

Technical Program Review  
**Nuclear Criticality Safety Program**  
March 26 – 27, 2019 • Amarillo, TX, USA

---

**Progress of Thermal Scattering Law  
Development and Evaluations  
at North Carolina State University**

**Cole Manring, Colby Sorrell, Ben Laramée  
Ayman I. Hawari**

**Nuclear Reactor Program  
Department of Nuclear Engineering  
North Carolina State University  
Raleigh, North Carolina, USA**

# Acknowledgement

---

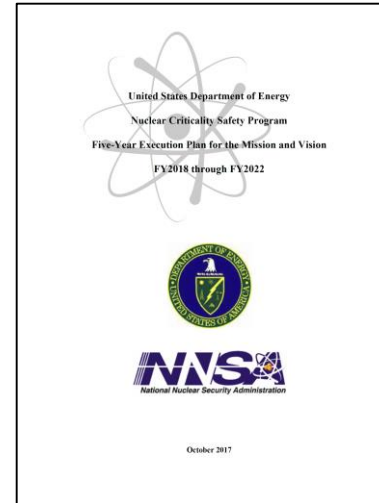
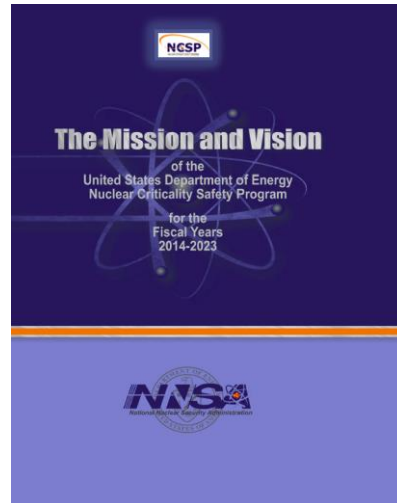
- ❑ The many graduate students, postdocs, and research staff at North Carolina State University
  
- ❑ Collaboration with LLNL and Bettis labs
  - David Heinrichs, Michael Zerkle, Jesse Holmes
  
- ❑ Funding
  - US NNSA Nuclear Criticality Safety program
  - US Naval Nuclear Propulsion Program

# FY 2018 / 2019

---

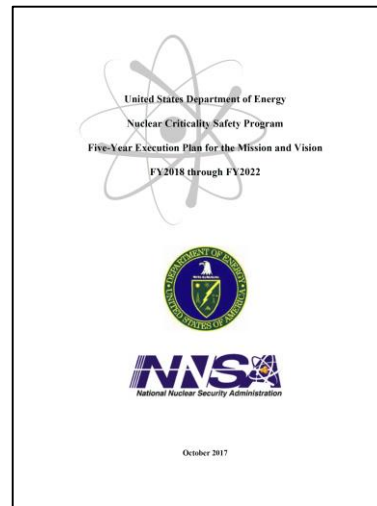
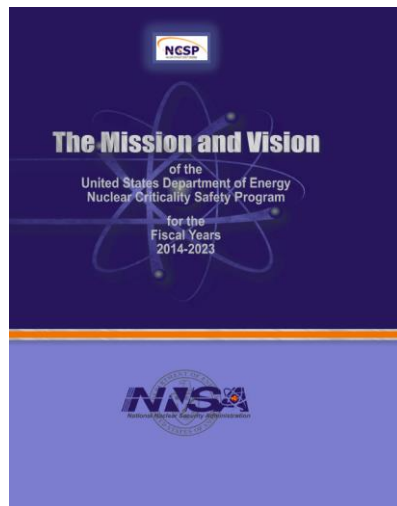
- ❑ 4 new TSL evaluations
  - 3 first-of-a-kind evaluations
  
- ❑ Modern predictive methods for thermal neutron cross section calculations based on the use of atomistic simulations
  - Ab initio lattice dynamics
  - Molecular dynamics (ab initio and classical)
    - ❑ New materials
    - ❑ All states of matter (solid, liquid, gas)
    - ❑ Imperfect structure
  
