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Critical Parameters of a Uranium Solution Slab-Cylinder System

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Critical parameters are reported for uranium-solution systems consisting of equally spaced vertical cylinders arranged in a square array resting on the bottom of a 20.3-cm-high square slab tank. Some of these systems were reflected externally. Both the cylinders and the slab contained uranyl-nitrate solution having 490 g of uranium (93.2 wt% ^{235}U)/liter.

A system of an 87-cm-high array of sixteen 11.0-cm-diam cylinders on an 11.4-cm-thick solution slab was critical. The slab alone was critical at 12.8 cm. Another critical system was a single 22.4-cm-diam cylinder of effectively infinite height on a solution slab 10.8-cm thick. The 22.4-cm diameter is 93.7% of the critical diameter for an infinite cylinder.

Monte Carlo calculations, simulating several typical experimental critical systems, yielded values for k_{eff} between 0.958 ± 0.012 and 0.986 ± 0.009.

INTRODUCTION

The criticality safety of interacting systems consisting of cylinders of fissile solution perpendicular to the surface of a fissile solution slab often must be evaluated. Examples of such systems are: (a) floor drains, (b) a leak in one or more of a group of vessels, and (c) disengagement sections of solvent extraction systems.

In the past, very conservative approximations[1] in solving problems such as these were used. The data presented here allow a more accurate evaluation of these situations. The data can also be used to determine the accuracy of computer programs.

The uranyl-nitrate solution contained 490 ± 30 g U/liter, where the uranium was enriched to 93.2 wt% ^{235}U. On externally reflected measurements, a 10.2-cm-thick Plexiglas[a] box-like reflector surrounded the slab-cylinder composite systems.

One auxiliary experiment evaluated the effect of passing a container of fissile solution through an array of cylinders. Another series of auxiliary measurements was made to determine the effect of raising the array of cylinders above the solution slab.

EXPERIMENTAL EQUIPMENT

The main experimental-equipment components were a set of right-circular cylinders, a slab tank, the uranyl-nitrate solution, and an external Plexiglas reflector. A typical reflected experimental configuration is shown in Fig. 1.

The dimensions of each Type-316 stainless-steel cylinder are given in Table I. The array of vertical cylinders rested on the bottom of the slab tank and was supported laterally by two mild steel plates. These 152-cm-square by 0.159-cm-thick plates were fastened to a low mass framework and positioned 36 and 95 cm above the slab-tank bottom. The cylinders were fixed in a square array with equal distances between the cylinder centerlines and half the distance between

[a]Trademark of Rohm and Haas Company, Philadelphia, Pennsylvania.

[1]C. L. SCHUSKE, *Application of Criticality Information to Y-12 Plant Problems*, Y-853, p. 37, Oak Ridge, Y-12 Plant (1952).

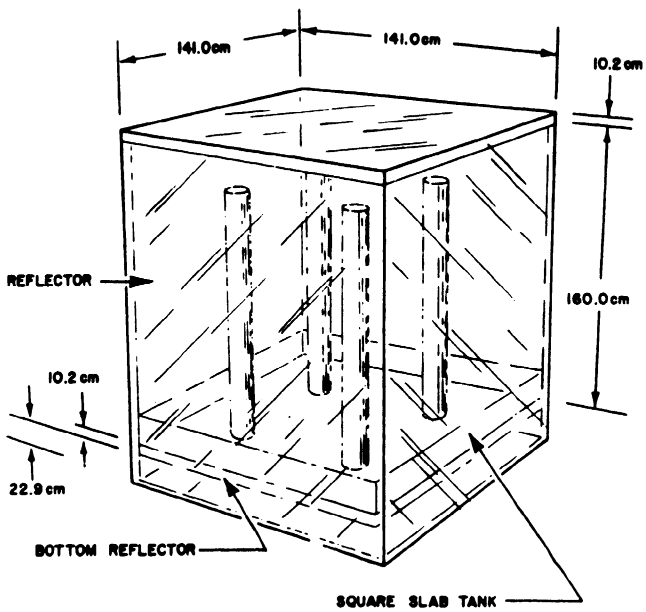

Fig. 1. Typical reflected experimental configuration.

