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Criticality of Slightly Enriched Uranium in Water-Moderated Lattices*

C. R. Richey, R. C. Lloyd and E. D. Clayton

General Electric Company, Hanford Atomic Products Operation, Richland, Washington

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Exponential and approach-to-critical experiments have been conducted with slightly enriched uranium in light water. The uranium was fabricated into elements of 0.175 to 1.66 in. diam with enrichments of 1.007 to 3.063 wt% U^{235}. Exponential measurements have also been made with natural uranium rods (0.925 in. diam) in light water. Analytical methods were used to correlate the experimental results and extend the data to include uranium rods containing 4.0 and 5.0 wt% U^{235}.

INTRODUCTION

With the development of commercial electrical-power producing reactors, increased quantities and varieties of reactor fuel with enrichments up to approximately 5 wt% U^{235} are being handled throughout the nuclear industry. The need for criticality data to establish procedures for safely handling these materials outside of reactor environments has, therefore, steadily increased. A large number of experiments has been conducted with slightly enriched uranium in water on which the criteria for safely handling low-enrichment reactor fuels are based. Many of the early exponential experiments with slightly enriched uranium rods were performed by Kouts, *et al*[1,2,3]. Kouts' initial measurements were with 0.750 in.-diam uranium rods containing 1.027 wt% U^{235}; subsequent experiments were conducted with 0.387 and 0.600 in.-diam rods at 1.027, 1.143, and 1.299 wt% U^{235} enrichment. Clayton (1955), utilizing rods of 0.925 and 1.66-in. diam, conducted a series of exponential experiments with 1.007 wt% U^{235} enriched uranium[4]. The safety criteria for handling reactor fuel enriched to approximately 1 wt% U^{235} are based on these two sets of data (Fig. 1). Rogers and Kinard have recently conducted exponential pile measurements utilizing 3.0 wt%-enriched uranium rods of 2.0- and 3.0-in. diam[5].

Since Clayton's initial experiments at the Hanford Laboratories with 1.007 wt% U^{235}-enriched rods, experiments have been conducted with water-moderated arrays consisting of 1) tubular fuel elements with enrichments ranging from 1.0 to 1.6 wt% U^{235} (1956-58), 2) uranium rods having 3.063 wt% U^{235} (1958-59) and 2.0 wt% U^{235} (1960), and 3) exponential experiments with 0.925 in.-diam rods to test the possibility of attaining criticality with natural uranium, water-moderated lattices. Although the results of these latter experiments are already familiar to many involved in criticality safety and have found application in establishing safety criteria of slightly enriched uranium[6],

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[1] H. KOUTS, G. PRICE, K. DOWNES, R. SHER and V. WALSH, "Exponential Experiments with Slightly Enriched Uranium Rods in Ordinary Water," *Proc. Intern. Conf. Peaceful Uses of Atomic Energy*, Vol. **5**, (1955).

[2] H. KOUTS and R. SHER, "Experimental Studies of Slightly Enriched Uranium, Water Moderated Lattices, Part I, 0.600 in. Diameter Rods," BNL-486, (September 1957).

[3] H. KOUTS, R. SHER, J. R. BROWN, D. KLEIN, S. STAIN, R. L. HELLENS, H. ARNOLD, R. M. BALL and P. W. DAVISON, "Physics of Slightly Enriched, Normal Water Lattices (Theory and Experiment)," *Proc. Second U. N. Intern. Conf. Peaceful Uses of Atomic Energy*, Vol. **12**, P1841 (1958).

[4] E. D. CLAYTON, "Exponential Pile Measurements in Water Moderated Lattices with Enriched Uranium Rods," HW-40930, (January 1956).

[5] W. B. ROGERS, Jr. and F. E. KINARD, "Material Buckling and Critical Mass of Uranium Rods Containing 3 wt% U^{235} in H_2O," *Nucl. Sci. Eng.*, **20**, 266 (1964).

[6] H. C. PAXTON, "Critical Data for Nuclear Safety Guidance," LAMS-2415, (February 1960); H. C. PAXTON *et al.*, "Critical Dimensions of Systems Containing U^{235}, Pu^{239} and U^{233}." TID-7028, (June, 1964).

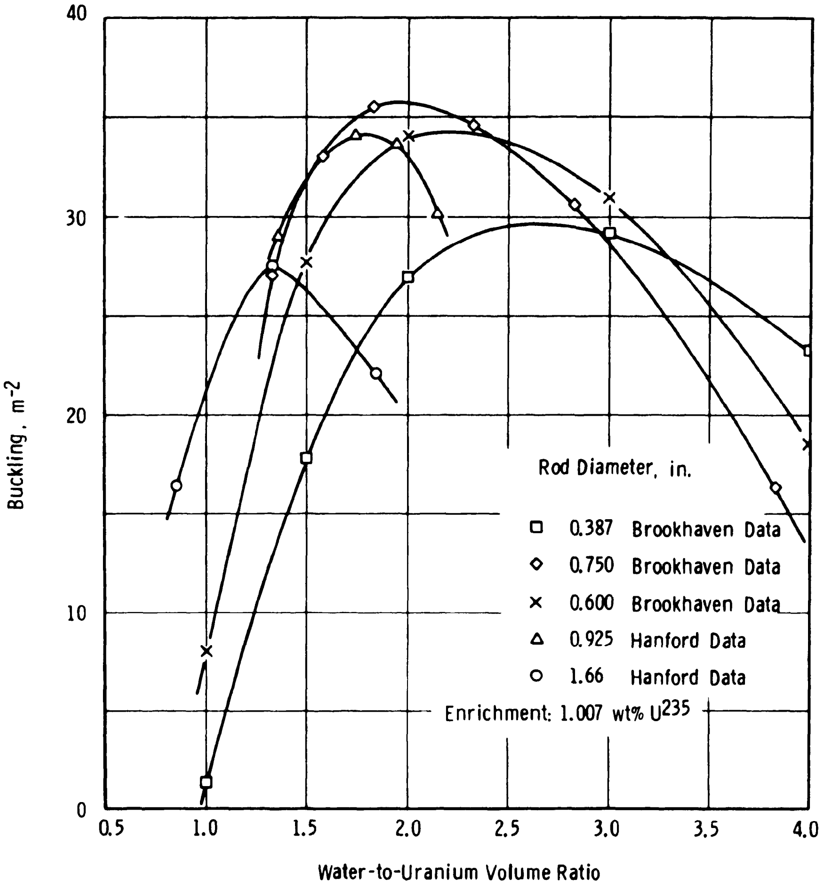

Fig. 1. Bucklings for 1.007 wt% U^{235}-enriched rods.

they have not previously appeared in the formal literature. (Previous dissemination of the data has been limited to regular Hanford progress reports and memoranda of limited distribution[7].) This paper presents a summary of these experiments. The results are combined with the earlier data of Kouts and Clayton to obtain maximum bucklings and minimum critical masses for uranium rods enriched to approximately 3 wt% U^{235}. Analytical methods used to correlate the experimental data are applied in extending the results to uranium containing 5 wt% U^{235}.