- ❑ Expanding the *FLASSH* thermal scattering analysis platform
  - Removed approximations such as incoherent approximation, cubic approximation, atom site approximation and SCT approximation
  
- ❑ Initiated the integration of the Generalized Nuclear Data Structure (GNDS) format into *FLASSH*

# Objectives



<b>Priority Needs */ Additional Needs</b>	Thermal scattering (Paraffinic Oil, HF, Silicone Oil, $\text{UO}_2\text{F}_2$ , $\text{PuH}_2$ , $\text{UH}_3$ , Paraffin, $\text{U}_3\text{O}_8$ , $\text{U}_3\text{Si}_2$ , UC, $\text{PuO}_2$ , etc.), $^{239}\text{Pu}$ , Fe, Cr, $^{237}\text{Np}$ , Pb, $^{55}\text{Mn}$ , Ti, $^{240}\text{Pu}$ / $^{233}\text{U}$ , Th, Be, $^{51}\text{V}$ , Zr, F, K, Ca, Mo, Na, La
<b>Completed Evaluations (FY)</b>	Minor Actinides (13), SiC(17), $\text{SiO}_2$ (17), $\text{C}_5\text{O}_2\text{H}_8$ (16), $\text{CH}_2$ (17), Be (17), BeO (17), Graphite (17), $\text{UO}_2$ (17), UN (17), $^{55}\text{Mn}$ (12), $^{58,60}\text{Ni}$ (14), $^{180,128,183,184,186}\text{W}$ (14), Ca (16), $^{59}\text{Co}$ (17), $^{63,65}\text{Cu}$ (17)

# Objectives



<p><b>Priority Needs */ Additional Needs</b></p>	<p>Thermal scattering (Paraffin, <del>Oil</del>, <del>HF</del> Silicone Oil, UO<sub>2</sub>F<sub>2</sub>, PuH<sub>2</sub>, UH<sub>3</sub>, Paraffin, U<sub>3</sub>O<sub>8</sub>, U<sub>3</sub>Si<sub>2</sub>, UC, PuO<sub>2</sub>, etc.), <sup>239</sup>Pu, Fe, Cr, <sup>237</sup>Np, Pb, <sup>55</sup>Mn, Ti, <sup>240</sup>Pu/<sup>233</sup>U, Th, <del>U</del>, <sup>51</sup>V, Zr, F, K, Ca, Mo, Na, La</p>
<p><b>Completed Evaluations (FY)</b></p>	<p>Minor Actinides (13), S<del>16</del> (17), Si<del>16</del> (17), C<del>16</del> (16), CH<del>16</del> (17), Be<del>17</del> (17), B<del>17</del> (17), Graduate (17), U<del>16</del> (17), U<del>16</del> (17), <sup>55</sup>Mn (12), <sup>58,60</sup>Ni (14), <sup>180,128,183,184,186</sup>W (14), Ca (16), <sup>59</sup>Co (17), <sup>63,65</sup>Cu(17)</p>

Completed light water (H<sub>2</sub>O)

Finalizing molten salt FLiBe

# Neutron Thermalization

Using first Born approximation combined with Fermi pseudopotential, it can be shown that the double differential scattering cross section has the form

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{1}{4\pi} \sqrt{\frac{E'}{E}} \left\{ \sigma_{coh} S(\vec{k}, \omega) + \sigma_{incoh} S_s(\vec{k}, \omega) \right\}$$

The scattering law  $S(\vec{k}, \omega)$  is composed of two parts

$$S(\vec{k}, \omega) = S_s(\vec{k}, \omega) + S_d(\vec{k}, \omega)$$

Van Hove's space-time formulation

$$I(\vec{k}, t) = \int G(\vec{r}, t) \exp(i\vec{k} \cdot \vec{r}) d\vec{r}$$

$$S(\vec{k}, \omega) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(\vec{r}, t) e^{i(\vec{k} \cdot \vec{r} - \omega t)} d\vec{r} dt$$

where  $G(\vec{r}, t)$  is the *dynamic pair correlation function* and can be expressed in terms of time dependent atomic positions.

