides a method for calculating subcritical limits using Sum-of-Fractions

Sum-of-Fractions (SoF)

- For a mixture of nuclides i with masses a_i and minimum subcritical mass A_i subcriticality is defined as:

$$\sum_i \frac{a_i}{A_i} \leq 1$$

- The minimum subcritical masses for all the nuclides is based on a system that is more reactive than the specific system evaluated

Method

- Establish a set of mixtures with ^{233}U, ^{235}U, and ^{239}Pu to determine bias, uncertainty in bias (calculational margin), and validation applicability
- Sensitivity analyses performed to determine bias and bias uncertainty
- Validation applicability determined from the upper and lower energy corresponding to the average lethargy causing fission (EALF)

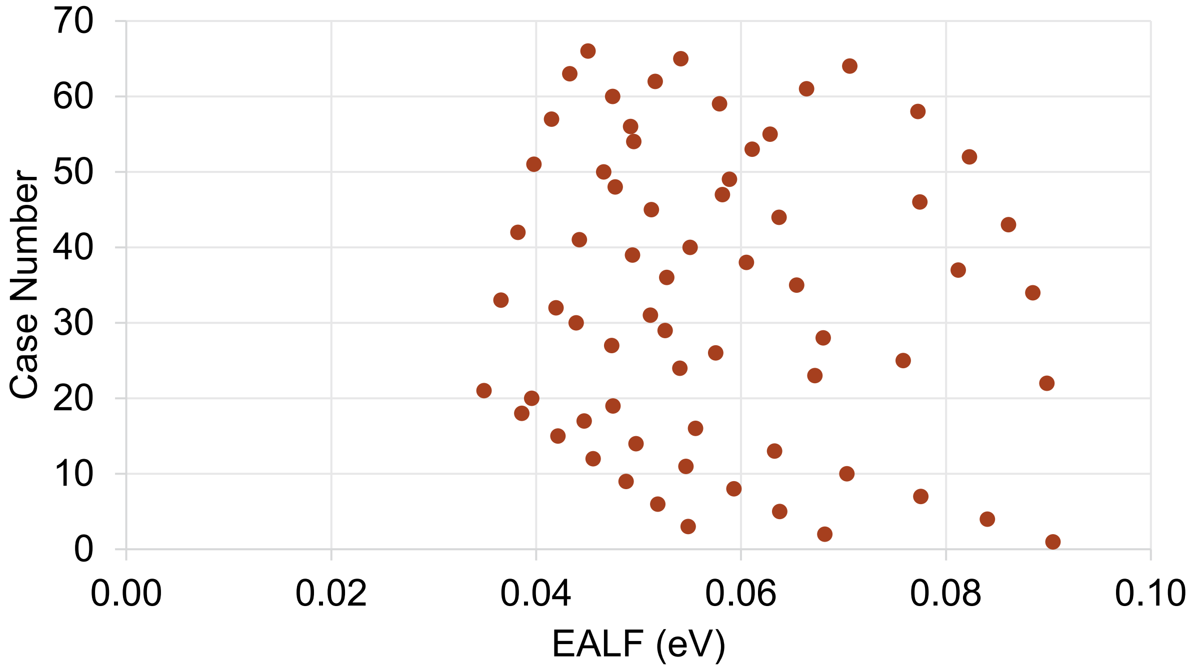
Method (cont.)

- Compute minimum subcritical masses for optimally moderated and fully reflected systems
  - Limited benchmarks to determine bias and bias uncertainty of actinides present in 8.15
  - Use of the ^{233}U, ^{235}U, and ^{239}Pu to establish
- Mixtures containing 5/6 subcritical mass of ^{233}U, ^{235}U, or ^{239}Pu and 1/6 of an 8.15 actinide
- Margin of subcriticality applied to nuclide masses
- Calculations using SCALE 6.2.4

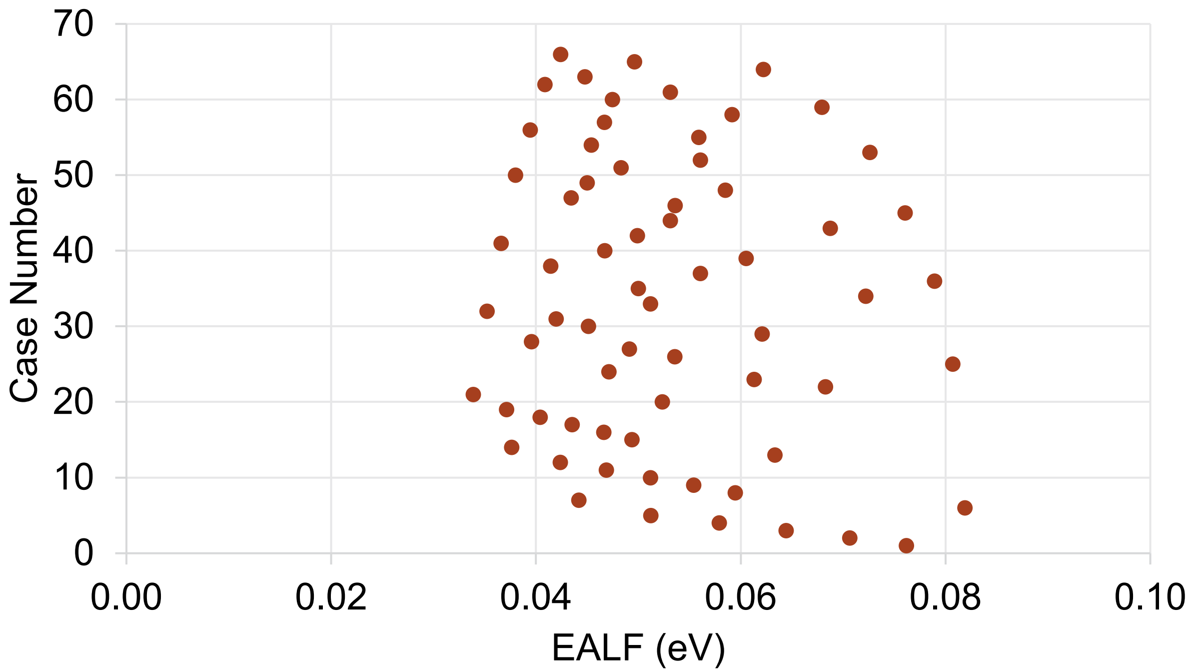
^{233}U, ^{235}U, and ^{239}Pu validation mixtures

- Mixtures used in validation selected to cover moderated systems
- Approximately 30g/L, 50g/L, and 80g/L systems
- All combinations of compositions of 0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1 volume fractions were computed
- EALF range for water system (0.035 - 0.09 eV)
- EALF range for polyethylene system (0.034 - 0.082 eV)