TABLE I

Cylinder Dimensions
Type-316 Stainless Steel

Inside Diameter (cm)	Bottom and Wall Thickness (cm)	Length (cm)
11.0	0.198	131
13.6	0.280	131
16.3	0.280	131
21.3	0.280	131
22.4	0.308	274
22.9	0.308	274
23.4	0.308	183
23.9	0.308	183

the outer cylinder centerlines and the slab-tank edge. These half-spacings were 60.3, 30.2, 20.1, and 15.1 cm for 1, 4, 9, and 16 cylinders, respectively.

The Type-316 stainless-steel slab tank was 120.7 cm square by 20.3 cm high inside with 0.15-cm-thick walls, and a 0.635-cm-thick bottom. This tank was supported by six mild steel 15-cm-diam pipes, 32 cm high, with a 0.6-cm-thick wall. These pipes rested on a mild steel table which was 152 cm square, 1.9 cm thick, and 137 cm above a concrete floor. The nearest concrete wall was > 3 m away. The Tygon[b] hoses, through which the cylinders were filled and drained, were submerged in the solution in the slab tank.

[b]Trademark of U.S. Stoneware Company, New York.

The properties of the uranyl-nitrate [$UO_2(NO_3)_2$] solution are reported in Table II. The uranium was enriched to 93.2 wt% ^{235}U. Since the solution properties were modified by the evaporation of some water, the solution concentrations are given for each measurement. The effects of the change in the solution properties were monitored in terms of critical solution slab thickness and are reported in Table III.

Two configurations were reflected with Plexiglas [methyl methacrylate, CH_2:C(CH_3)COO(CH_3)] as shown in Fig. 1. The average distance between the inside surfaces of the reflector and slab-tank sides and bottom was 0.3 cm.

TABLE II

Fissile Solution Properties
[$UO_2(NO_3)_2$]

	Test Beginning	End of Unreflected Measurements	Test End
Concentration (g U/liter)	466	499	520
Density (g/cm^3)	1.636	1.685	1.704
Normality	0.59	0.70	0.77
H/^{235}U atomic ratio[a]	50	46	43

[a]Derived from laboratory analyses.

UNCERTAINTY OF MEASUREMENTS

The uncertainty[c] of the solution slab thickness was based on considerations of small deviations of the slab-tank bottom from a horizontal plane, the solution level instrumentation, and measurements of the accuracy of extrapolation of reciprocal multiplication data to the critical slab thickness. The uncertainty of the solution height in the array of cylinders was based on solution-level instrumentation and on observation of the high-solution mark in the cylinders. These experimental uncertainties are: ±0.3 cm of slab thickness in the 11.0-cm-diam cylinders and ±0.2 cm of slab thickness in all other diameters. The uncertainties on array solution height are: ±3.0 cm for the 11.0-, 13.6-, 16.3-, and 21.3-cm-diam cylinders, excepting the single 21.3-cm-diam cylinder, which was ±5.0 cm; and ±2.0 cm for the single 22.4-, 22.9-, 23.4-, and 23.9-cm cylinder diameters.

[c]The quoted uncertainties are for the two sigma confidence levels.

