Nuclear Criticality Safety Engineer Training
Module 2 [1]

Neutron Interactions

LESSON OBJECTIVE

In this module the ideas of neutron interactions, neutron flux density and neutron cross sections are introduced.

NEUTRON FLUENCE

Imagine a region through which neutrons are moving. This could be pictured as a location near a beehive on a warm sunny day and the bees likened to neutrons. There is an important difference in that neutrons move in straight lines with constant velocities until they hit something and the bees dart about in ever changing speeds and directions on their own wills. Lets count the number of neutrons (or bees) that pass through a region and define the fluence of neutrons as

$$\Phi \equiv \frac{\lim}{\Delta A \to 0}\left(\frac{\Delta N}{\Delta A}\right)$$

where ΔN is the number of neutrons passing through a volume element whose cross sectional area is ΔA. The neutron fluence has the dimensions of neutrons per unit area, or more commonly, neutrons/cm^2. Next calculate a fluence rate, i.e.

$$\phi \equiv \frac{\lim}{\Delta t \to 0}\left(\frac{\Delta \Phi}{\Delta t}\right),$$

which has dimensions of neutrons per unit area per unit time, commonly given as neutrons/cm^2-s. This quantity is the fluence rate or flux density, commonly called the neutron flux. This is an awkward concept to understand rigorously and most texts devote a section or two to explaining flux density and deriving concepts related to it. The nuclear criticality specialist rarely encounters neutron flux density directly. However, the criticality specialist frequently encounters interaction rates, such as fission rates, absorption rates or leakage rates. Interaction rates are the products of neutron flux densities and interaction cross sections. Neutron interaction cross sections are discussed next.

[1] Developed for the U. S. Department of Energy Nuclear Criticality Safety Program by T. G. Williamson, Ph.D., Westinghouse Safety Management Solutions, Inc., in conjunction with the DOE Criticality Safety Support Group.

CROSS SECTION

Consider the nuclear cross section from an experimental view. Imagine a narrow, parallel beam (flux density with all particles moving in the same direction) of radiation incident on a plane, thin layer of material, as shown in Figure 1. Imagine a detector behind the material plane which will detect some of the particles which have passed through the material. The experiment is to note the number of particles counted by the detector as the thickness of material in the beam is changed. Figure 2 shows a typical result of this type of measurement.

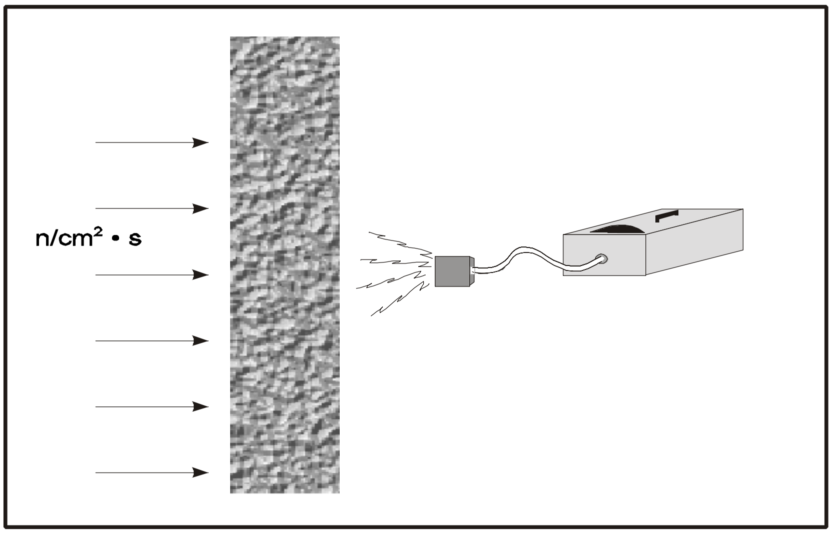

Figure 1. Neutron Beam Incident on Thin Layer of Material

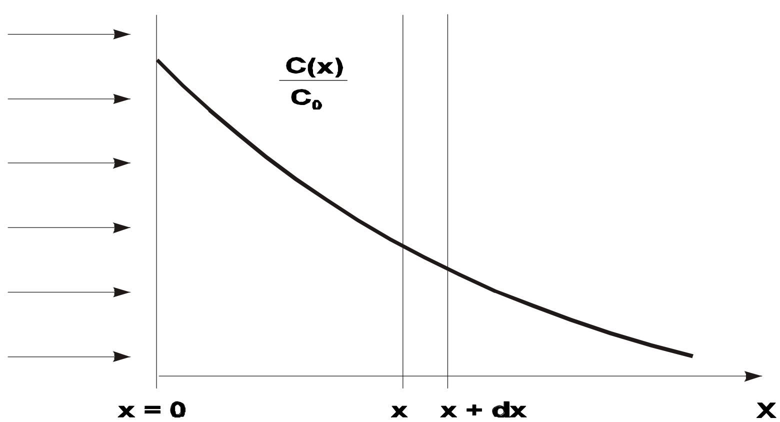

Figure 2. Typical Results of Neutron Beam Attenuation Measurements

In this experiment it is observed that the number of particles measured by the detector can be well fit by an exponential function. The argument of this function is related to the amount of material in the beam; i.e.

$$C(x)/C_0 = \exp(-\,\Sigma\, x)$$

where $C(x)/C_0$ is the detector response as the thickness of material, x, in the radiation beam is changed. The quantity Σ, with dimensions of reciprocal length, is given a value which best fits the data. In the limit of small material thickness, Σ is a property of the material for a given interaction, in this case for removal of a particle from the beam. In the limit of small path length, Σ is seen to be the probability per unit path length that a particle will undergo an interaction. It is reasonable to expect that this interaction probability should be proportional to the number of targets, or atoms, in the material. That is

$$\Sigma\ (cm^{-1}) = \sigma\ (cm^2) \times N\ (atoms/cm^3)$$

where the proportionality constant, σ, has the units of cm^2, an area. This quantity, σ, is called the *nuclear cross section*. In this case it is the cross section for any interaction that removes a particle from the beam. If the experiment were made with a beam of neutrons, it is reasonable to expect that the interaction rate of neutrons in this material should be related to the probability for interaction and the number of neutrons. Specifically the interaction rate per unit volume of target material is

$$\begin{aligned} \text{Interaction Rate} &= \phi \times \Sigma \\ &= (\text{neutrons/cm}^2\text{-sec}) \times (\text{interaction probability/neutron-cm}) \\ &= \text{interactions/cm}^3\text{-sec.} \end{aligned}$$

In this experiment, if the number of neutrons in the beam is known, i.e., the neutron flux density, and the cross section has been measured, then the interaction rate can be calculated. For this beam experiment, with a thin absorber, an interaction is any nuclear collision which removes a neutron from the beam.