EALF Values for Validation Cases

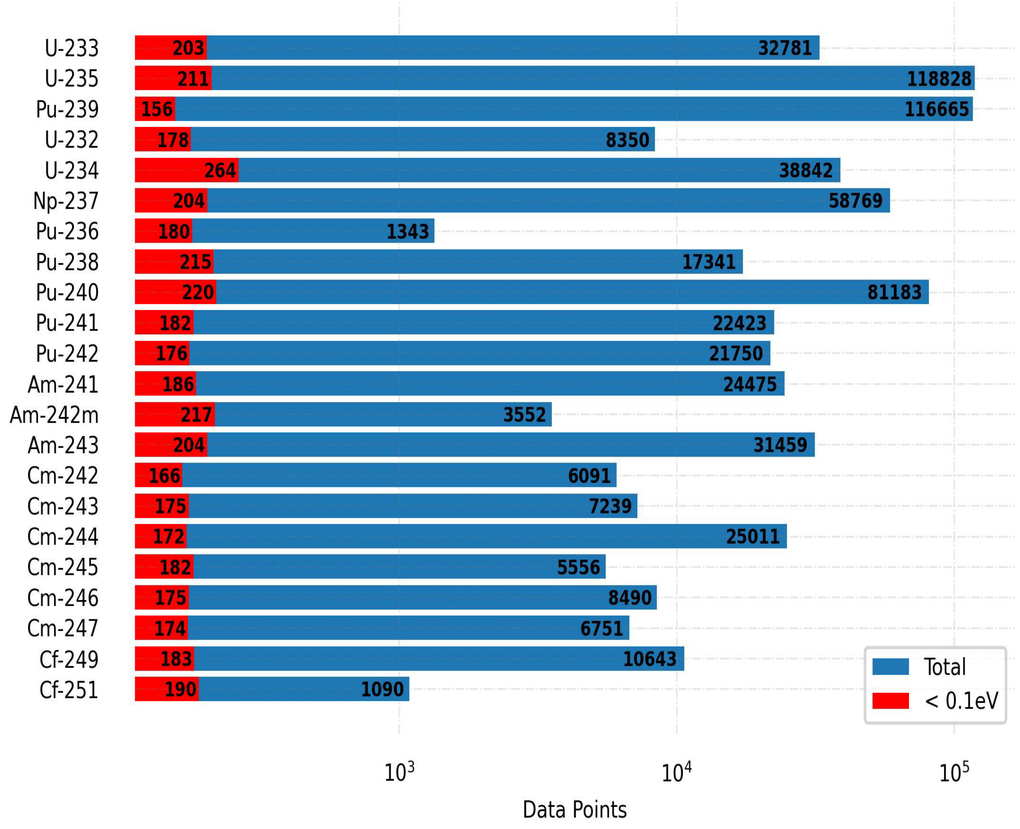


Water Moderated and Reflected

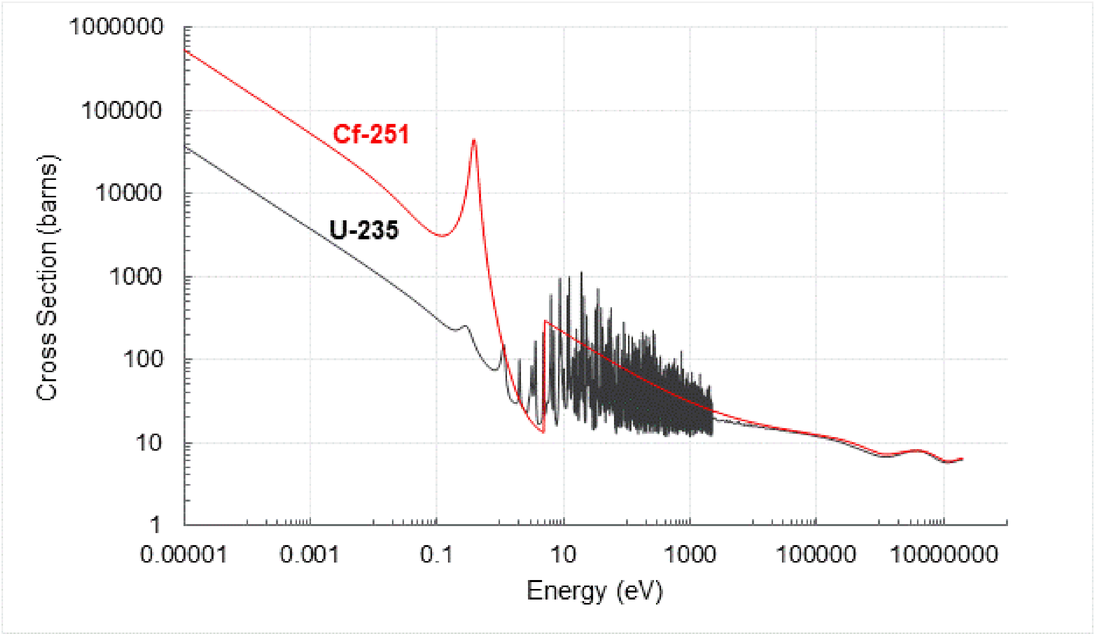


Polyethylene Moderated and Reflected

Available Cross Section Data ENDF/B VII.1

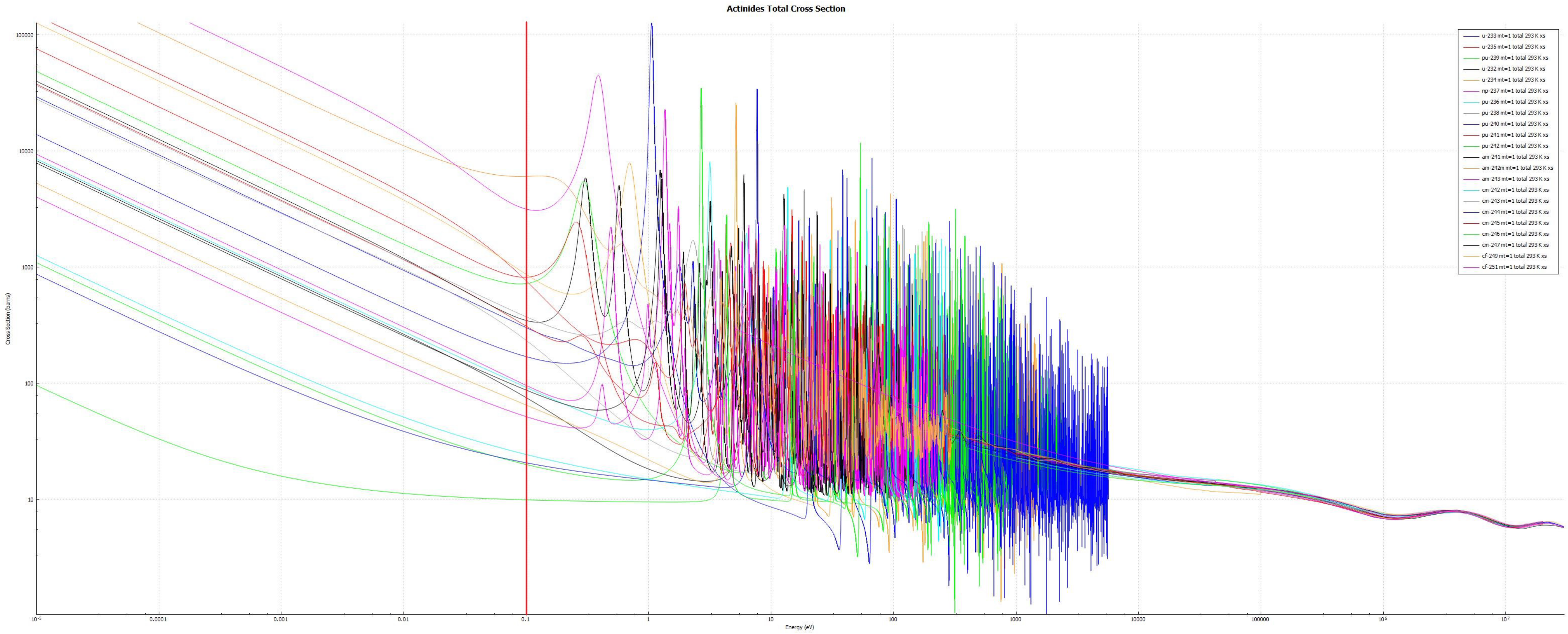


Number of data points for total cross sections



^{235}U and ^{251}Cf total cross sections

Total Cross Section of Nuclides



Sensitivity Study

- TSUNAMI-1D used for generating sensitivity data for application models
  - Sensitivities confirmed by direct perturbation calculations
  - Adequacy of MG 1D method confirmed by comparisons with CE TSUNAMI
- VALID and NEA used as source for benchmarks
  - c_k threshold of 0.8 used
- Calculational margin generated via c_k trending
  - 0.980 for water moderated and reflected systems
  - 0.965 for polyethylene moderated and reflected systems