$$S_s(\alpha, \beta) = k_B T \cdot S_s(\vec{k}, \omega)$$

$$\left. \frac{d^2 \sigma}{d\Omega dE'} \right|_{inelastic} = \frac{\sigma}{2k_B T} \sqrt{\frac{E'}{E}} S_s(\alpha, \beta)$$

Since 1960s  
**GASKET**  
**NJOY/LEAPR**  
**INCOHERENT**  
**APPROXIMATION**

$$\beta = \frac{E - E'}{k_B T} \quad \text{Energy transfer}$$

$$\alpha = \frac{(E + E' - 2\sqrt{EE'} \cos \theta)}{k_B T} \quad \text{Momentum transfer}$$

The scattering law (TSL) is the Fourier transform of a Gaussian correlation function

$$S_s(\alpha, \beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\beta t} e^{-\gamma(t)} dt$$

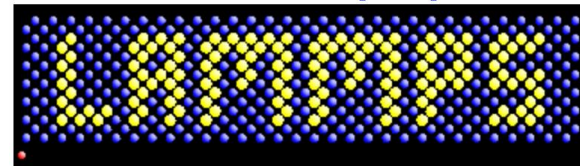
$$\gamma(t) = \frac{\alpha}{2} \int_{-\infty}^{\infty} \frac{\rho(\beta)}{\beta \sinh(\beta/2)} [1 - e^{-i\beta t}] e^{\beta/2} d\beta$$

$\rho(\beta)$  – density of states (e.g., phonon frequency distribution)

# Thermal Scattering Law Analysis

---

- Key development in the last 20 years is the use of atomistic simulations methods to support the evaluation process
  - Produce data necessary to calculate the TSL including
    - DOS for evaluation of TSL
    - Direct access to TSL using correlation analysis