The interaction rate is the primary quantity of interest to the criticality specialist. We care about fission rates, neutron production rates, neutron absorption rates, leakage rates, etc. We may not particularly care about flux densities and cross sections as long as the product of the two quantities gives the correct interaction rates. Also, interaction rates are the only things that can be measured in nuclear experiments. While cross sections are the topic of discussion, keep in mind the significance of interaction rates.

There are two types of neutron interactions with matter: absorption and scattering. The meaning of *neutron absorption* is just as the name implies - the neutron is absorbed and disappears. As a result of the neutron absorption process, other products may be formed. For example,

Fission reactions	(n,f)	products are fission products, neutrons, gammas rays, etc.
Capture reaction	(n,γ)	products are gamma rays, typically the gamma rays have energies comparable to the neutron binding energy, 1 to10 MeV
Capture reaction	(n,p), (n,α), etc.	product is a charged particle, a proton and an alpha particle in these two examples.

Cross sections for each of these reactions may be designated by subscripts, superscripts or by symbols in parentheses. For example, σ_f might designate a fission cross section, σ_f^{235} the fission cross section for ^{235}U, σ_c a capture cross section (capture may include fission), and σ (n,p) a neutron capture followed by a proton emission.

Neutron scattering means that the neutron energy and direction of motion are changed; i.e, after a scattering reaction the neutron has a different velocity vector. Neutron scattering can be one of two kinds, elastic or inelastic.

In an *elastic scattering* reaction the kinetic energy and momentum of the neutron-nucleus system are conserved. In an *inelastic scattering* reaction the kinetic energy of the system is not conserved because some of the neutron kinetic energy is transferred to excitation energy of the scattering nucleus. The excitation energy is then released as gamma-ray energy.

Each nuclide has characteristic cross sections, for each interaction type. These cross sections vary with the neutron energy. Look at the Evaluated Nuclear Data File (ENDF) cross section plots at the end of this lesson, which are from the Brookhaven National Laboratory Nuclear Data Center (http://www.nndc.bnl.gov/nndc). The BNL Nuclear Data Center is a source for up-to-date cross sections in the United States. The notation on the cross section curves in this module, ENDF/B-VI, means Evaluated Nuclear Data Files, version six. Version six is the latest cross section set available today (1999) and is used by many criticality specialists. Older versions are still widely used. For example ENDF/B-V is widely used with the Los Alamos National Laboratory Monte Carlo code MCNP. Version five is the basis for the 238-group cross section set used with the Oak Ridge National Laboratory SCALE package. The older set, ENDF/B-IV, is the basis for the 27-group cross section set in SCALE. It is important to note that, as long as the code package and cross section set are properly validated against critical experiments, it is satisfactory to use older evaluations of the cross sections. In addition to the ENDF cross sections there are similar modern evaluations of cross sections from the European community and from Japan.

Some features of these cross section curves are noted below. These figures are all plots of the neutron interaction cross sections in barns (10^{-24} cm^2) versus the neutron energy in electron volts (some axes scaled by factors of 10^3 as needed for clarity). The curves are plotted on log-log scales, and cover a wide range of values in both energy and cross section.

Figures 3 through 6: Fission Cross Sections of ^{239}Pu and ^{235}U

- Energy range 0.01 eV to 10 MeV
- Cross section range 1 to 1000 barns
- Cross section is large in the low energy range, E < 1 eV
- Cross section increases as neutron energy decreases in the low energy range
- Cross section is relatively small and flat in the high energy range, E > 100 keV
- Wide swings in the cross section in the intermediate energy range,
  1 eV < E < 100 keV
- Well defined resonances in the low intermediate range, 1 eV < E < 100 eV
- Unresolved resonances in the upper intermediate range, 100 eV < E < 100 keV

Figures 7 through 9: ^{238}U Elastic Scattering and Total Cross Sections

- Energy range 10^{-5} eV to 10 MeV
- Cross section range 0.1 to 10^{4} barns
- Cross section is essentially flat except in resonance region.
- Defined resonances from about 4 eV to 4 keV
- A large absorption resonance at 6.7 eV
- Resonances in the scattering cross section with dips in the cross section at energies just below the resonances

Figure 10: Hydrogen Total Cross Section

- Energy range 10^{-5} eV to 10^{8} eV
- Cross section range 0.1 to 1000 barns
- Cross section is flat at about 20 barn from 1 eV to 10 keV then decreases
- No resonances
- All scattering - not evident from this figure but absorption is about 10^{-3} of total

Figure 11: Carbon Total Cross Section

- Energy range 10^{-5} eV to 10^{9} eV
- Cross section range 0.3 to 5 barns
- Cross section is flat at about 5 barn below 0.05 MeV
- No resonances until about 2 MeV
- All scattering - not evident from this figure but absorption is about 10^{-3} of total

Figure 12: Boron Total Cross Section

- Energy range 10^{-5} eV to 10 MeV
- Cross section range 1.5 to 2 x 10^{5} barns
- Cross section varies as 1/v (neutron speed) from 0.01 eV to 100 eV
- No resonances until 0.3 MeV
- All absorption, (n,α) - not evident from this figure but scattering is about 10^{-3} of total

Some summary statements about neutron cross sections from these figures:

- Cross section values range over many orders of magnitude
- Absorption cross sections generally increase as neutron energy decreases
- Scattering cross sections generally are constant in the low energy range, vary in the intermediate resonance range, and decrease in the high energy range
- The lowest energy resonance is in the MeV range for low atomic weight nuclides and in the eV range for high atomic weight nuclides
- A few elements, like boron, lithium, cadmium and gadolinium have very high neutron absorption cross sections.