Minimum Subcritical Masses

Nuclide	Water (g)	Poly (g)	8.15 (g)
U-233	500	250	-
U-235	700	400	-
Pu-239	450	250	-
U-232	3300	2900	1000
U-234	100000	90000	59000
Np-237	47000	44000	35000
Pu-236	1000	600	600
Pu-238	6300	5800	5100
Pu-240	31000	29000	20000
Pu-241	250	100	185
Pu-242	59000	55000	50000
Am-241	56000	52000	24000
Am-242m	21	11	11
Am-243	120000	108000	65000
Cm-242	9800	9000	6000
Cm-243	200	100	90
Cm-244	20000	19000	11000
Cm-245	58	33	23
Cm-246	65000	60000	16000
Cm-247	1100	650	500
Cf-249	58	33	10
Cf-251	27	15	5

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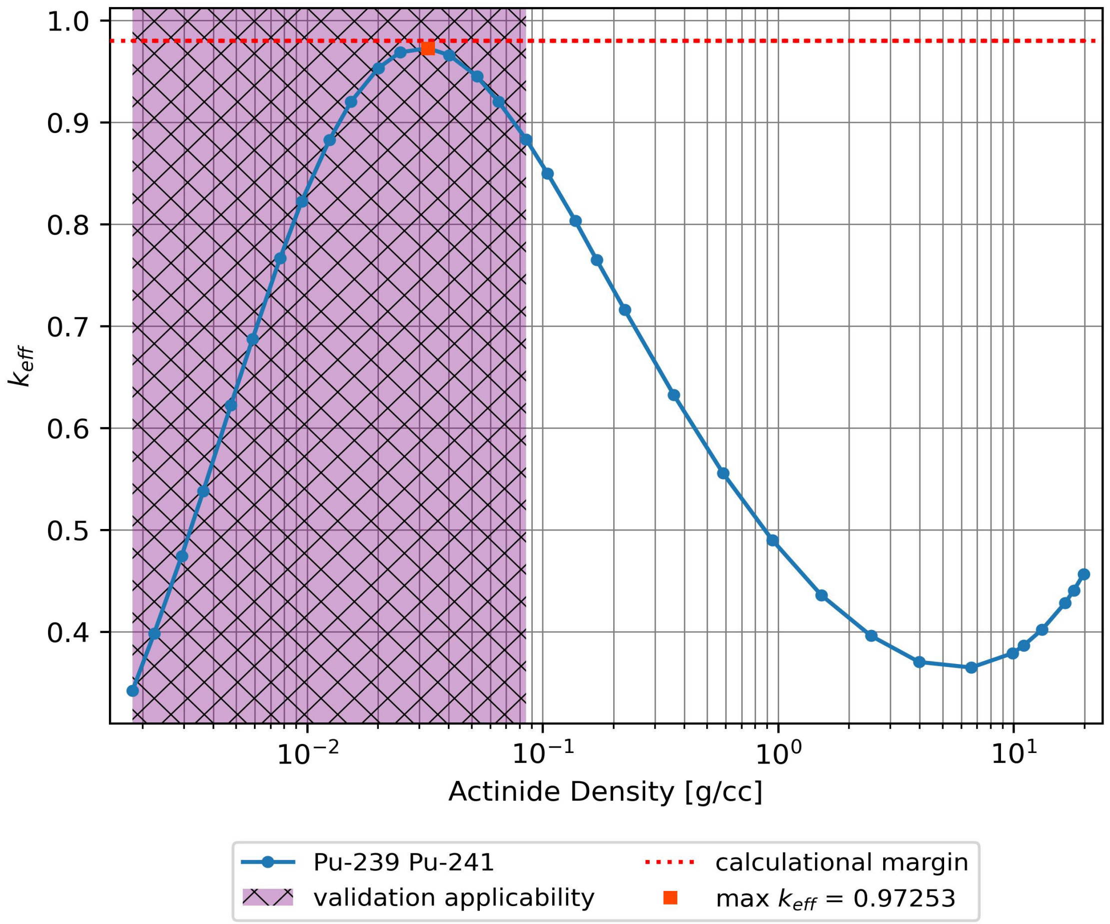
Two Nuclide Mix

Water Moderated and Reflected			
	Peak k_{eff}		
Nuclide	U-233	U-235	Pu-239
U-232	0.94332	0.95189	0.95796
U-234	0.69109	0.67946	0.68568
Np-237	0.68676	0.67217	0.68742
Pu-236	0.96967	0.96872	0.97081
Pu-238	0.66242	0.61224	0.66893
Pu-240	0.68269	0.65830	0.67526
Pu-241	0.97086	0.97123	0.97253
Pu-242	0.75328	0.73399	0.74228
Am-241	0.66117	0.65010	0.65946
Am-242m	0.96814	0.96539	0.96490
Am-243	0.68339	0.67944	0.68780
Cm-242	0.86555	0.88714	0.89897
Cm-243	0.96894	0.96540	0.96855
Cm-244	0.77865	0.81193	0.83088
Cm-245	0.96499	0.96368	0.96333
Cm-246	0.86374	0.87935	0.88773
Cm-247	0.97451	0.97275	**0.97582***
Cf-249	0.96597	0.96284	0.96381
Cf-251	0.97096	0.96770	0.96947