TABLE III

Critical Slab Thicknesses

Critical Slab[a] Thickness (cm)	Estimated[b] Accuracy (cm)	Uranium Concentrations (g/liter)	Condition of Slab
Slab With Minimum Reflector			
12.7	±0.3	465	Clean slab tank only
12.7	±0.3	465	Clean slab tank only
12.7	±0.3	465	Slab tank plus 16 dummy cylinders and hoses
12.6	±0.3	465	Clean slab tank only
12.7	±0.3	465	Slab tank plus 0.5 cm of precipitate
12.8	±0.2	495	Slab tank plus 1 cm of precipitate
13.0	±0.2	500	Clean slab tank only
13.0	±0.2	510	Clean slab tank only
13.0	±0.2	520	Slab tank including 1 cm of precipitate.
Reflected Slab[c]			
10.3	±0.2	505	Clean slab tank only

[a]Includes any precipitate present in the slab tank.
[b]Repeatability of measurements was ±0.1 cm.
[c]Reflected slab tank without cylinders, as shown in Fig. 1.

DISCUSSION OF MEASUREMENTS OF SLAB-CYLINDER SYSTEMS

Perturbing Effect of Equipment

As shown by the data in Table III, the perturbing effects of the precipitate in the slab-tank bottom and of immersing the lower portion of the cylinders and the fill and drain hoses in the solution slab, was too small to be measured. The change of the solution properties, because of water evaporation and precipitation from the solution, was monitored by critical slab-thickness measurements. An average of two earlier critical slab thicknesses was 12.7 cm and the average of three later critical slab thicknesses was 13.0 cm. This indicates that the changing solution properties affected the critical slab thickness by 0.3 cm.

DTF calculations[2] were performed to show how well the 120.7-cm-square slab approximates an infinite slab. Comparing the calculated thickness, 14.34 cm, of an infinite unreflected critical slab to the calculated thickness, 14.66 cm, of a 120.7-cm-square unreflected critical slab indicates that the thickness of the experimental slab is 98% of the thickness of an infinite slab.

[2]B. G. CARLSON, W. J. WORLTON, W. GUBER, and M. SHAPIRO, *DTF Users Manual*, UNC Physics/Math 3321, United Nuclear Corporation, White Plains, New York, Vol. I (November 1963) and Vol. II (May 1964).

Minimum Reflector Configuration

Minimum reflector[d] measurements were made of the critical slab thickness as a function of the solution height in the cylinders of the array for (a) slab plus 1-, 4-, 9-, or 16-cylinder arrays with cylinder diameters of 11.0, 16.3, or 21.3 cm; (b) slab plus sixteen 13.6-cm-diam cylinders in the array; (c) slab plus a single cylinder of 23.4- or 23.9-cm diameter; (d) slab plus a single cylinder of 22.4-, or 22.9-cm diameter and effectively infinite solution height. The experimental data, with the corresponding uranyl-nitrate concentrations, are reported in Table IV.

The 100-cm-high, 11.0-cm-diam cylinder arrays had little effect on the critical slab thickness. For example, one cylinder changed the critical slab thickness by $< 2\%$ and 16 cylinders changed the critical slab thickness by only 10%.

Array Spaced Above Slab

Critical slab thicknesses were measured as a function of solution height in a sixteen, 16.3-cm-diam cylinder array when the array bottom was suspended 14.1 cm ± 0.5 and 28.2 cm ± 0.5 above the slab-tank bottom. Because of these fixed spacings, the distance from the top of the solution

[d]"Minimum Reflector" refers to the system with only the unavoidable reflection of the experimental fixtures and floors and walls of the experimental area.