SLIGHTLY ENRICHED URANIUM RODS

Measurements

Both exponential pile and neutron multiplication (critical approach) measurements were conducted to obtain critical buckling and critical mass data for 2.0 and 3.063 wt% U^{235}-enriched uranium rods moderated with water. Experiments with the 3.063 wt% U^{235}-enriched uranium utilized four rod diameters: 0.175, 0.300, 0.600 and 0.925 in. The 2.0 wt% U^{235}-enriched uranium measurements utilized rods of two diameters: 0.600 and 0.925 in. These experiments were conducted in a cylindrical water tank 4 ft in diam and 5 ft high. Each fuel rod was positioned in a Plexiglas tube and charged into the water-filled experimental vessel in the desired array. Sufficient water surrounded each lattice array to provide an effectively infinite water reflector. The reflector was maintained on the top and bottom surfaces by loading solid Plexiglas rods above and below each fuel rod.

Subcritical source-neutron multiplication measurements were conducted to determine the critical number of fuel rods in each experimental array; i.e. the critical mass in cylindrical geometry. The neutron source consisted of a single 0.25 g Ra-Be pencil located near the center of the fuel loading. Three BF_3 proportional neutron counters with their corresponding scaling circuits provided measurements of the apparent source-neutron multiplication as stepwise fuel additions were made. The critical number of fuel rods was determined from a least squares analysis of the

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[7] R. C. LLOYD, "Summary Listing of Subcritical Measurements of Heterogeneous Water-Uranium Lattices made at Hanford," HW-65552, (June, 1960).

reciprocal multiplication data between 85 and 96% of criticality.

Since the critical mass is determined directly, it is not dependent upon knowing the extrapolation length, λ, as is the critical buckling, i.e.

$$B^2 = \left(\frac{2.4048}{R_c+\lambda}\right)^2 + \left(\frac{\pi}{H_c+2\lambda}\right)^2 . \qquad (1)$$

where R_c and H_c denote the dimensions of the critical core. Each lattice core was idealized as a cylinder having as its radius $R_c = b\sqrt{N_c}$ with N_c being the critical number of fuel rods and πb^2 the area of a single lattice cell.

To evaluate λ and hence the critical buckling, exponential pile measurements were conducted for each experimental array. Four 0.5 g Ra-Be neutron sources placed below the fuel core provided the necessary neutron flux. The neutron monitor utilized in the axial traverse was a BF_3 proportional counter. Each monitored position was so selected that corrections for deviation from the fundamental mode were less than 10%. The relaxation length of the axial flux, L_1, is related to the critical buckling by

$$B^2 = \left(\frac{2.4048}{R+\lambda}\right)^2 + \frac{1}{L_1^2} . \qquad (2)$$

Equating Eqs. (1) and (2) gives the critical buckling and extrapolation length.

The extrapolation lengths for the 0.175, 0.300, and 0.600 in.-diam, 3.063 wt% U^{235}-enriched uranium rods are recorded in Fig. 2. Two core heights were used in each measurement with the latter two rod sizes, λ being the average of the two resulting values. Large uncertainties in the effective core radii (3.063 wt% U^{235}-enriched uranium) prohibited any useful results from being obtained from the exponential pile measurements with the 0.925 in.-diam rod lattices. This was due to the small number of fuel rods making up these particular fuel cores. It was therefore necessary to obtain λ for the 0.925 in.-diam rod lattices by extrapolating the results for the three smaller rods. The criticality parameters for the 3.063 wt% U^{235}-enriched uranium rods are given in Table I and the critical bucklings are plotted in Fig. 3. The minimum critical mass and the maximum buckling are plotted as a function of rod diameter in Fig. 4. For uranium rods having 3.063 wt% U^{235}, the maximum obtainable buckling was concluded to be about 156 m^{-2} (occurring at a rod diam of 0.4 in.). When the critical mass curve was extrapolated by theoretical methods (described later in this paper), a minimum critical mass of about 76.2 kg U (2.33 kg of U^{235}) was obtained.

The experimental data for the 2.0 wt% U^{235}-enriched rods are summarized in Table II and the critical bucklings recorded in Fig. 5. The maximum bucklings for the 0.925 and 0.600 in.-diam rods are about 110 m^{-2} and 112 m^{-2}, respectively,

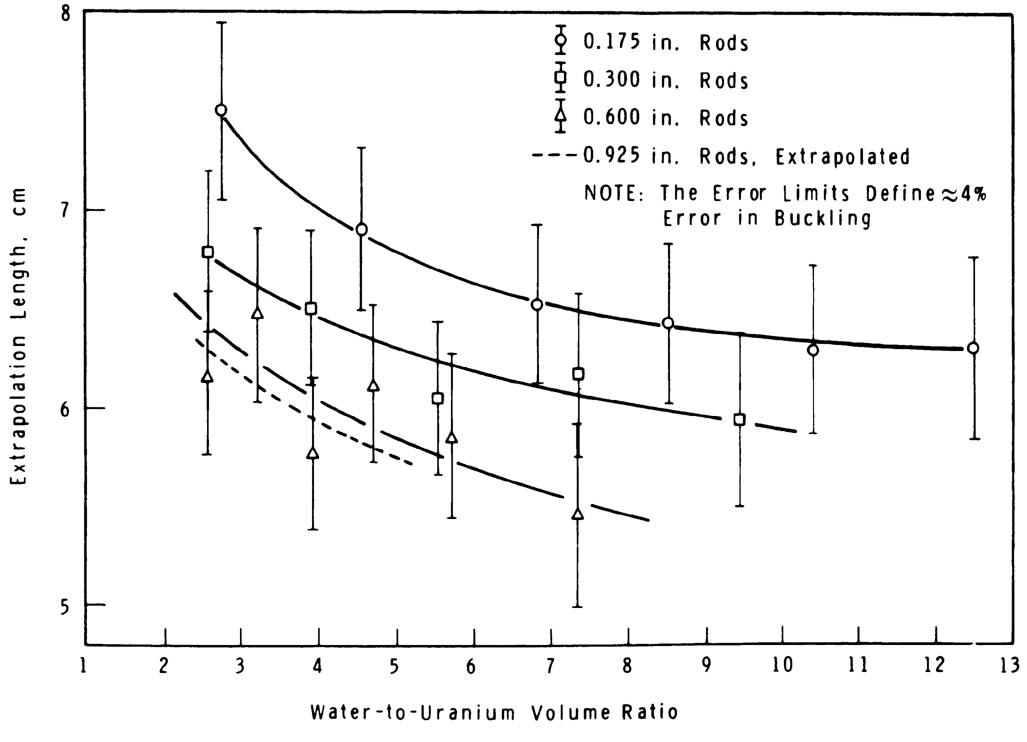

Fig. 2. Extrapolation lengths for 3.063 wt% U^{235}-enriched rods.