It is convenient to think of the neutron energy range for fissile elements in three energy intervals.

Thermal	$E < 1$ eV
Intermediate	1 eV $< E <$ 100 keV
Fast	100 keV $< E$

These are approximate values for the energy ranges and other authors may suggest different boundaries. In the thermal range the cross sections are smooth and increase with decreasing neutron energy; in the intermediate range resonances dominate; and cross sections in the fast range are smaller than in the thermal range with not much resonance structure.

SUMMARY

The concept of a nuclear cross section as a probability of a nuclear interaction per unit of path length was introduced. The interaction rate was defined as the product of the neutron flux density and the macroscopic cross section. The shapes of several cross sections were considered.

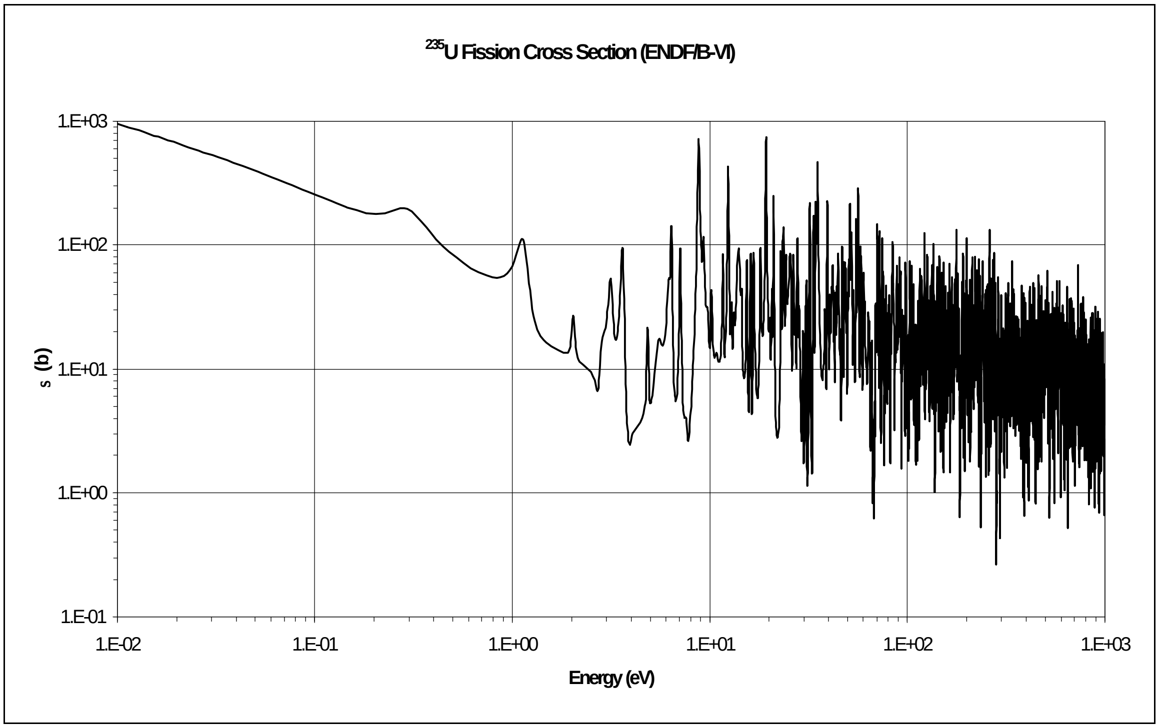

Figure 3. Low Energy ^{235}U Fission Cross Section

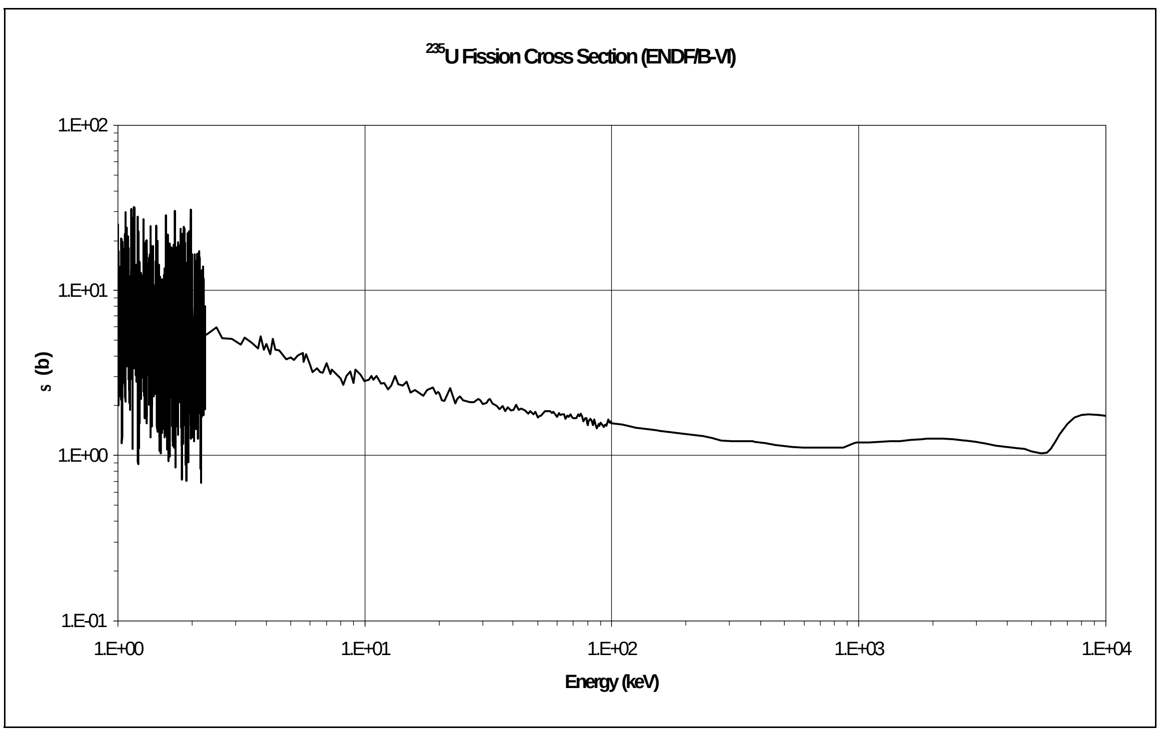

Figure 4. High Energy ^{235}U Fission Cross Section

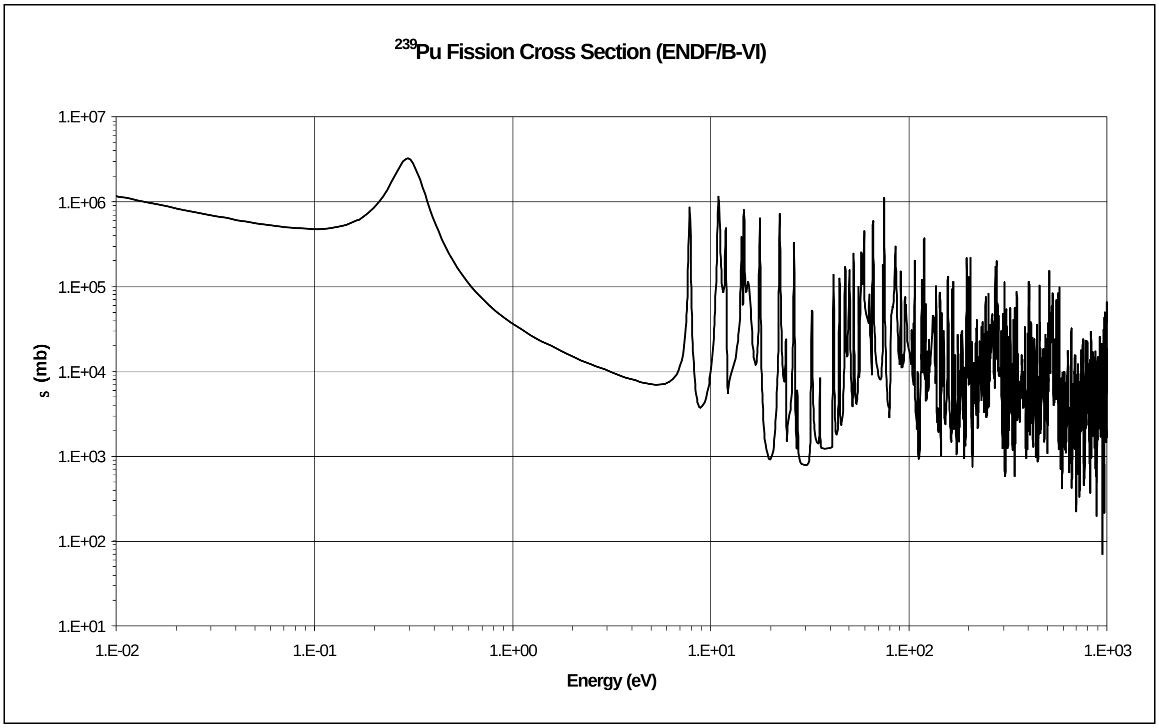

Figure 5. Low Energy ^{239}Pu Fission Cross Section

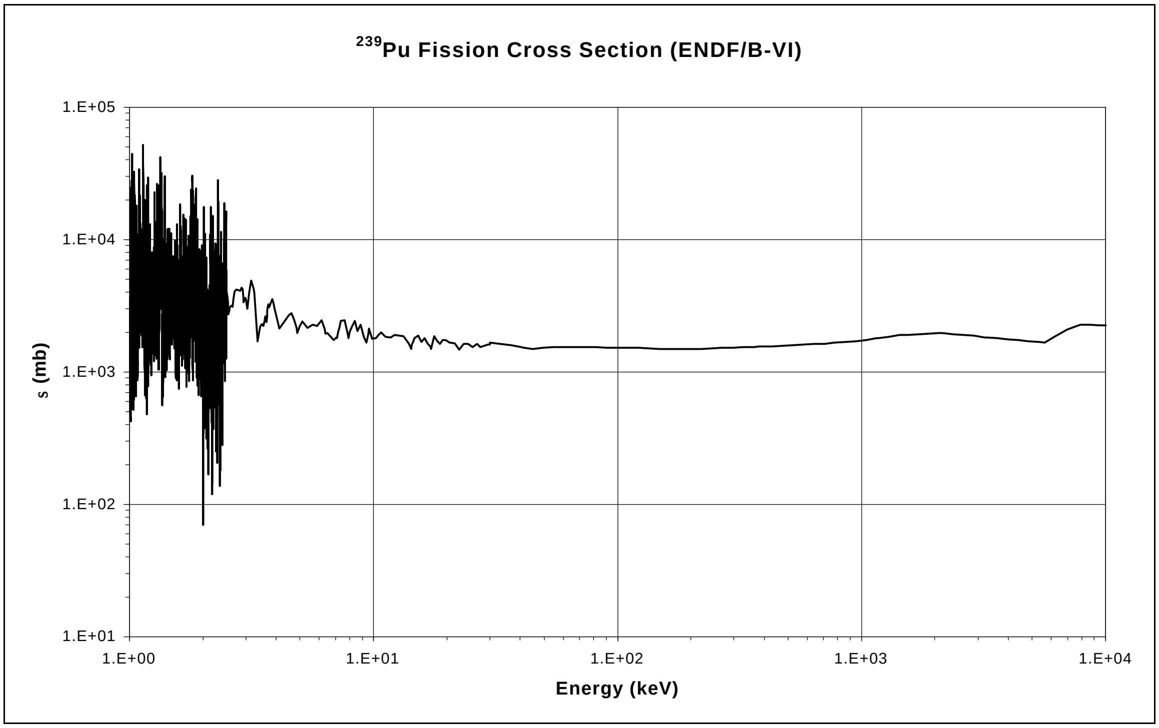

Figure 6. High Energy ^{239}Pu Fission Cross Section

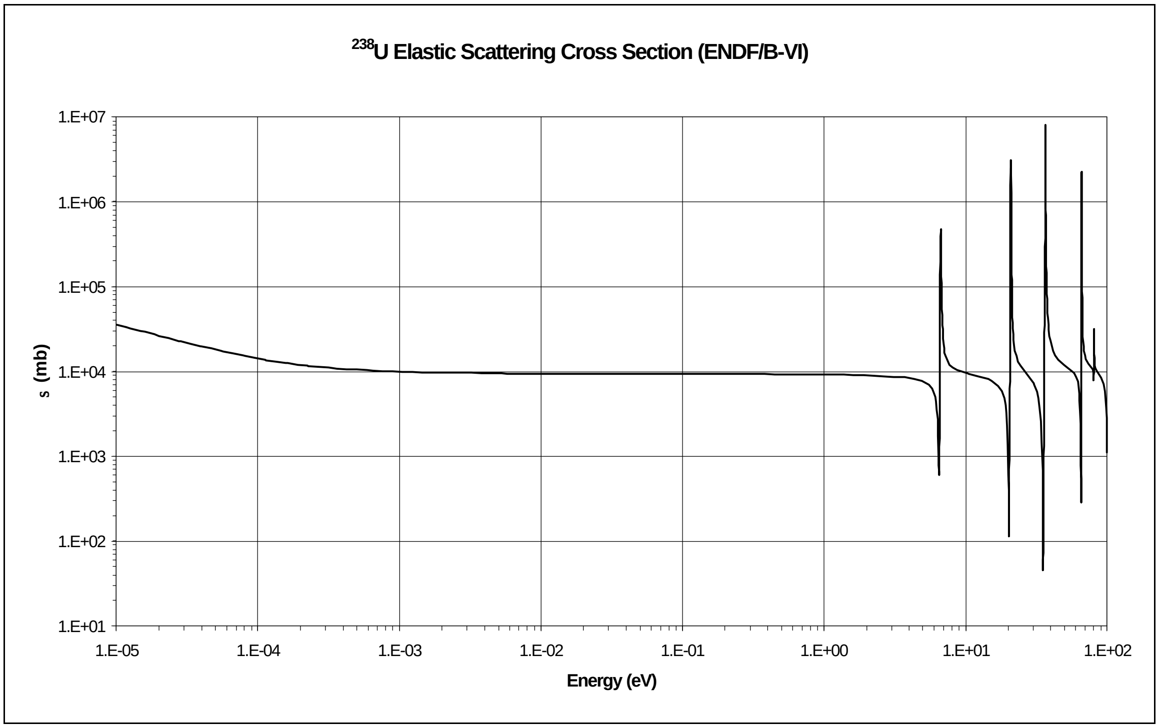