TABLE IV

Experimental Data for Critical Slab-Array Configuration With Minimum Reflector

Cylinder Diameter (cm)	Array Configuration Number of Cylinders	Slab Thickness (cm)	Array[a] Height (cm)	Uranium Concentration (g/liter)
11.0	1	12.6	87	465
		12.6	37	
	4	12.3	88	465
		12.4	5	
		12.5	0	
	9	12.1	88	465
		12.2	39	
		12.3	18	
		12.6	2	
	16	11.4	88	465
		11.7	38	
		11.9	19	
		12.5	2	
		12.7	0	
13.6	16	10.0	108	520
		10.4	69	
		11.3	26	
		12.3	4	
16.3	1	12.2	88	470
		12.2	40	
		12.3	19	
		12.3	14	
		12.5	6	
	4	12.0	88	480
		12.2	25	
		12.3	12	
		12.6	1	
	9	10.5	90	485
		11.0	40	
		11.8	14	
		12.4	3	
		12.7	0	
	16	0	78	495
		2.6	64	
		5.5	50	
		7.7	37	
		10.0	20	
		11.4	8	
		12.4	3	
		12.6	0	
21.3	1	11.1	88	500
		11.1	39	
		12.4	3	
	4	8.8	91	500
		10.1	39	
		11.0	19	
		11.8	10	
		12.7	2	
	9	0	47	500
		1.5	43	
		5.9	32	
		8.6	23	
		10.0	15	
		11.6	7	
		12.7	1	
	16	0	26	500
		2.7	22	
		7.5	14	
		9.1	11	
		9.5	9	
		11.8	3	
22.4	1	10.8	108	505
22.9	1	10.1	110	505
23.4	1	8.9	111	525
		10.3	33	
		11.4	15	
23.9	1	0	112	525
		7.7	66	
		9.7	34	
		11.9	8	

[a]Solution height in cylinders of array, measured from top of solution slab.

slab to the bottom of the array varied with the solution-slab thickness.

The experimental measurements are reported in Table V. The uncertainties in the data are ±0.2 cm in the slab thickness and ±3 cm in the solution height in the array except for the one measurement at 0.3-cm solution height in the array where the uncertainty is ±0.3 cm.

The data shown in Fig. 2 were derived from the experimental data to show the effect of constant spacing between the solution-slab top and the array bottom. The uncertainties in Fig. 2 are ±0.2, ±0.4, ±0.5, and ±0.7 cm of slab thickness and ±3, ±4, ±5, and ±7 cm of solution height in the array for the 0-, 5-, 15-, and 25-cm spacings, respectively. These uncertainties are different from the experimental-data uncertainties because

TABLE V

Slab-Array* Critical Configuration with Array Suspended above the Slab

Spacing Between Top of Solution Slab to Bottom of Array (cm)	Slab Thickness (cm)	Solution Height[a] in Cylinders (cm)
9.4	4.7	66
6.9	7.3	49
3.6	10.5	19
2.7	11.4	8
20.4	7.8	67
18.2	10.0	49
16.7	11.4	19
16.4	11.7	10
28.2	12.4	0.3

*Sixteen 16.3-cm-diam cylinders in the minimum reflector array at 485 g U/liter.

[a]This height is measured from the bottom of the cylinders of the array.

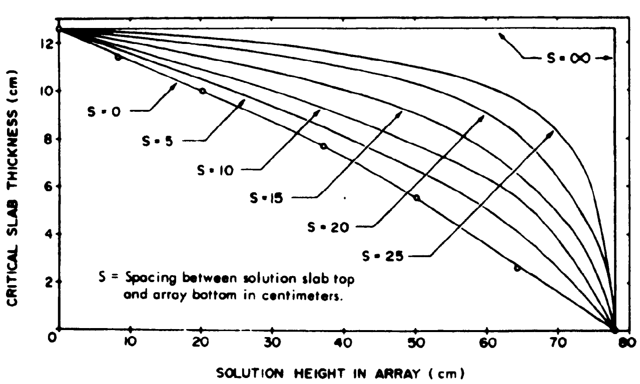

Fig. 2. Constant spacing data for sixteen 16.3-cm-diam cylinders in the array with minimum reflector.

of the interpolation required to provide the constant spacing values.

As seen in Fig. 2, when most of the system reactivity is in the slab, the critical slab thickness is little affected by the spacing between the array and the slab, and vice versa. At a 40-cm solution height in the array, each successive 5-cm spacing out to a 25-cm spacing produces nearly the same change in the critical slab thickness. At the 25-cm spacing, the critical slab thickness is 90% of the critical slab thickness at an infinite spacing. Therefore, the effect of each 5-cm-spacing increment must decrease rapidly with the next few increments.