TABLE I

Summary of 3.063 wt% U^{235}-Enriched Uranium Rod Measurements

H_2O:U, Vol	H_c, cm	Critical No. Rods	R_c, cm	$\bar{\lambda}$, cm	B^2, m^{-2}	Crit Mass Cyl, kg of U^{235}	Computed Crit Mass Sphere, kg of U^{235}
0.175 in.-Diam Rods							
2.24	59.69	1786.0 ± 6.3	16.90 ± 0.03	7.50	114.77 ± 0.24	9.57 ± 0.03	7.78 ± 0.02
4.06	59.69	873.3 ± 1.5	14.78 ± 0.01	6.91	141.21 ± 0.11	4.68 ± 0.01	3.57 ± 0.01
6.29	59.69	628.3 ± 1.1	15.04 ± 0.01	6.53	142.92 ± 0.11	3.37 ± 0.01	2.56 ± 0.01
8.00	59.69	569.7 ± 1.0	15.92 ± 0.02	6.43	134.58 ± 0.21	3.05 ± 0.01	2.38 ± 0.01
9.89	59.69	554.2 ± 2.2	17.27 ± 0.04	6.30	123.01 ± 0.36	2.97 ± 0.01	2.38 ± 0.01
11.96	59.69	572.5 ± 1.0	19.15 ± 0.02	6.31	108.09 ± 0.14	3.07 ± 0.01	2.56 ± 0.01
0.300 in.-Diam Rods							
2.06	81.28	624.0 ± 0.8	16.70 ± 0.02	6.80	136.07 ± 0.18	13.39 ± 0.02	6.46 ± 0.01
3.41	40.64	387.5 ± 0.9	15.75 ± 0.02	6.51	150.98 ± 0.21	4.16 ± 0.01	3.81 ± 0.01
3.41	81.28	298.1 ± 1.7	13.82 ± 0.04			6.40 ± 0.04	
5.00	40.64	296.9 ± 0.9	16.09 ± 0.03	6.05	153.52 ± 0.32	3.19 ± 0.01	2.90 ± 0.01
5.00	81.28	230.0 ± 0.7	14.16 ± 0.03			4.94 ± 0.02	
6.84	40.64	271.9 ± 0.8	17.59 ± 0.03	6.17	137.58 ± 0.26	2.92 ± 0.01	2.71 ± 0.01
6.84	81.28	199.9 ± 1.1	15.09 ± 0.05			4.29 ± 0.02	
8.92	40.64	285.7 ± 0.4	20.29 ± 0.03	5.94	119.86 ± 0.20	3.07 ± 0.01	2.88 ± 0.01
8.92	81.28	203.8 ± 0.8	17.14 ± 0.04			4.37 ± 0.02	
0.600 in.-Diam Rods							
2.06	40.64	152.1 ± 0.5	16.45 ± 0.03	6.16	148.34 ± 0.30	6.55 ± 0.02	6.01 ± 0.02
2.06	81.28	114.9 ± 0.8	14.30 ± 0.06			9.89 ± 0.07	
2.71	40.64	118.6 ± 0.1	15.97 ± 0.02	6.52	148.55 ± 0.20	5.10 ± 0.01	4.68 ± 0.01
2.71	81.28	91.2 ± 0.6	14.01 ± 0.04			7.85 ± 0.05	
3.41	40.64	104.5 ± 0.3	16.36 ± 0.04	5.77	154.35 ± 0.42	4.50 ± 0.01	4.10 ± 0.02
3.41	81.28	79.9 ± 0.3	14.30 ± 0.04			6.88 ± 0.02	
4.18	40.64	98.9 ± 0.2	17.24 ± 0.04	6.13	141.15 ± 0.37	4.26 ± 0.01	3.93 ± 0.02
4.18	81.28	74.1 ± 0.2	14.92 ± 0.04			6.37 ± 0.02	
5.18	40.64	100.3 ± 0.3	18.97 ± 0.05	5.86	129.84 ± 0.38	4.32 ± 0.01	4.03 ± 0.02
5.18	81.28	73.8 ± 0.2	16.27 ± 0.04			6.35 ± 0.02	
6.84	40.64	122.0 ± 0.4	23.57 ± 0.06	5.46	105.76 ± 0.28	5.25 ± 0.02	4.90 ± 0.02
6.84	81.28	82.4 ± 0.2	19.37 ± 0.04			7.09 ± 0.02	
0.925 in.-Diam Rods[a]							
1.89	40.64	72.1 ± 0.1	16.98 ± 0.03	6.37	140.68 ± 0.27	7.37 ± 0.01	6.90 ± 0.02
1.89	60.96	60.0 ± 0.3	15.50 ± 0.06			9.21 ± 0.05	
2.29	40.64	63.5 ± 0.1	17.01 ± 0.02	6.23	142.12 ± 0.17	6.50 ± 0.01	6.07 ± 0.01
2.29	60.96	52.4 ± 0.2	15.45 ± 0.05			8.04 ± 0.03	
2.72	40.64	58.6 ± 0.5	17.35 ± 0.09	6.12	140.27 ± 0.80	5.99 ± 0.05	5.60 ± 0.05
2.72	60.96	49.2 ± 0.1	15.90 ± 0.02			7.54 ± 0.02	
3.17	40.64	56.7 ± 0.1	18.08 ± 0.04	6.00	135.36 ± 0.33	5.80 ± 0.01	5.45 ± 0.02
3.17	60.96	46.9 ± 0.3	16.44 ± 0.07			7.20 ± 0.05	
3.89	40.64	60.4 ± 0.4	20.21 ± 0.08	5.86	121.08 ± 0.52	6.18 ± 0.04	5.85 ± 0.04
3.89	60.96	48.6 ± 0.2	18.13 ± 0.05			7.46 ± 0.03	

[a] $\bar{\lambda}$ for the 0.925 in.-Diam rods was extrapolated from a plot of $\bar{\lambda}$ as a function of rod diameter.