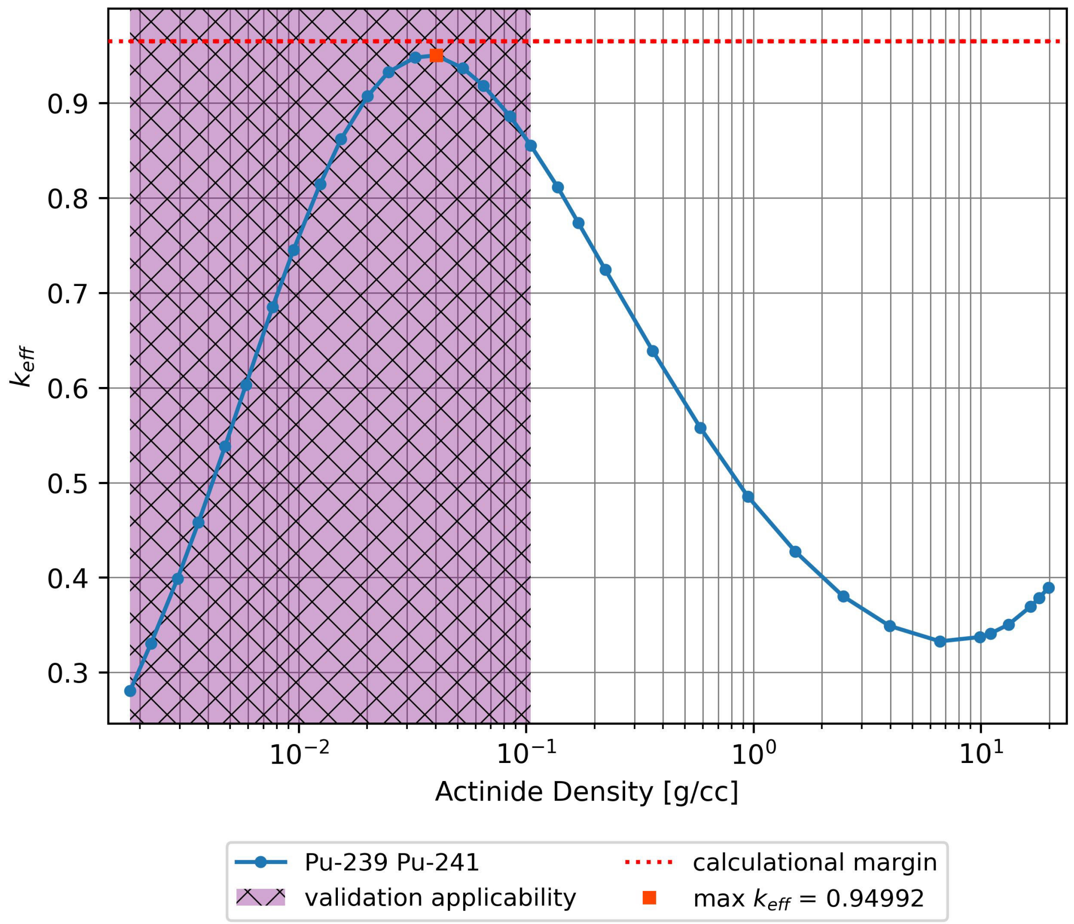
* Max k_{eff}

Polyethylene Moderated and Reflected			
	Peak k_{eff}		
Nuclide	U-233	U-235	Pu-239
U-232	0.91658	0.95326	0.95647
U-234	0.65125	0.64905	0.65243
Np-237	0.65894	0.65255	0.66233
Pu-236	0.93244	0.96100	0.95938
Pu-238	0.63373	0.60226	0.63869
Pu-240	0.64961	0.63653	0.64521
Pu-241	0.91864	0.95049	0.94992
Pu-242	0.70907	0.70673	0.71211
Am-241	0.64059	0.63493	0.64037
Am-242m	0.92694	0.95331	0.95197
Am-243	0.65611	0.65725	0.66339
Cm-242	0.79546	0.85636	0.86932
Cm-243	0.92375	0.95166	0.95109
Cm-244	0.71021	0.75365	0.77373
Cm-245	0.92479	0.95217	0.94976
Cm-246	0.80311	0.85438	0.86153
Cm-247	0.93589	**0.96355***	0.96285
Cf-249	0.92555	0.95271	0.95025
Cf-251	0.93112	0.96004	0.95600