Figure 7. Low Energy ^{238}U Elastic Scattering Cross Section

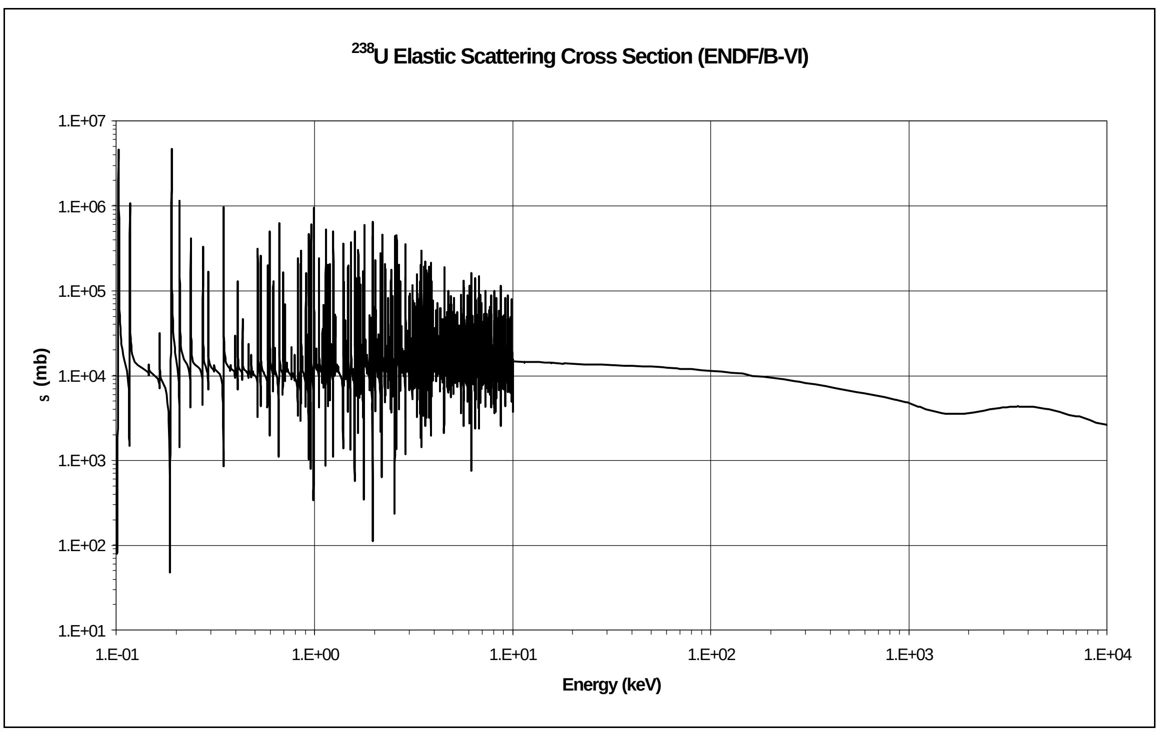

Figure 8. High Energy ^{238}U Elastic Scattering Cross Section

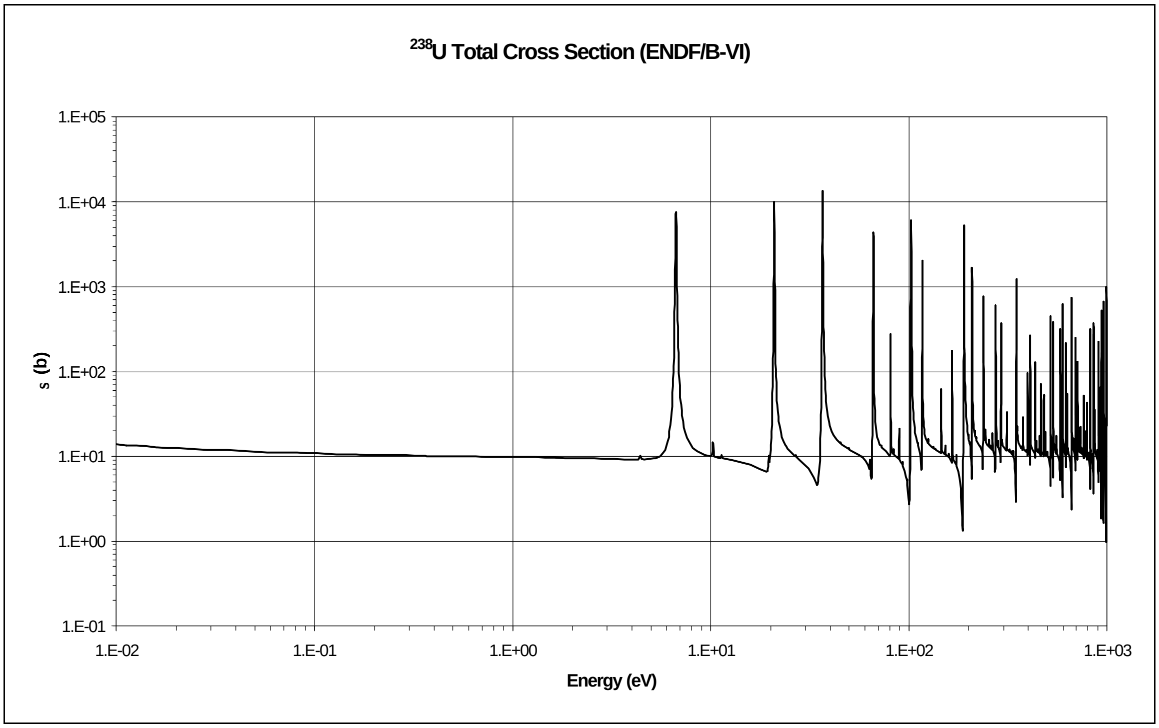

Figure 9. ^{238}U Total Cross Section

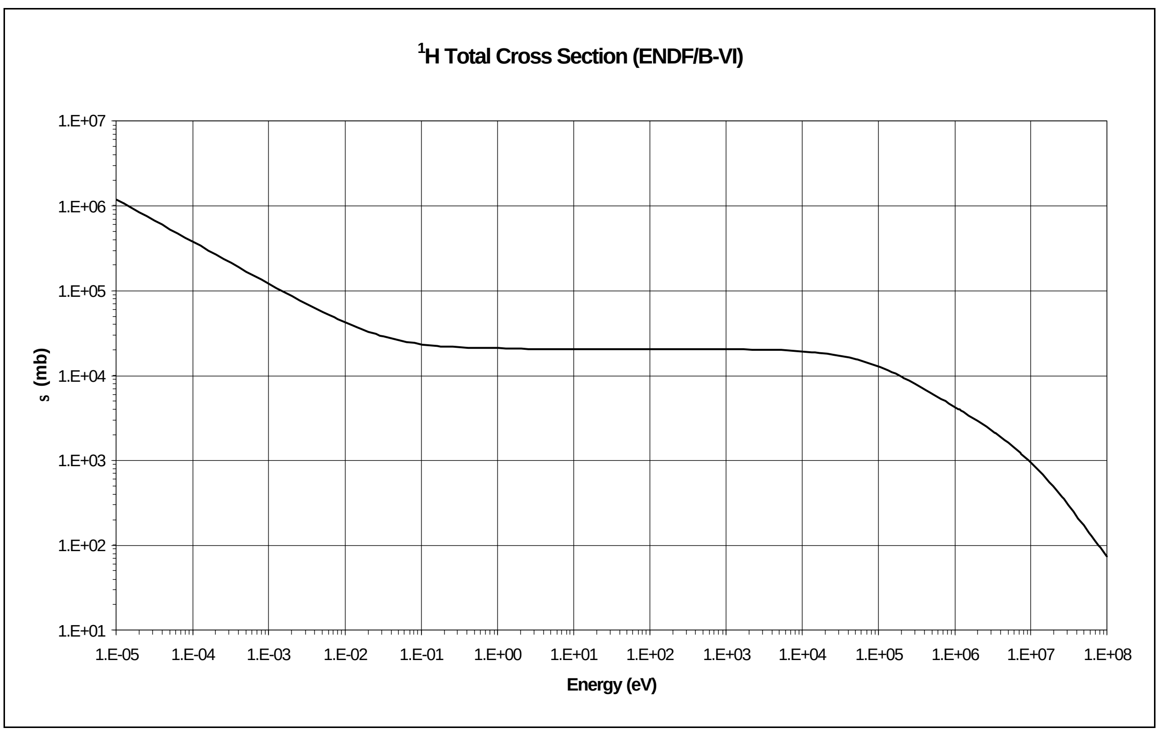

Figure 10. ^{1}H Total Cross Section

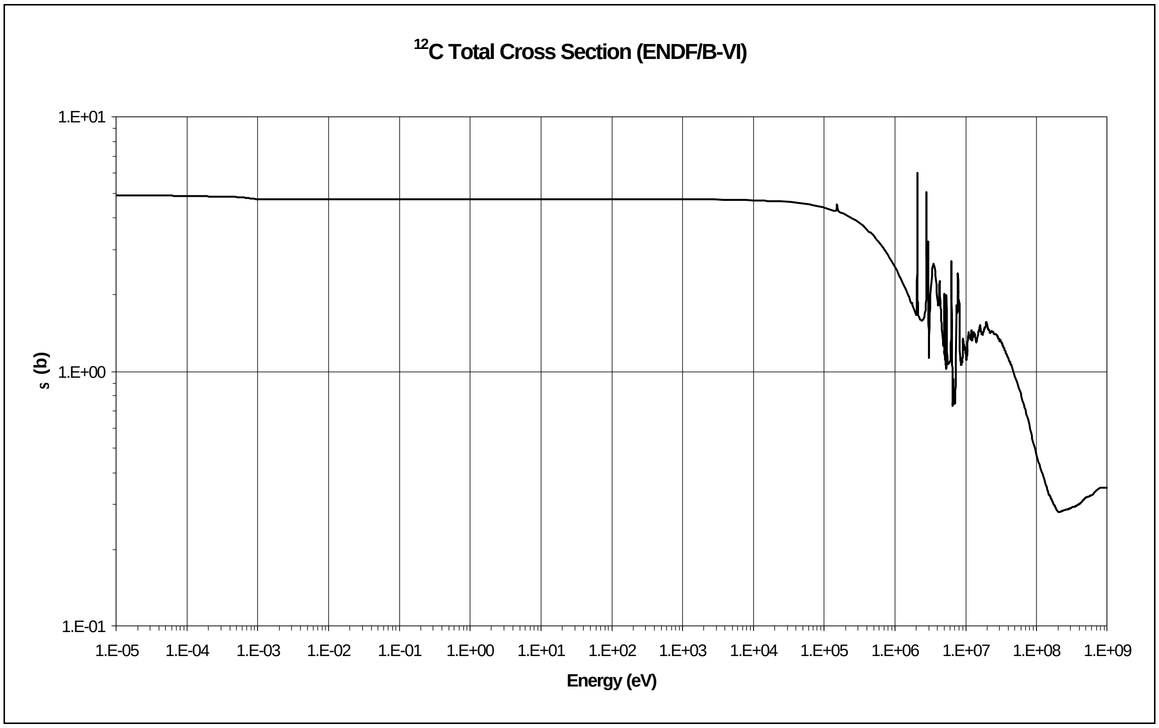

Figure 11. ^{12}C Total Cross Section

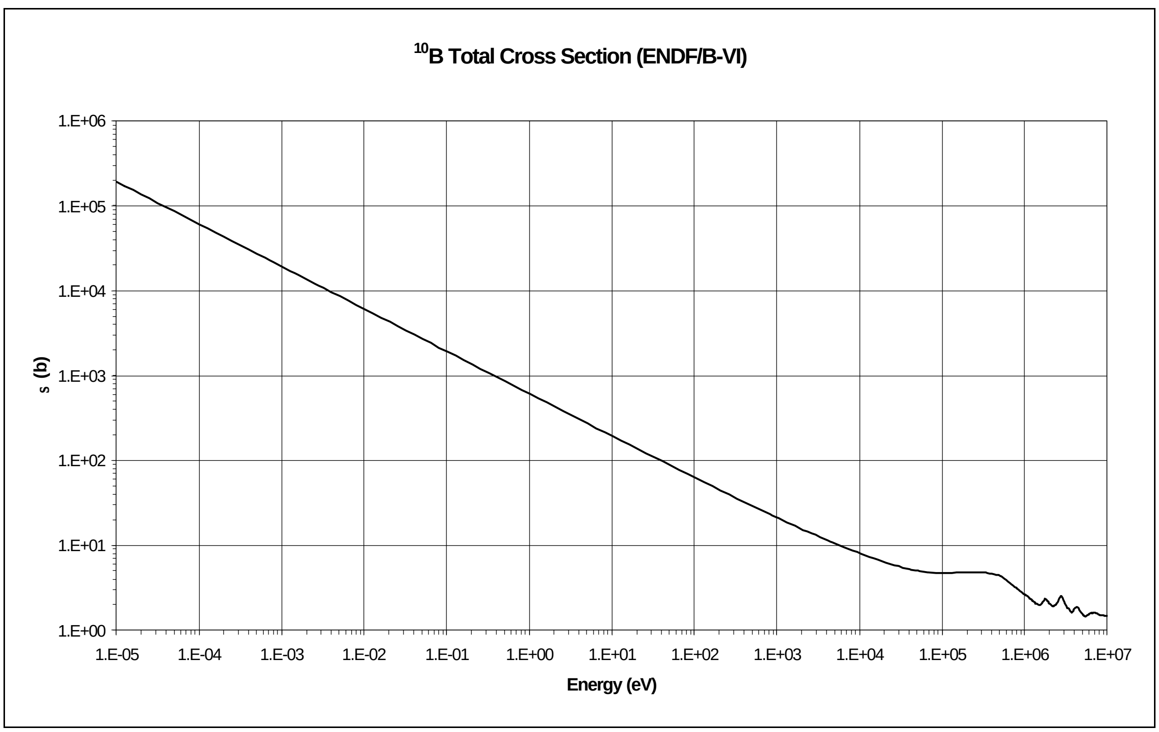

Figure 12. ^{10}B Total Cross Section

PROBLEMS

1. On Feb. 12, 1957 there was an inadvertent criticality at the Godiva assembly at Los Alamos National Laboratory. Godiva was a bare ^{235}U metal sphere that went prompt critical and experienced a short power burst which was terminated by disassembly of the core. (See DOE/NCT-04, "A Review of Criticality Accidents" by W. R. Stratton.) The assembly was remotely controlled and no one was injured. The critical mass was about 54 kg uranium metal enriched to 93.7% ^{235}U. The burst had a width of approximately 100 μs and the estimated yield was 1.2 x 10^{17} fissions. This was a fast neutron spectrum burst. Assume a fission cross section for ^{235}U of 1.3 barns at 2 MeV. For this burst, estimate the fast neutron