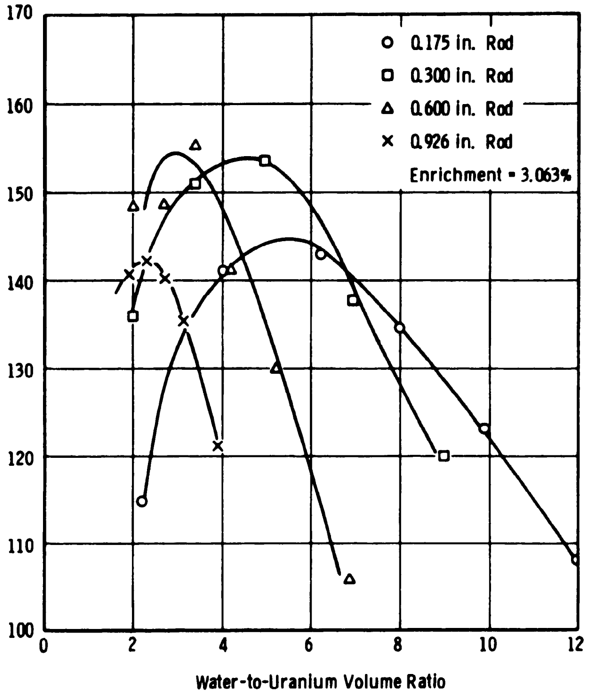

Fig. 3. Buckling for 3.063 wt% U^{235}-enriched uranium rods.

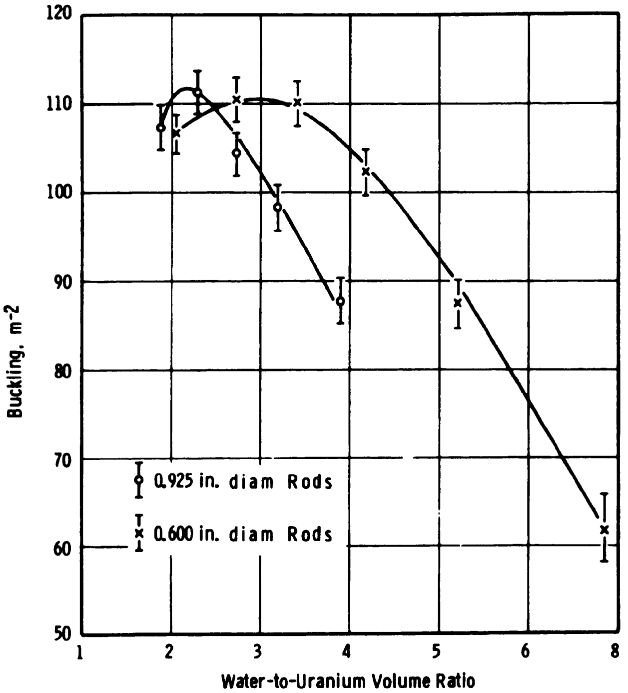

Fig. 5. Buckling for 2.00 wt% U^{235} uranium rods in water.

at H_2O-to-U volume ratios of 2.2 and 2.9. The minimum critical mass for the 0.925-in. rods is 313 kg U and 231 kg U for the 0.600-in. rods; these minimum critical masses occur at H_2O-to-U volume ratios of 2.8 and 3.8, respectively.

Error limits quoted for the critical bucklings are standard errors associated with the determination of the critical radius. In addition, the error in the extrapolation length would contribute an uncertainty of 3 to 5% in the critical buckling.

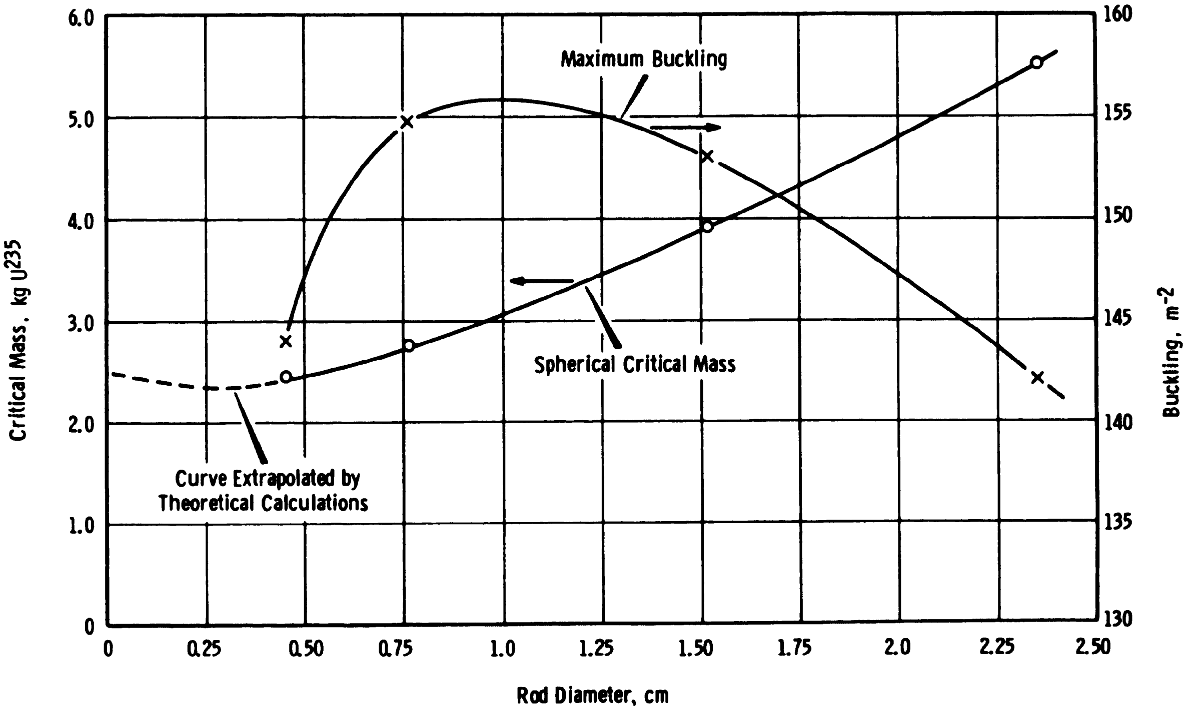

Fig. 4. Maximum bucklings and minimum critical masses for 3.063 wt% U^{235} uranium rods in water.