^{239}Pu and ^{241}Pu Mix

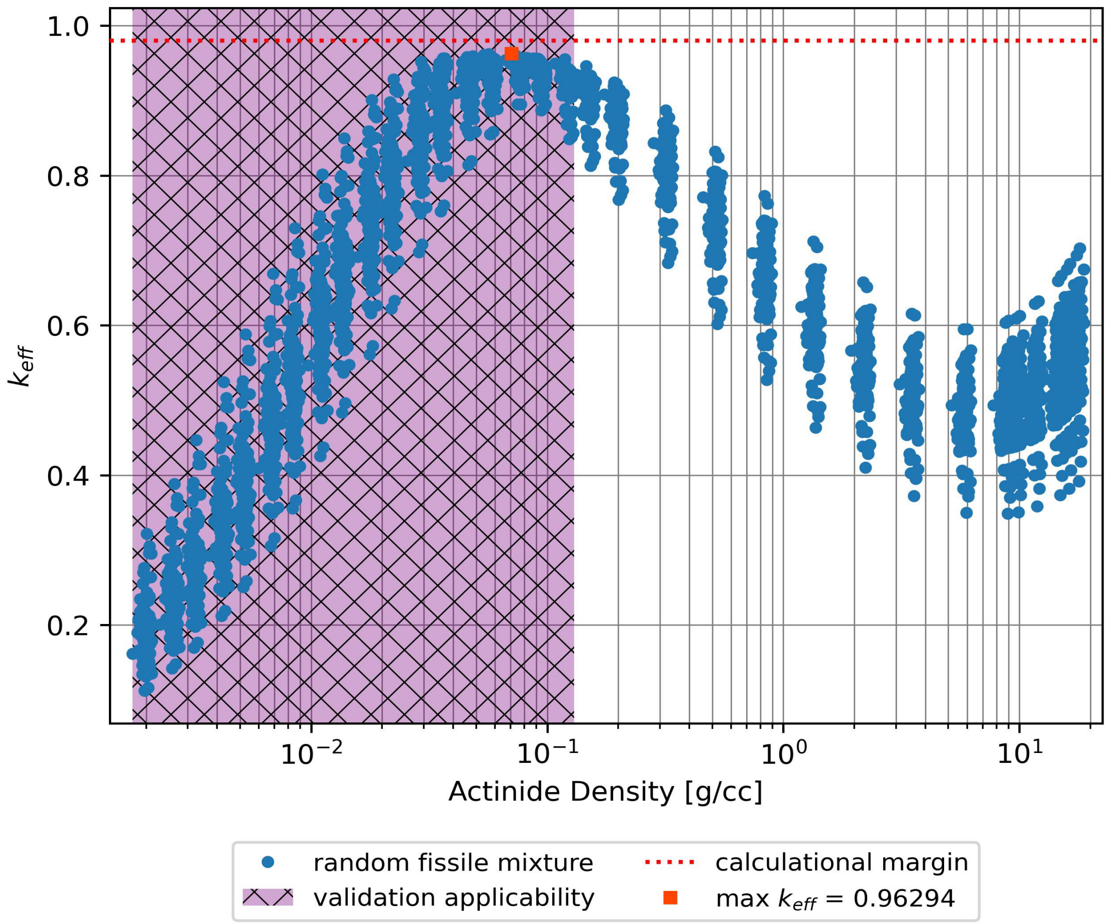


Water Moderated and Reflected

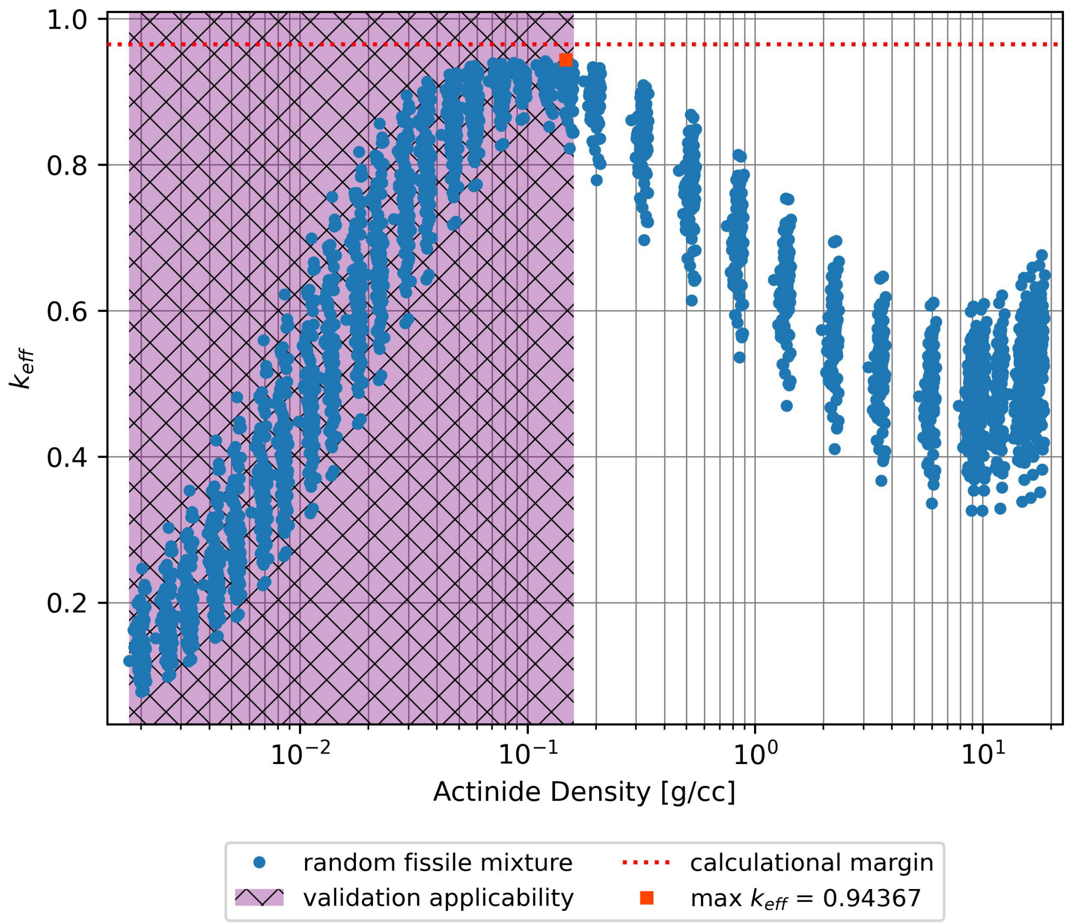


Polyethylene Moderated and Reflected

Random Mix of Fissile Nuclides

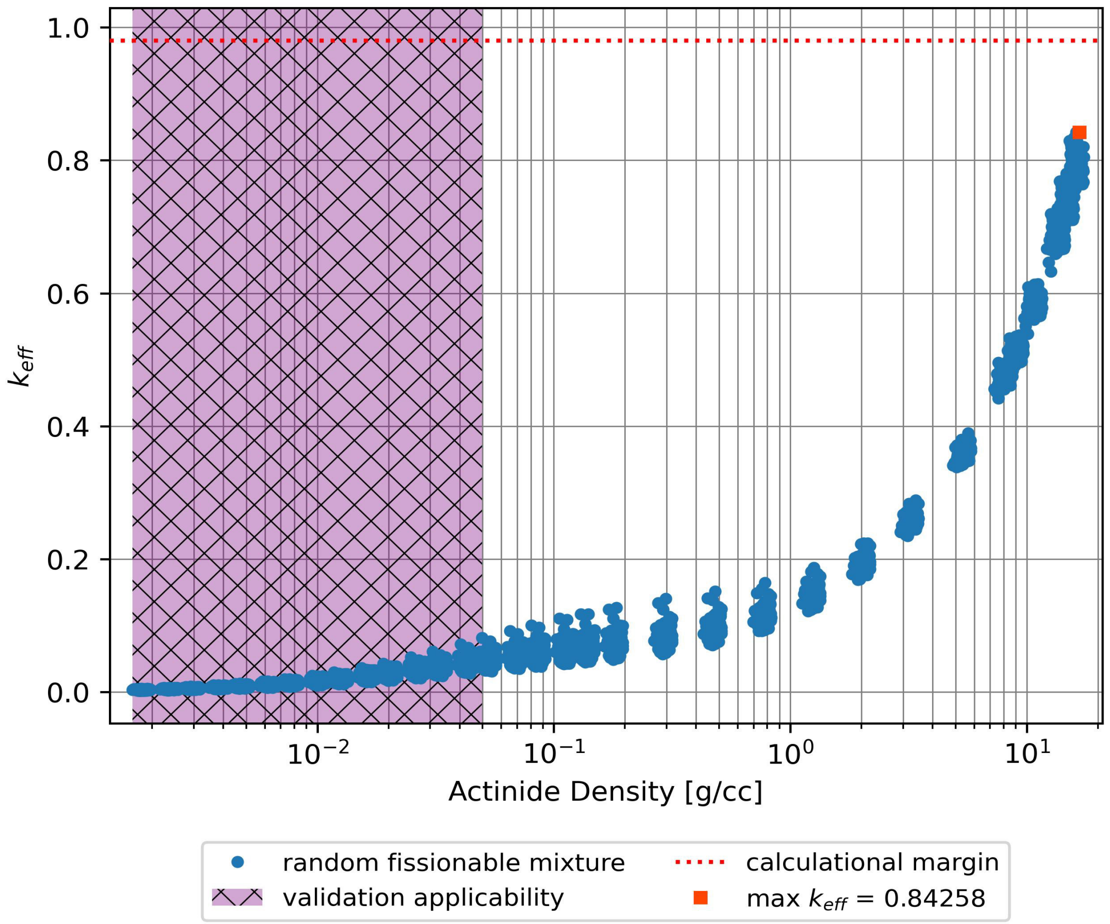


Water Moderated and Reflected

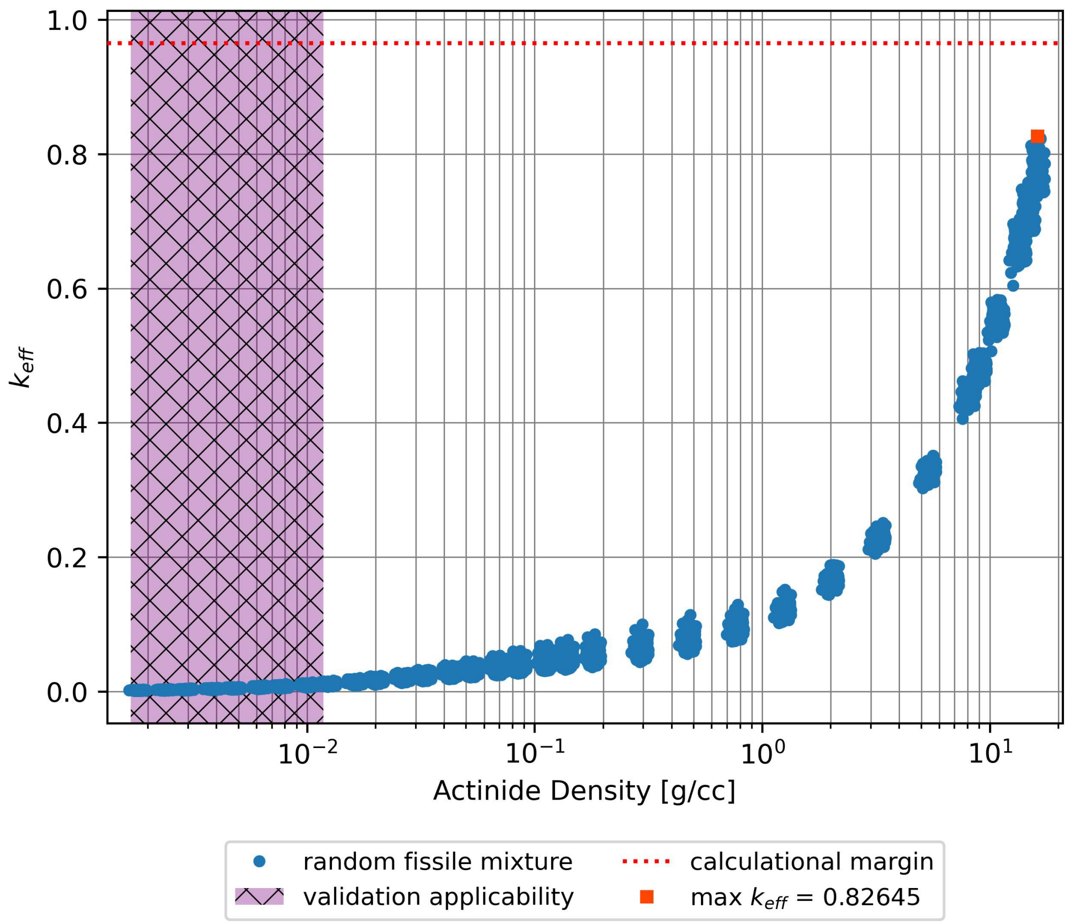


Polyethylene Moderated and Reflected

Random Mix of Fissionable Nuclides



Water Moderated and Reflected



Polyethylene Moderated and Reflected

Margin of Subcriticality

- Due to the lack of benchmarks and uncertainties in cross sections a mass penalty is applied via a subcritical factor (Appendix C of ANSI/ANS-8.15)
- Subcritical factors were applied to the ANSI/ANS-8.15 nuclides for water moderated and reflected systems
- Extension of the subcritical factors for nuclides in 8.15 can be made for polyethylene moderated and reflected systems
- Minimum subcritical mass between the computed and 8.15 values is used for each nuclide