The critical slab thickness, 12.4 cm, of the zero array solution height is lower than the minimum-reflector critical slab thickness, 12.8 cm, because of the reflection of the suspended hardware which included ~0.3 cm of undrained solution in the array. Comparing these two slab thicknesses provides an upper limit of 0.4 cm on slab thickness for the reflector saving of the suspended array hardware.

Reflected Measurements

Three series of reflected measurements were performed with 13.6-cm-diam cylinders in the array. The first series comprised 9 and 16 cylinder arrays with a constant reflector thickness of 10.2 cm on all six sides. The second series was a 16 cylinder array with a constant 10.2-cm-thick bottom reflector and varied side and top reflector thicknesses. The last series was a 16 cylinder array with the top and sides unreflected and varied bottom reflector thicknesses. The data for all three series are reported in Table VI.

The reflector saving was obtained by comparison of the minimum reflected, sixteen 16.3-cm-diam cylinder array data to the fully reflected, sixteen 13.6-cm-diam cylinder array data. For this experimental configuration, the reflector saving was an ~2.4-cm decrease in each cylinder diameter of the array when the array was nearly equilateral.

Comparing the 10.2-cm-thick reflected critical slab thickness to the minimum-reflected critical slab thickness yields a reflector saving of 2.5 cm on critical slab thickness for the complete reflector. The slab reflector saving due to the 10.2-cm-thick bottom reflector can be obtained by noting that at corresponding array solution heights, there is a constant 1.7-cm difference between the critical slab thickness for the bottom reflector only and the minimum reflector data. This 1.7 cm is the reflector saving for the bottom reflector. The reflector saving due to the sides and top is then the total reflector saving minus the bottom-reflector-only saving, or 0.8 cm.

Pass-Through Measurements

The critical solution height of an array of sixteen 13.6-cm-diam cylinders reflected by the 10.2-cm-thick reflector was determined with and without two 2-liter bottles of uranyl-nitrate solution near the array center, as indicated in Fig. 3. The two polyethylene bottles were 11.4-cm o.d., 0.08-cm wall thickness, 22.9 cm tall, and each contained 2 liters of solution. The bottom of the bottles was positioned 37 cm above the bottom of the array.

The critical solution height of the array was 88 cm ± 1 with the solution bottles and 92 cm ± 1 without the bottles. The 4-cm decrease in solution height is equivalent to removing 9.3 liters of solution from the top of the array and inserting 4 liters of solution near the array center.

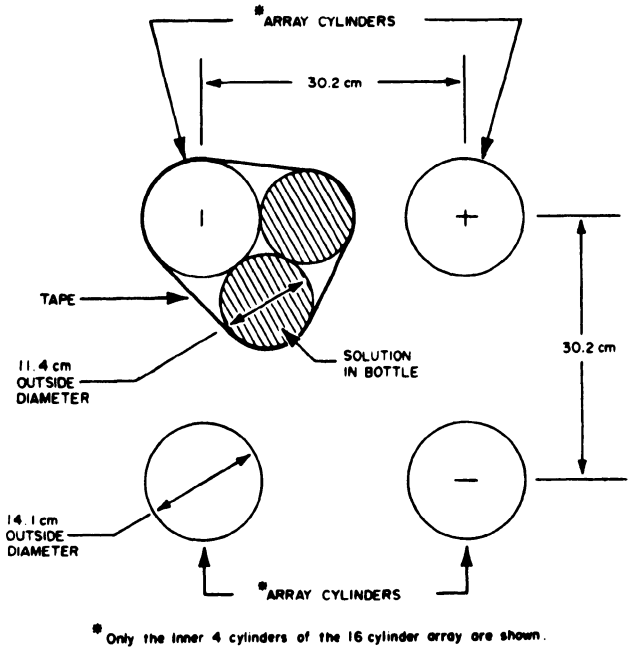

Fig. 3. Placement of solution bottles in the array.