TABLE II

Summary of 2.0 wt% U^{235}-Enriched Uranium Rod Measurements

H_2O:U, Vol	H_c, cm	Critical No. Rods	R_c, cm	$\bar{\lambda}$, cm	B^2, m^{-2}	Crit Mass Cyl, kg of U^{235}	Computed Crit Mass Sphere, kg of U^{235}
0.925 in.-Diam Rods							
1.89	40.64	117.6 ± 0.4	21.70 ± 0.04	6.46	107.35 ± 0.15	7.85 ± 0.03	7.43 ± 0.03
1.89	81.28	82.4 ± 0.6	18.16 ± 0.07			10.99 ± 0.02	
2.29	40.64	103.7 ± 0.4	21.73 ± 0.04	5.94	111.31 ± 0.20	6.92 ± 0.03	6.51 ± 0.03
2.29	81.28	72.5 ± 0.1	18.17 ± 0.03			9.67 ± 0.03	
2.72	40.64	99.9 ± 0.5	22.66 ± 0.06	6.20	104.51 ± 0.15	6.66 ± 0.04	6.29 ± 0.04
2.72	81.28	68.5 ± 0.2	18.76 ± 0.07			9.14 ± 0.02	
3.17	40.64	102.2 ± 0.7	24.26 ± 0.08	6.07	98.29 ± 0.21	6.81 ± 0.04	6.39 ± 0.04
3.17	81.28	68.2 ± 0.2	19.83 ± 0.04			9.10 ± 0.03	
3.89	40.64	119.0 ± 0.6	28.37 ± 0.07	5.44	87.77 ± 0.10	7.94 ± 0.04	7.18 ± 0.03
3.89	81.28	74.4 ± 0.4	22.43 ± 0.06			9.92 ± 0.05	
0.600 in.-Diam Rods							
2.06	40.64	254.4 ± 0.4	21.28 ± 0.02	6.84	106.60 ± 0.62	7.10 ± 0.01	6.79 ± 0.01
2.06	81.28	176.3 ± 0.2	17.71 ± 0.01			9.83 ± 0.01	
2.71	40.64	197.9 ± 0.3	20.64 ± 0.03	6.78	110.47 ± 0.60	5.52 ± 0.01	5.27 ± 0.02
2.71	81.28	139.0 ± 0.3	17.29 ± 0.02			7.76 ± 0.02	
3.41	40.64	178.9 ± 0.5	21.38 ± 0.04	6.32	110.12 ± 0.51	4.98 ± 0.02	4.73 ± 0.02
3.41	81.28	124.2 ± 0.3	17.84 ± 0.02			6.93 ± 0.02	
4.18	40.64	178.9 ± 1.0	23.20 ± 0.08	6.18	102.18 ± 0.10	4.99 ± 0.03	4.72 ± 0.03
4.18	81.28	120.7 ± 0.5	19.04 ± 0.04			6.73 ± 0.03	
5.18	40.64	204.3 ± 0.3	27.07 ± 0.02	6.14	87.69 ± 0.15	5.70 ± 0.01	5.28 ± 0.03
5.18	81.28	128.0 ± 0.6	21.42 ± 0.05			7.14 ± 0.03	
6.84	40.64	174.2 ± 0.7	28.17 ± 0.06	5.72	61.85 ± 0.17	9.73 ± 0.04	8.10 ± 0.04

Theoretical Analysis

Criticality parameters necessary to establish standards for criticality control in handling, storing and shipping uranium rods enriched to about 1.0 and 3.0 wt% U^{235} have been determined experimentally. However, at other enrichments there is uncertainty in the available parameters. It is necessary, therefore, to obtain the criticality parameters for other enrichments by calculational methods.

The usual two-group critical equation for a bare reactor was assumed to be valid:

$$k_{\infty} = \eta\epsilon f p = (1 + B^2 L^2)(1 + B^2 \tau), \tag{3}$$

where the notation is that of Glasstone and Edlund[8]. Traditional models found in Glasstone and Edlund were used to calculate the terms in Eq. (3). Also, since others[9,10] have previously made similar correlations, only a brief outline of the computational methods are given.

Thermal cross sections averaged over a Wigner-Wilkins spectrum[11], for an equivalent homogenization of the unit lattice cell, were used to compute η, f and L^2; the thermal flux used to homogenize the lattice cell was that from a one-velocity P_3 calculation[12]. Hellstrand's semiempirical expression was used to obtain the resonance shielding in closely packed lattices[13,14]. The interaction of first-flight collisions between neighboring fuel rods was included in the ϵ calculation. This was done by using the Bonalumi method to calculate first-flight collision probabilities in con-

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[8] S. GLASSTONE and M. C. EDLUND, *The Elements of Nuclear Reactor Theory*, D. Van Nostrand Company, Inc., (1952).

[9] J. BENGSTON and R. L. HELLENS, "A Survey of the Beginning of Life Characteristics of Uranium-Fueled, Water-Moderated Lattices," CEND-137, (1961).

[10] C. L. BROWN, "A Semi-Empirical Method of Estimating Material Bucklings for Slightly Enriched Uranium-Water Lattices," HW-68405, (February 1961).

[11] E. P. WIGNER and J. E. WILKINS, Jr., "Effects of the Temperature of the Moderator on the Velocity Distribution of Neutrons with Numerical Calculations for H as Moderator," AECD-2275, (1948).

[12] D. D. MATSUMOTO and C. R. RICHEY, "Program on the 709 Digital Computer of the P_3 Approximation to the Boltzmann Transport Equation in Cylindrical Geometry," HW-60781, (June, 1959).

[13] E. HELLSTRAND, "Measurements of the Effective Resonance Integral in Uranium Metal and Oxide," *J. Appl. Phys.*, **28**, (1957).

[14] S. M. DANCOFF and M. GINSBURG, "Surface Resonance Absorption in a Close-Packed Lattice," CP-2157, (September 1944).

centric annuli, isotropically reflecting the first-flight neutrons that escaped to the equivalent unit cell boundary[15]. The Fermi age was calculated by the empirical expression

$$\tau = \frac{29(1-P_i)+73P_i}{1+1.9V_u/V_{H_2O}} \cdot \frac{V_{\mathrm{Total}}}{V_{H_2O}} \quad , \qquad (4)$$

where P_i denotes the probability that a neutron terminates its first flight with an inelastic scattering event. The symbols, V_u, V_{H_2O}, and V_{Total}, denote the volume of uranium, water, and unit cell, respectively.