Subcritical Masses for Sum-of-Fractions in Water Moderated and Reflected Systems

Nuclide	Computed Mass (g)	factors	Computed w/ factors (g)	ANSI-8.15 (g)	SoF Mass (g)
U-233	500	1	500	--	500
U-235	700	1	700	--	700
Pu-239	450	1	450	--	450
U-232	3300	0.5	1650	1000	1000
U-234	100000	0.5	50000	59000	50000
Np-237	47000	0.7	32900	35000	32900
Pu-236	1000	0.5	500	600	500
Pu-238	6300	0.7	4410	5100	4410
Pu-240	31000	0.7	21700	20000	20000
Pu-241	250	0.7	175	185	175
Pu-242	59000	0.7	41300	50000	41300
Am-241	56000	0.5	28000	24000	24000
Am-242m	21	0.5	10.5	11	10.5
Am-243	120000	0.5	60000	65000	60000
Cm-242	9800	0.5	4900	6000	4900
Cm-243	200	0.5	100	90	90
Cm-244	20000	0.5	10000	11000	10000
Cm-245	58	0.5	29	23	23
Cm-246	65000	0.5	32500	16000	16000
Cm-247	1100	0.5	550	500	500
Cf-249	58	0.5	29	10	10
Cf-251	27	0.5	13.5	5	5

Californium Production

- Subcritical mass using the SoF method: 768 grams
- Subcritical mass using the most reactive fissile mass subcritical limit from Cf-251: 5 grams

Nuclide	Mass (g)	Mass with Margin of Subcriticality (g)
pu-238	1.192114	0.463458
pu-239	0.715268	0.278075
pu-240	663.8882	258.0997
pu-241	2.384228	0.926916
pu-242	99.30309	38.60604
am-241	11.68272	4.541887
am-243	75.69923	29.42958
cm-244	215.7726	83.88588
cm-245	50.66484	19.69696
cm-246	809.6838	314.7806
cm-247	44.70427	17.37967
cf-249	1.072903	0.417112
cf-251	0.238423	0.092692
Total	1977.002	768.5986

Conclusions

- No instances of the use of the SoF method resulted in k_{eff} values exceeding the calculational margin
- When utilizing the SoF include all fissile nuclides
- Provides additional flexibility than ANSI/ANS-8.15 for mixtures
- Extend validation applicability by extending validation

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NCSP
NUCLEAR CRITICALITY SAFETY PROGRAM

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References

- SCALE Code System, ORNL/TM-2005/39, Version 6.2.4 (April 2020).
- ANSI/ANS-8.15-1981, "Nuclear Criticality Control of Special Actinide Elements," American Nuclear Society, La Grange Park, IL, 1981.
- ANSI/ANS-8.15-2014, "Nuclear Criticality Control of Special Actinide Elements," American Nuclear Society, La Grange Park, IL, 2014.

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Thank you

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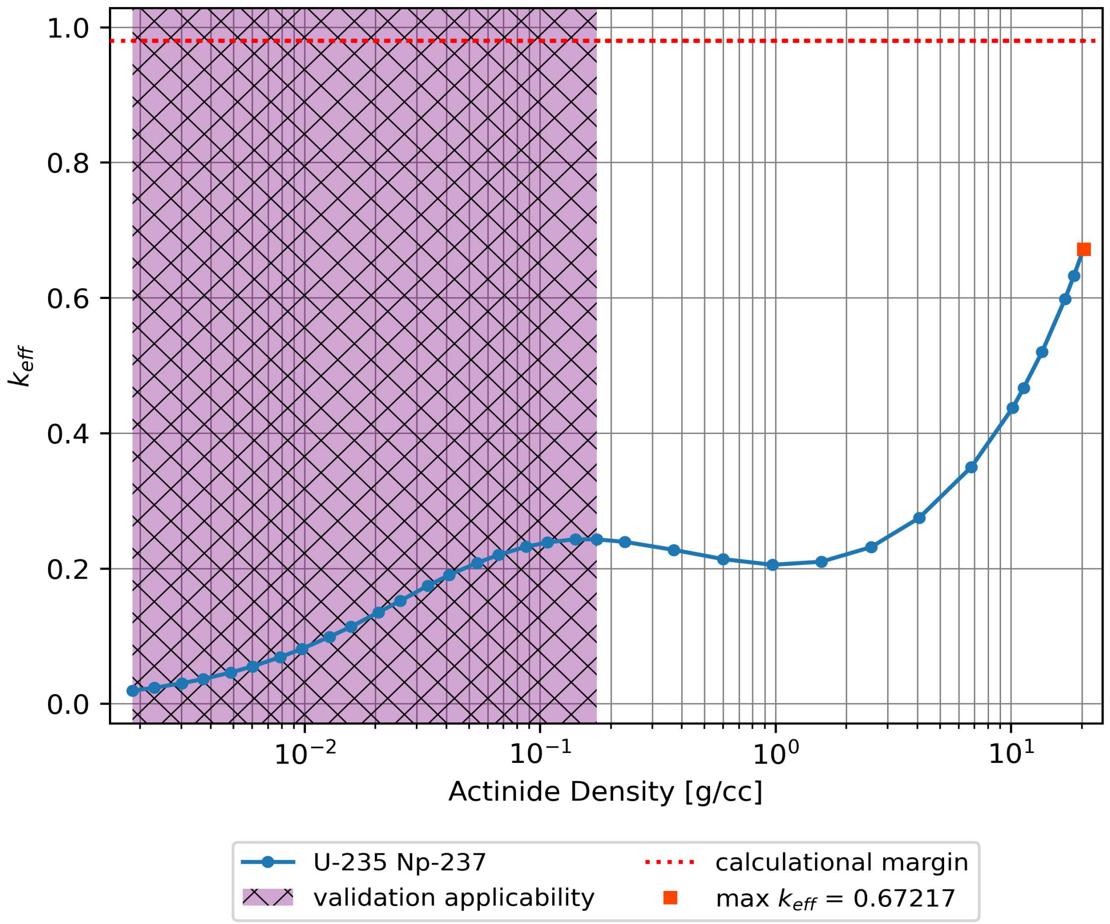
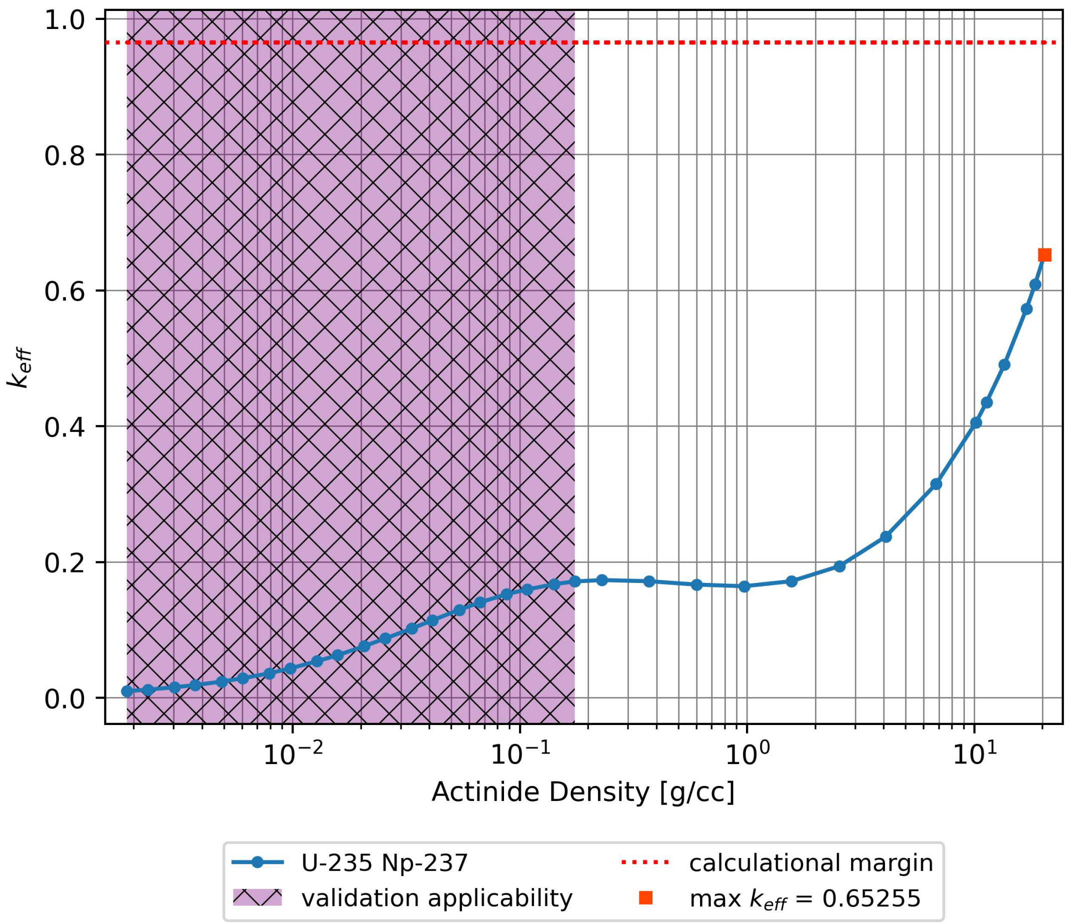
^{235}U and ^{237}Np Mix