fluence and flux density, the number and fraction of the ^{235}U atoms burned in the excursion, the decrease in mass of the assembly from the conversion of mass to energy and the energy release in units of watt-seconds and pounds of high explosive material equivalent (4.18 x 10^9 joules/ton TNT).

2. Many neutron absorption cross sections in the low energy range vary as the reciprocal of the neutron speed, 1/v, that is the cross section can be expressed as

$$\sigma_a(E) = \frac{\sigma_0}{v} = \frac{\sigma_0}{\sqrt{E}}$$

The absorption cross section for boron is noted to be 1/v over a wide range. Read some values from the ^{10}B cross section curve and satisfy yourself that this is true. Draw a 1/v line on the ^{235}U and the ^{239}Pu curves in the low energy region. Are these 1/v absorbers? Why don't the ^{238}U and C curves have a 1/v shape?

PROBLEM SOLUTIONS

1. From page 3, the fission rate per unit volume of target is given by the product of the neutron flux, ϕ, and the macroscopic fission cross section, Σ, where

$$\Sigma\ (cm^{-1}) = \sigma\ (cm^2) \times N\ (atoms/cm^3)$$

The total fission rate is calculated by multiplying the per-volume rate by the volume of the assembly (or total number of atoms). This is also the total number of fissions in the burst divided by the length of the burst.