Calculations were made for arrays in which the critical bucklings were determined experimentally. Typical results of the correlation between experiment and theory are given in Figs. 6 and 7. The data recorded in Fig. 6 are for the 0.600 in.-diam rods having enrichments ranging from 1.027 to 3.063 wt% U^{235}. The experimental data recorded for the three lower enrichments in Fig. 6 are those measured by Kouts[2]. Figure 7 shows the correlation for several rod sizes at a single enrichment, 3.063 wt% U^{235}.

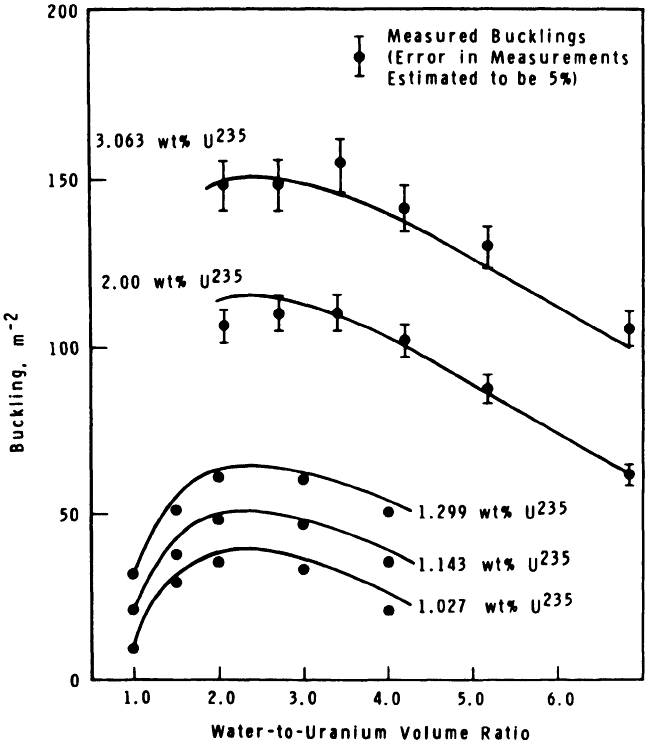

Fig. 6. Theoretical bucklings for slightly enriched 0.600-in. rods in water.

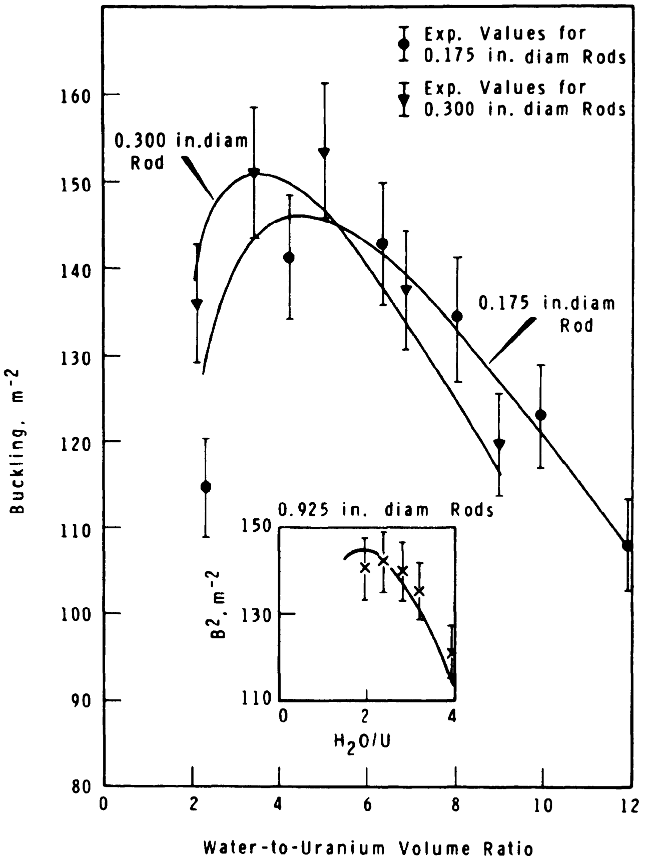

Fig. 7. Theoretical bucklings for 3.063 wt% U^{235}-enriched rods.

Utilizing the above correlation, the principal parameters defining criticality for uranium rods in water have been calculated for enrichments ranging from 1.0 to 5.0 wt% U^{235}. The maximum critical bucklings are plotted as a function of rod diameter in Fig. 8; the minimum critical mass is given as a function of rod diameter in Fig. 9. The extrapolation lengths used in calculating the minimum masses were obtained by interpolating and extrapolating the experimentally determined values of λ.

NATURAL URANIUM MEASUREMENTS

The lower limit on the enrichment at which criticality is possible in light-water-moderated systems is usually assumed to be that of natural uranium (wt% $U^{235} = 0.712$). The consequences, however, of optimum rod size and spacing in water are not known definitely. A number of exponential experiments were performed at the Oak Ridge National Laboratory to ascertain the possibility of obtaining criticality with natural uranium

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[15] A. BONALUMI, "Neutron First Flight Collision Probabilities in Reactor Physics," *Energia Nucleare*, **8**, 5; 326-336 (1961).

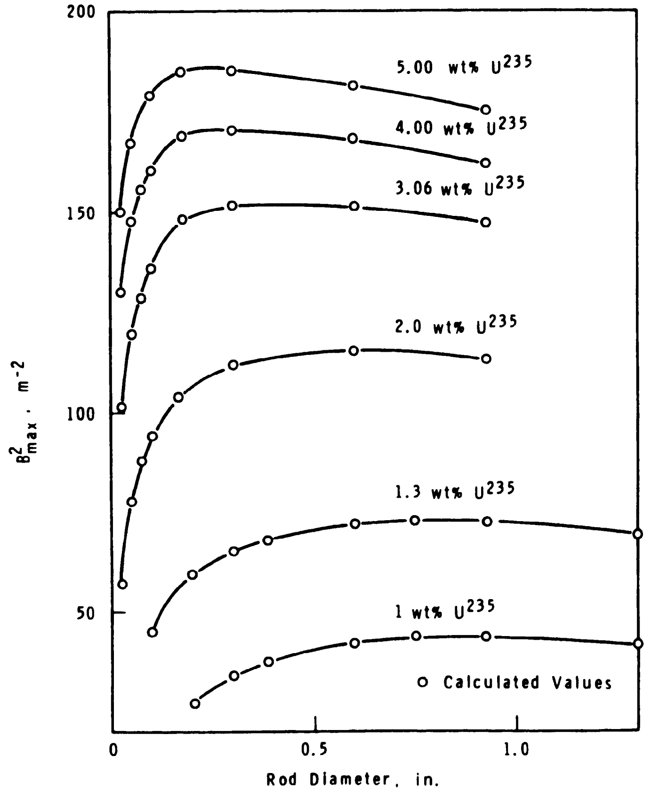

Fig. 8. Maximum buckling as a function of rod diameter.

in water[1]. It was concluded from these experiments that criticality would be possible after applying corrections for neutron capture by the aluminum tubing and spacers. As an experimental check on this conclusion, a number of exponential measurements were performed at Hanford with 0.925 in.-diam rods encased in thin-walled ($\frac{1}{32}$ in.) Plexiglas tubes. The use of Plexiglas tubes eliminated the loss by neutron capture in aluminum tubing. The resulting bucklings recorded in Fig. 10 were obtained from the measured relaxation lengths utilizing extrapolation lengths inferred from the experiments by Kouts[1,3]. For the 0.925-in. rods, the maximum obtainable buckling is approximately 0.3 m^{-2} at a water-to-uranium volume ratio of 1.6. The bounds of accuracy in the measurements are ± 2.5 m^{-2}, leaving the possibility of achieving criticality uncertain.