ANALYSIS

KENO Calculations

Calculations were performed on some typical slab-cylinder configurations using the KENO[3] code. The Rocky Flats version of the code uses the 16-group Hansen-Roach[4] cross sections and

[3]G. E. WHITESIDES and N. F. CROSS, *KENO, A Multigroup Monte Carlo Criticality Program*, CTC5, Oak Ridge Computer Technology Center (1969).

[4]G. E. HANSEN and W. H. ROACH, *Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies*, LAMS-2543, Los Alamos Scientific Laboratory (1960).

TABLE VI

Reflected Slab-Array Critical Configuration Data
(The array consisted of 13.6-cm-diam cylinders.)

Number of Cylinders	Top and Side Reflector Thickness (cm)	Slab Tank Bottom Reflector Thickness (cm)	Slab Thickness (cm)	Array[a] Height (cm)	Uranium Concentration (g/liter)
9	10.2	10.2	7.1 ± 0.2	112 ± 1	505
			8.8 ± 0.2	42 ± 1	
			9.5 ± 0.2	20 ± 1	
			10.3 ± 0.2[b]	0 ± 0	
16	10.2	10.2	0 ± 0.2	92 ± 1	505
			3.8 ± 0.2	61 ± 4	
			8.1 ± 0.2	26 ± 1	
			10.3 ± 0.2[b]	0 ± 0	
16	7.6	10.2	0 ± 0.5	96 ± 1	510
			4.7 ± 0.2	59 ± 2	
			7.9 ± 0.2	26 ± 2	
			10.5 ± 0.2	0 ± 2	
16	5.1	10.2	0 ± 0.5	105 ± 1	510
			3.4 ± 0.2	75 ± 1	
			7.4 ± 0.2	36 ± 1	
			10.6 ± 0.2	2 ± 1	
16	2.5	10.2	0 ± 0.5	149 [c]	515
			3.3 ± 0.2	114 ± 1	
			6.0 ± 0.2	72 ± 1	
			8.6 ± 0.2	30 ± 1	
			9.8 ± 0.2	9 ± 1	
16	0	10.2	8.3 ± 0.2	110.0 ± 0.6	515
			8.7 ± 0.2	67.5 ± 0.6	
			9.6 ± 0.2	27.5 ± 0.6	
16	0	5.1	8.7 ± 0.2	109.0 ± 0.5	515
			9.0 ± 0.2	69.5 ± 0.5	
			9.9 ± 0.2	26.0 ± 0.5	
			11.0 ± 0.2	7.5 ± 0.5	
16	0	0	10.0 ± 0.2	108 ± 1	520
			10.4 ± 0.2	69 ± 1	
			11.3 ± 0.2	26 ± 1	
			12.3 ± 0.2	4 ± 1	

[a] Array height measured from top surface of solution in slab.
[b] These measurements were made with the slab tank alone.
[c] Extrapolated from a solution height of 120 cm.

TABLE VII

Calculated $\overline{k}_{eff}$ for Critical Experimental Assemblies

Number of Cylinders	Uranium Content (g/liter)	Slab Thickness (cm)	Cylinder Solution Height (cm)	Cylinder Radius (cm)	$\overline{k}_{eff} \pm \sigma$ Reflection from Table Only	$\overline{k}_{eff} \pm \sigma$ Reflection from Room and Table
0	485	13.0	---	---	0.979 ± 0.009	0.981 ± 0.013 (15)[a]
1	520	8.9	111.0	11.68	0.975 ± 0.010	
4	500	8.8	91.0	10.67	0.977 ± 0.010	
9	500	8.6	22.6	10.67	0.958 ± 0.012	
16	520	10.0	108.0	6.79	0.961 ± 0.010 (23)[a]	0.969 ± 0.010 (21)[a]
16	495	0.0	78.0	8.13	0.986 ± 0.009 (26)[a]	

[a] Value in parentheses is the number of batches averaged. If no value is given, 30 batches were used.