Given that

$F_{tot} = 1.2 \times 10^{17}$ fissions	m(U) = 54 kg
$t = 1.0 \times 10^{-4}$ s	^{235}U wt. fraction = 0.937
$\sigma = 1.3$ barns	

and assuming that

$A_{235} = 235.0439$ g	$N_a = 6.0221 \times 10^{23}$ at/mole

the total number of ^{235}U atoms in the assembly is

$$N_{235} = (N_a / A_{235}) \times m(U) \times wt.fr.(^{235}U)$$

$$= 1.2964 \times 10^{26} \text{ atoms } (^{235}U)$$

The neutron flux is then

$$\phi = (F_{tot} / t) / (N_{235} \times \sigma)$$

$$= 7.12 \times 10^{18}\ n/cm^2\text{-}s$$

The fluence, Φ, is given by the product of the time, t, and the flux, ϕ, or

$$\Phi = 7.12 \times 10^{14}\ n/cm^2$$

The fraction of ^{235}U atoms destroyed in the burst is the ratio of the number of fissions to the total number of atoms initially present,

$$(F_{tot} / N_{235}) = 9.257 \times 10^{-10}$$

a really small fraction of the initial material.

The most obvious method to calculate the decrease in mass is to multiply the mass of each uranium atom times the number of fissions.

$$F_{tot} = (N_a / A_{235}) \times m_{fiss}$$

or

$$m_{fiss} = 4.68 \times 10^{-5} \text{ g}$$