ENRICHED URANIUM TUBES

For a given enrichment, it is not known which has the least critical volume (maximum buckling) and the minimum critical mass, solid or tubular fuel elements. A number of exponential measurements were made with various tubular elements, but the number of measurements was not suffi-

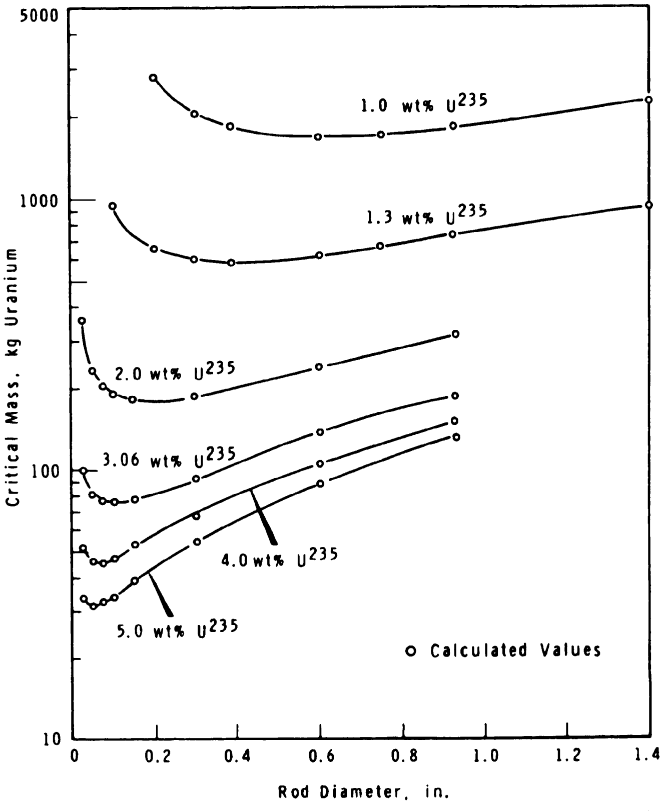

Fig. 9. Minimum critical mass as a function of rod diameter.

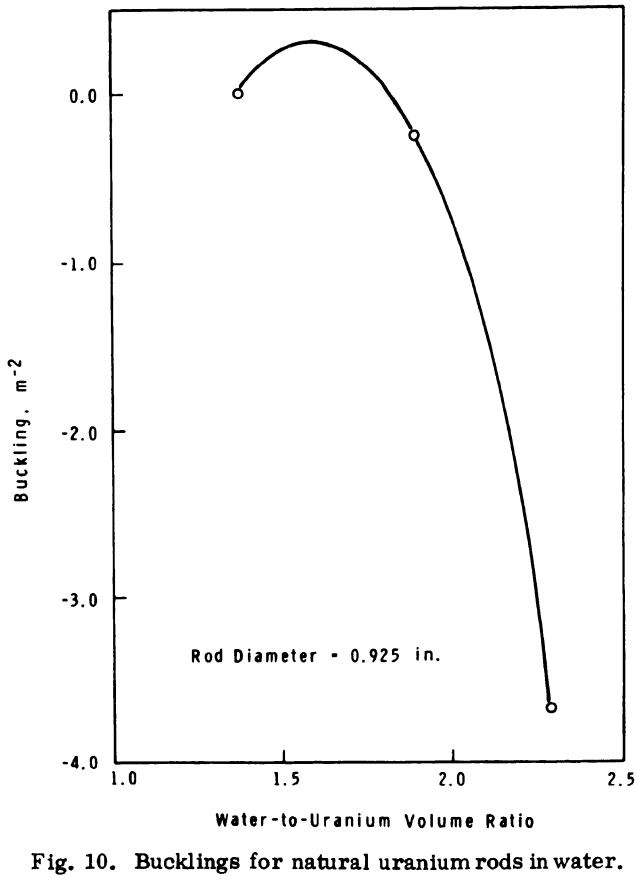

Fig. 10. Bucklings for natural uranium rods in water.

cient to determine which fuel elements (solid or tubular) have either the largest buckling or minimum critical mass. The usefulness of the experiments, other than for establishing the criticality parameters of the enriched fuel element in question, has been in verifying theoretical calculations.

The exponential measurements with the tubular elements varied in enrichments from 1.0 to 1.6 wt% U^{235}. The outer diameters of the tubular elements varied between 1.34 and 1.66 in.; the inner diameters were between 0.46 and 0.94 in. A summary of the experimental results is given in Table III. The critical bucklings were determined from the measured relaxation lengths using extrapolation lengths inferred from the experiments by Kouts[1,3]. Errors in the bucklings were obtained from uncertainties in the measured relaxation lengths. An additional error of approximately 5% in the buckling results from an uncertainty in the extrapolation length and the equivalent cylindrical radius of the fuel loadings.

The theoretical models used to correlate the enriched uranium rod experiments were applied in calculating the critical bucklings for the tubular fuel elements. These results, recorded in Table III, are larger than the experimental bucklings by approximately 10 m^{-2}; this difference corresponds to an error of about 3% in k_{∞}. It is noted that the calculated bucklings do not reproduce the observed difference when the central annulus contains water and when it is void. The theoretical calculations, as applied to criticality safety problems, are of value in that they conservatively underestimate the critical mass.