TABLE VIII

Constants in Eq. (2) for Critical Arrays with Minimum Reflector

Number of Cylinders	C_{TD}[a] (cm^2)	Diameter (D_0)[b] (cm)	Height (H)[c] (cm)
1	5.36	24.7	78
		25.1	48
		26.1	26
4	6.38	23.0	78
		23.5	48
		24.0	26
9	8.18	20.3	78
		21.3[d]	48
		22.9	26
16	10.56	16.3[d]	78
		17.4	48
		21.3[d]	26

[a] C_{TD} = hyperbola curvature

[b] D_0 = diameter of each cylinder in the critical array

[c] H = solution in the array

[d] These are experimental data points. The other values were obtained from Eq. (2).

treats all scatterers except hydrogen as isotropic. In terms of the KENO geometry routine, the slab-cylinder geometry was treated as an array of cylinders reflected by a slab of fissile solution. Also included in the geometry description were the tank bottom and the table above which the tank was supported.

The results of the calculations are summarized in Table VII. In all cases, the first five batches were discarded to eliminate source distribution effects. The calculated values are typical of a uranyl-nitrate solution system with a stainless-steel tank, steel table, and the 16-group Hansen-Roach cross sections.[5]

Empirical Data Analysis

The data, reported in Table IV, may be empirically fitted to a mathematical equation for easy and accurate interpolation. The equilateral hyperbola was chosen because the equation is simple, fits the data, has two interpretable asymptotes, and has been used previously.[6,7]

[5]G. E. WHITESIDES, Personal Communication (January 1970).

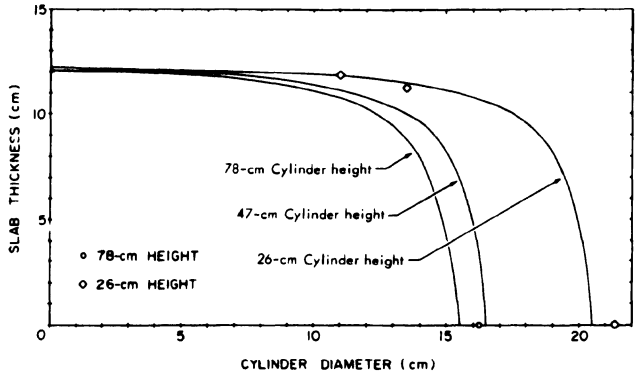

Fig. 4. Solution slab thickness vs cylinder diameter from hyperbolic fit for 16 cylinders in the array at various array solution heights.

Using the experimental values of critical slab thickness, T, and diameter of the cylinders in the array, D, as the variables, the experimental data were fit to the equation of the equilateral hyperbola for array solution heights of 78, 48, and 26 cm. The orthogonal asymptotes of the hyperbola are the critical slab thickness, 12.8 cm, with no array and the diameter, D_0, of the cylinders when the array alone is critical. The equation is:

$$(T - 12.8)(D - D_0) = C_{TD} \quad . \qquad (1)$$

The hyperbola curvature, C_{TD}, and the diameter, D_0, of each cylinder in the critical array are reported in Table VIII at various solution heights, H, in the array. Using these values, Eq. (1), which fits all experimental data to within ±5%, is plotted in Fig. 4 for a typical case.

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[6]C. L. SCHUSKE and J. W. MORFITT, *An Empirical Study of Some Critical Mass Data*, Y-533, Oak Ridge, Y-12 Plant (1949).

[7]C. L. SCHUSKE, B. B. ERNST, and H. W. KING, *Empirical Analysis of Critical Bare Arrays of Cylinders Containing Enriched $UO_2(NO_3)_2$*, RFP-315, The Dow Chemical Company, Rocky Flats Division (1963).