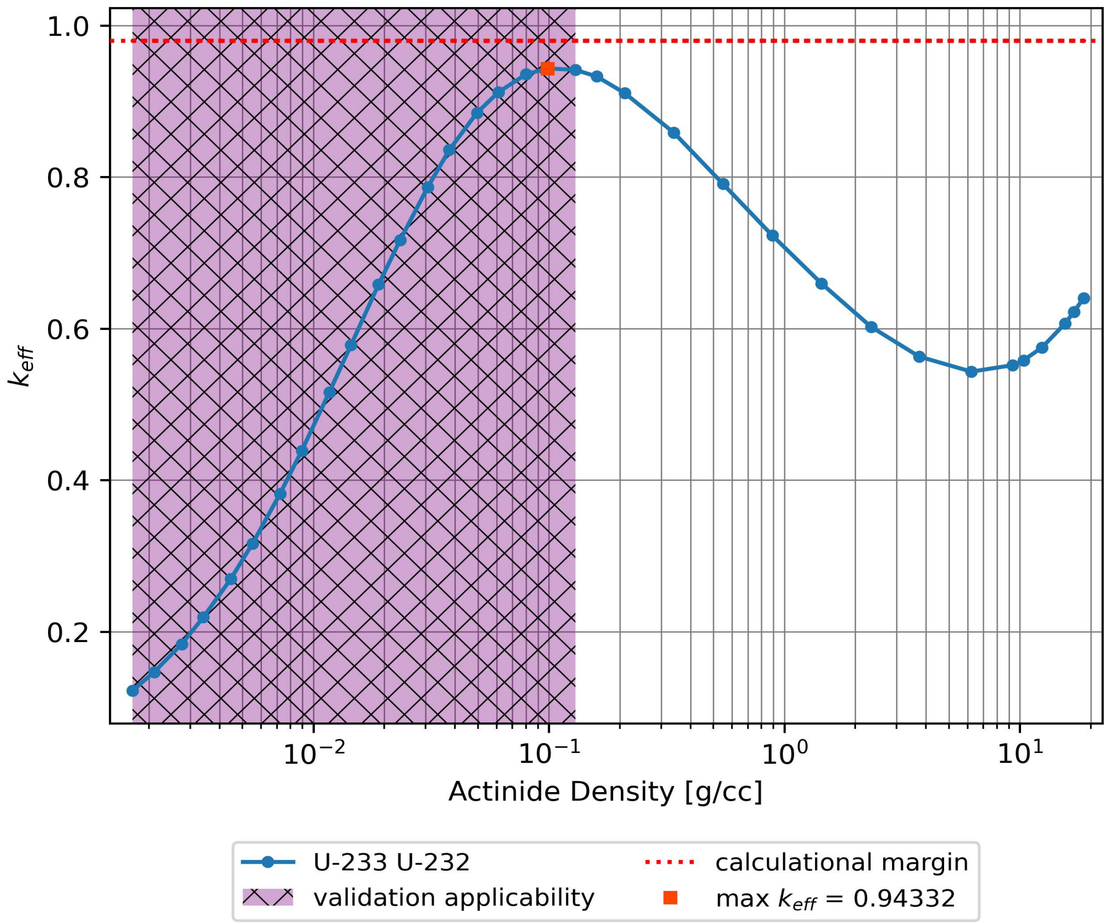


Water Moderated and Reflected

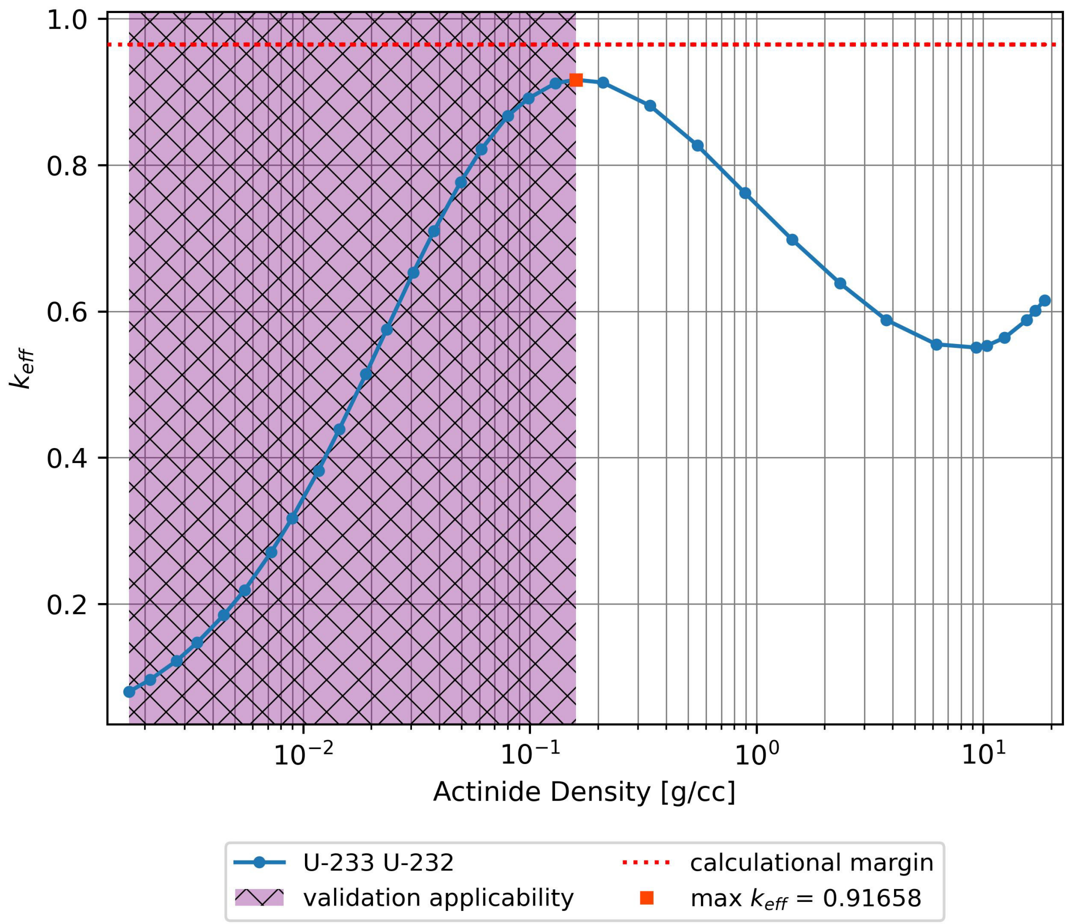


Polyethylene Moderated and Reflected

^{233}U and ^{232}U Mix



Water Moderated and Reflected

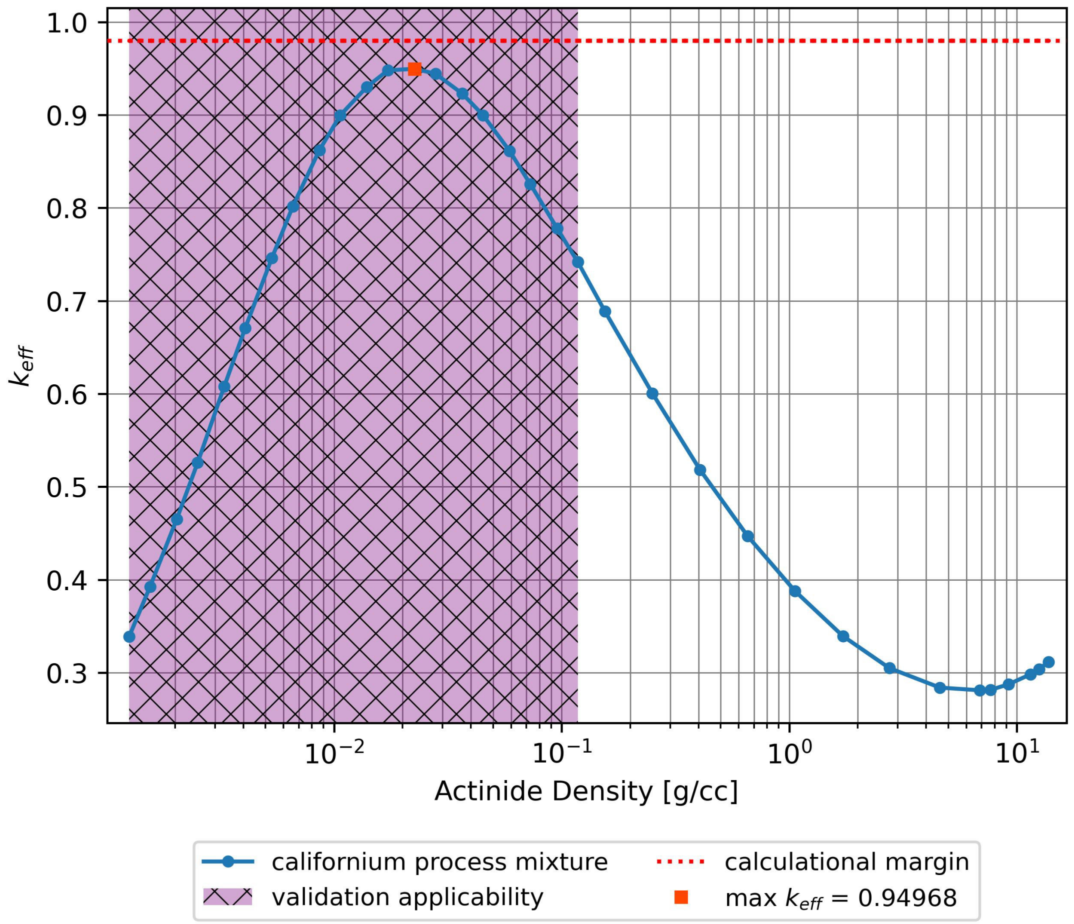


Polyethylene Moderated and Reflected

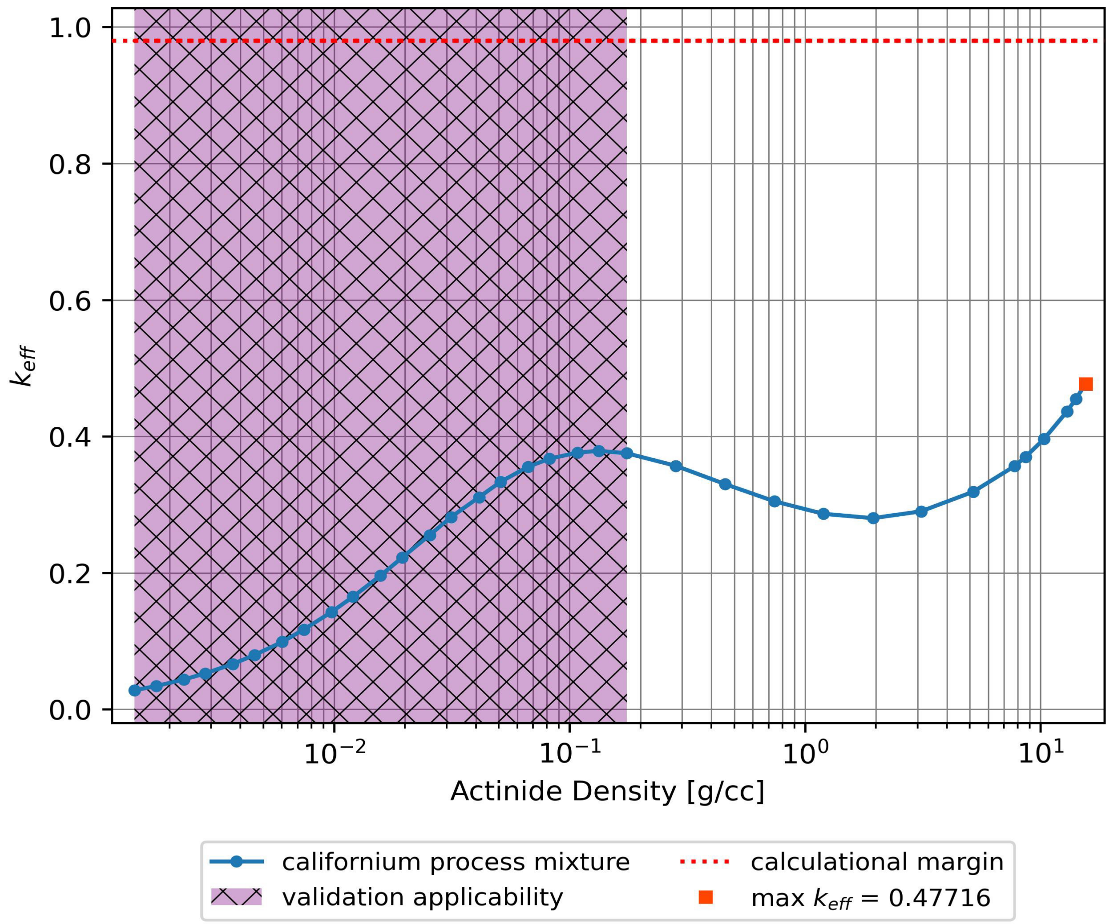
Subcritical Masses for Sum-of-Fractions in Polyethylene Moderated and Reflected Systems

Nuclide	Computed Mass (g)	factors	Computed w/ factors (g)	ANSI 8.15 (g)	SoF Mass (g)
U-233	250	1	250	--	250
U-235	400	1	400	--	400
Pu-239	250	1	250	--	250
U-232	2900	0.5	1450	1000	1000
U-234	90000	0.5	45000	59000	45000
Np-237	44000	0.7	30800	35000	30800
Pu-236	600	0.5	300	600	300
Pu-238	5800	0.7	4060	5100	4060
Pu-240	29000	0.7	20300	20000	20000
Pu-241	100	0.7	70	185	70
Pu-242	55000	0.7	38500	50000	38500
Am-241	52000	0.5	26000	24000	24000
Am-242m	11	0.5	5.5	11	5.5
Am-243	108000	0.5	54000	65000	54000
Cm-242	9000	0.5	4500	6000	4500
Cm-243	100	0.5	50	90	50
Cm-244	19000	0.5	9500	11000	9500
Cm-245	33	0.5	16.5	23	16.5
Cm-246	60000	0.5	30000	16000	16000
Cm-247	650	0.5	325	500	325
Cf-249	33	0.5	16.5	10	10
Cf-251	15	0.5	7.5	5	5

Cf Production



Fissile only



Fissile and fissible