To determine the effect, if any, of neutron streaming within the void fuel cores, measurements were made with the center annuli of the 1.37-in. OD, 0.48-in. ID, 1.44 wt% U^{235} fuel elements filled with bismuth powder ($\rho = 5.5$ g/cm^3). Bismuth was chosen for its low capture cross section and moderating power ($\sigma_a \approx 34$ mb and $\xi = 9.5 \times 10^{-3}$) while being a significant neutron scatterer ($\sigma_s \approx 9$ b). Since on a relative scale the uncertainty of each buckling is about ± 0.30 m^{-2}, the bismuth in the core had a negligible effect on the critical buckling. The calculated difference in

TABLE III

Experimental Results for Slightly Enriched Tubular Fuel Elements

	Water in Central Annulus					Central Annulus Dry				
Spacing, in.	H_2O:U, Vol	λ, cm	B^2 Exp, m^{-2}	B^2 Calc, m^{-2}	Computed Spherical Critical Mass, kg of U^{235}	H_2O:U, Vol	λ, cm	B^2 Exp, m^{-2}	B^2 Calc, m^{-2}	Computed Spherical Critical Mass, kg of U^{235}
1.66 in. OD, 0.94 in. ID, 1.007 wt% U^{235}: Outer Al Cladding 0.028 in. Thick; Inner 0.032 in.										
2.05	1.27	7.66	21.33 ± 0.09	28.96	70.69 ± 0.44	0.87	8.25	14.34 ± 0.42	19.39	134.05 ± 5.89
2.20	1.65	7.34	29.20 ± 0.15	35.79	36.64 ± 0.28	1.25	7.67	23.78 ± 0.06	34.05	51.14 ± 0.19
2.45	2.33	6.94	27.45 ± 0.14	33.74	33.61 ± 0.26	1.93	7.14	25.29 ± 0.13	35.82	38.19 ± 0.29
2.70	8.09	6.68	15.31 ± 0.05	22.01	64.12 ± 0.42	2.69	6.80	16.19 ± 0.20	24.34	67.23 ± 1.25
1.34 in. OD, 0.50 in. ID, 1.25 wt% U^{235}: Outer Al Cladding 0.028 in. Thick, Inner 0.020 in.										
1.85	1.31	7.63	46.48 ± 0.19	55.42	22.56 ± 0.14	1.18	7.77	44.50 ± 0.35	55.47	24.11 ± 0.29
2.00	1.73	7.29	51.96 ± 0.14	60.33	16.43 ± 0.06	1.59	7.38	49.69 ± 0.23	61.50	17.45 ± 0.12
2.10	2.02	7.12	50.22 ± 0.16	59.16	16.08 ± 0.07	1.89	7.19	49.57 ± 0.09	60.49	16.13 ± 0.05
2.20	2.33	6.96	48.15 ± 0.49	55.59	15.97 ± 0.25	2.20	7.02	47.09 ± 0.10	56.87	16.31 ± 0.05
2.40	2.99	6.72	37.09 ± 0.10	43.90	21.54 ± 0.09	2.86	6.76	36.96 ± 0.10	44.87	21.34 ± 0.09
1.37 in. OD, 0.48 in. ID, 1.44 wt% U^{235}: Outer and Inner Al Cladding 0.049 in. Thick										
2.00	1.51	7.44	60.60 ± 0.07	68.11	15.29 ± 0.03	1.31	7.61	52.03 ± 0.05	69.10	19.76 ± 0.03
2.10	1.79	7.23	60.98 ± 0.07	70.20	13.88 ± 0.03	1.59	7.37	56.56 ± 0.06	71.37	15.72 ± 0.03
2.20	2.07	7.07	60.48 ± 0.07	69.06	13.02 ± 0.03	1.87	7.17	57.08 ± 0.06	70.21	14.34 ± 0.03
2.40	2.69	6.81	52.37 ± 0.06	60.41	14.50 ± 0.03	2.49	6.89	51.31 ± 0.14	61.30	14.93 ± 0.06
2.60	3.36	6.63	36.96 ± 0.08	46.67	23.03 ± 0.07	3.16	6.66	37.97 ± 0.15	47.21	21.93 ± 0.13
2.00[a]						1.31[a]	7.61	52.53 ± 0.12		19.43 ± 0.07
2.20[a]						1.87[a]	7.17	57.38 ± 0.07		14.18 ± 0.03
1.394 in. OD, 0.464 in. ID, 1.466 wt% U^{235}: Outer and Inner Al Cladding 0.049 in. Thick										
2.00	1.39	7.52	59.47 ± 0.13	68.34	16.61 ± 0.06					
2.10	1.65	7.32	63.60 ± 0.05	71.71	13.56 ± 0.01					
2.20	1.92	7.14	63.25 ± 0.09	71.43	12.72 ± 0.03					
2.40	2.51	6.86	56.93 ± 0.06	63.93	13.27 ± 0.01					
2.60	3.15	6.67	43.66 ± 0.18	50.83	18.34 ± 0.10					
1.394 in. OD, 0.464 in. ID, 1.6 wt% U^{235}: Outer and Inner Al Cladding 0.049 in. Thick										
2.00	1.38	7.53	66.05 ± 0.07	75.91	14.91 ± 0.02	1.25	7.65	60.33 ± 0.14	76.70	17.46 ± 0.06
2.10	1.64	7.35	69.41 ± 0.04	79.45	12.56 ± 0.02	1.51	7.45	64.83 ± 0.10	80.50	14.11 ± 0.03
2.20	1.91	7.14	70.00 ± 0.07	79.25	11.49 ± 0.02	1.79	7.25	65,89 ± 0.27	80.29	12.74 ± 0.08
2.40	2.50	6.88	64.50 ± 0.04	71.74	11.50 ± 0.02	2.37	6.93	61.36 ± 0.10	72.53	12.54 ± 0.03
2.60	3.14	6.68	51.25 ± 0.10	58.48	15.09 ± 0.05	3.01	6.70	49.37 ± 0.02	58.94	16.08 ± 0.02

[a] Bismuth in central annulus

the thermal utilization was negligible between bismuth in the core and the core being void. Unless the age or the fast effect are appreciably changed by inelastic scattering in the bismuth, it can be concluded that the effect of neutron streaming within the void central annulus is small.

SUMMARY

Exponential and approach-to-critical experiments were conducted with slightly enriched uranium in light water. Analytical methods used to correlate the experimental data were used to calculate the maximum bucklings and minimum critical masses for 5 wt% U^{235}-enriched uranium rods in water. The largest critical buckling obtainable is 185.5 m^{-2} for 0.25 in.-diam rods; the minimum critical mass is 31.3 kg U for 0.05 in.-diam rods. Caution should be used in the application of the 5 wt% U^{235} data; however, where theoretical calculations were compared to experiment, they were found to be conservative in most cases; i.e. the calculated maximum bucklings were larger and the calculated minimum critical masses were smaller than the experimentally determined values.

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