However, this is the decrease in the ^{235}U mass, not the change in the mass of the assembly. The mass lost per fission is the mass equivalent of the energy released per fission. The value of 200 MeV per fission that is often used in calculations is actually the recoverable energy, which is of interest for reactors. A check of any nuclear engineering text (e.g., *Nuclear Reactor Theory* by Lamarsh) shows that the total energy released per fission is approximately 207 MeV. The mass equivalent of this energy for the entire burst is

$$\begin{aligned}\Delta m &= 1.2 \times 10^{17} \text{ fiss} * 207 \text{ Mev/fiss} \\ &= 2.48 \times 10^{19} \text{ MeV}\end{aligned}$$

Using the conversion factor of 1.783×10^{-30} kg/MeV, the change in assembly mass is

$$\Delta m = 4.4 \times 10^{-8} \text{ g} .$$

Again using the value of 207 MeV of total energy released per fission, this excursion released approximately

$$\begin{aligned}E &= (1.2 \times 10^{17} \text{ fissions}) \times 207 \text{ MeV/fission} \\ &= 2.48 \times 10^{19} \text{ MeV} \\ &= 3.97 \times 10^{6} \text{ joules} \\ &= 1.9 \text{ lb(TNT)}\end{aligned}$$

using the conversion factors of 1.60×10^{-13} j/MeV and 4.18×10^{9} joules per ton of TNT.

2. These cross sections are plotted on log-log scales. On log-log plots of cross section versus energy, a cross section proportional to the reciprocal of the square root of the energy should be a straight line with slope of -1/2. The cross section will decrease one order of magnitude for each two-order decrease in energy. The ^{10}B cross section is approximately 2000 barns at 10^{-5} eV. It is approximately 200 barns at 10^{-3} eV, 20 barns at 10^{-1} eV, and so forth indicating that it appears to behave as 1/v in the energy range from 10^{-5} eV to 10^{4} eV.

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The ^{235}U fission cross section is approximately 1000 barns at 10^{-2} eV. If it behaved as 1/v it would be about 100 barns at about 1 eV. The cross section at 1 eV is below this 1/v shape. There is resonance structure in the low energy range which alters the shape from 1/v. In a similar manner the ^{239}Pu fission cross section has a large resonance at about 3 eV which causes the cross section to deviate from the 1/v shape.

The ^{238}U and C curves do not evidence a 1/v shape because the plots show the total cross sections which, for these nuclides, are dominated by scattering, not absorption. Scattering cross sections are generally constant with energy, not 1/v.