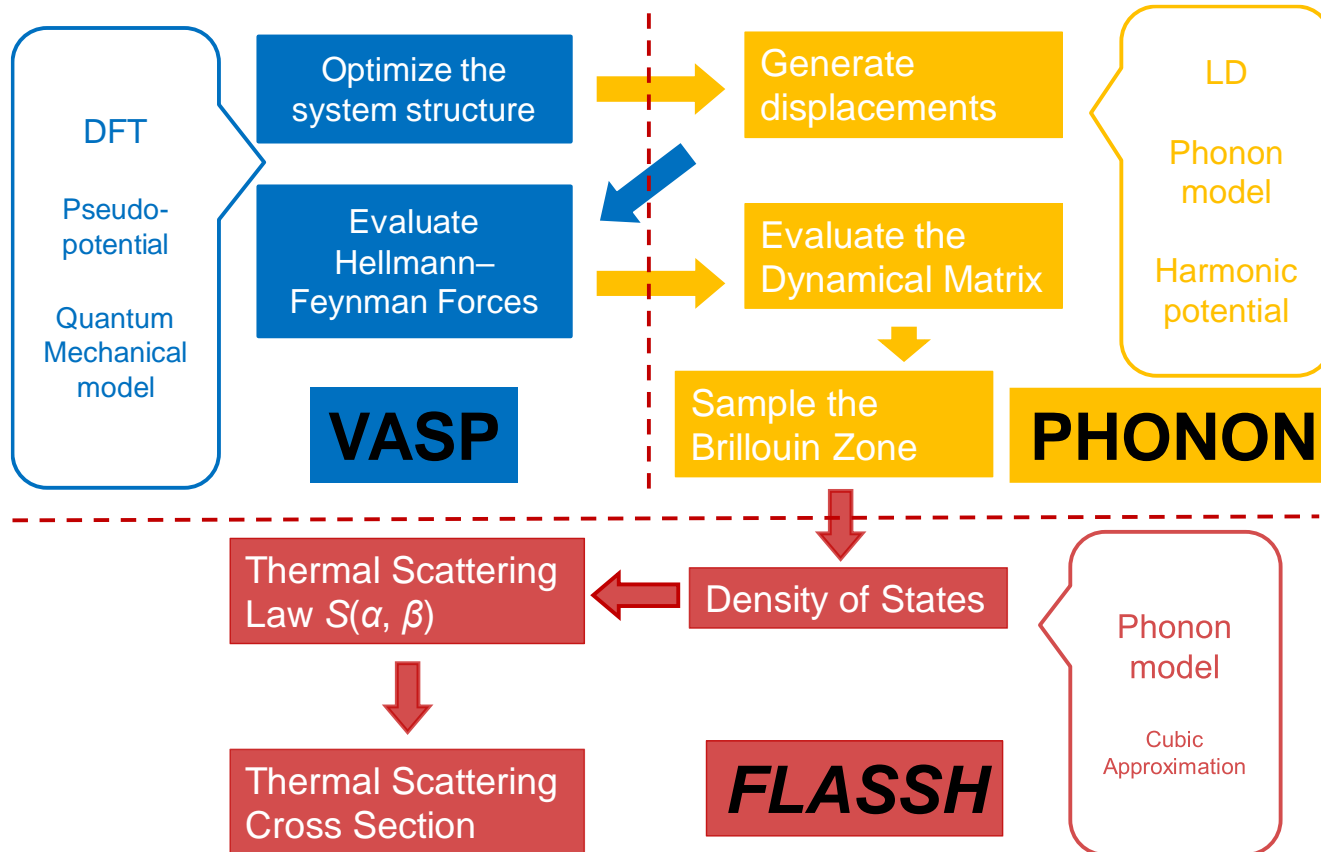
# ENDF/B-VIII TSL Evaluations

Material	ENDF Library Name	Evaluation Basis	Institution
<b>Beryllium metal</b>	tsl-Be-metal.endf	DFT/LD	NCSU
<b>Beryllium oxide (beryllium)</b>	tsl-BeinBeO.endf	DFT/LD	NCSU
<b>Beryllium oxide (oxygen)</b>	tsl-OinBeO.endf	DFT/LD	NCSU
<b>Light water (hydrogen)</b>	tsl-HinH2O.endf	MD	CAB
<b>Light water ice (hydrogen)</b>	tsl-HinIceIh.endf	DFT/LD	BAPL
<b>Light water ice (oxygen)</b>	tsl-OinIceIh.endf	DFT/LD	BAPL
<b>Heavy water (deuterium)</b>	tsl-DinD2O.endf	MD	CAB
<b>Heavy water (oxygen)</b>	tsl-OinD2O.endf	MD	CAB
<b>Polymethyl Methacrylate (Lucite)</b>	tsl-HinC5O2H8.endf	MD	NCSU
<b>Polyethylene</b>	tsl-HinCH2.endf	MD	NCSU
<b>Crystalline graphite</b>	tsl-graphite.endf	MD	NCSU
<b>Reactor graphite (10% porosity)</b>	tsl-reactor-graphite-10P.endf	MD	NCSU
<b>Reactor graphite (30% porosity)</b>	tsl-reactor-graphite-30P.endf	MD	NCSU
<b>Silicon carbide (silicon)</b>	tsl-CinSiC.endf	DFT/LD	NCSU
<b>Silicon carbide (carbon)</b>	tsl-SiinSiC.endf	DFT/LD	NCSU
<b>Silicon dioxide (alpha phase)</b>	tsl-SiO2-alpha.endf	DFT/LD	NCSU
<b>Silicon dioxide (beta phase)</b>	tsl-SiO2-beta.endf	DFT/LD	NCSU
<b>Yttrium hydride (hydrogen)</b>	tsl-HinYH2.endf	DFT/LD	BAPL
<b>Yttrium hydride (yttrium)</b>	tsl-YinYH2.endf	DFT/LD	BAPL
<b>Uranium dioxide (oxygen)</b>	tsl-OinUO2.endf	DFT/LD	NCSU
<b>Uranium dioxide (uranium)</b>	tsl-UinUO2.endf	DFT/LD	NCSU
<b>Uranium nitride (nitrogen)</b>	tsl-NinUN.endf	DFT/LD	NCSU
<b>Uranium nitride (uranium)</b>	tsl-UinUN.endf	DFT/LD	NCSU

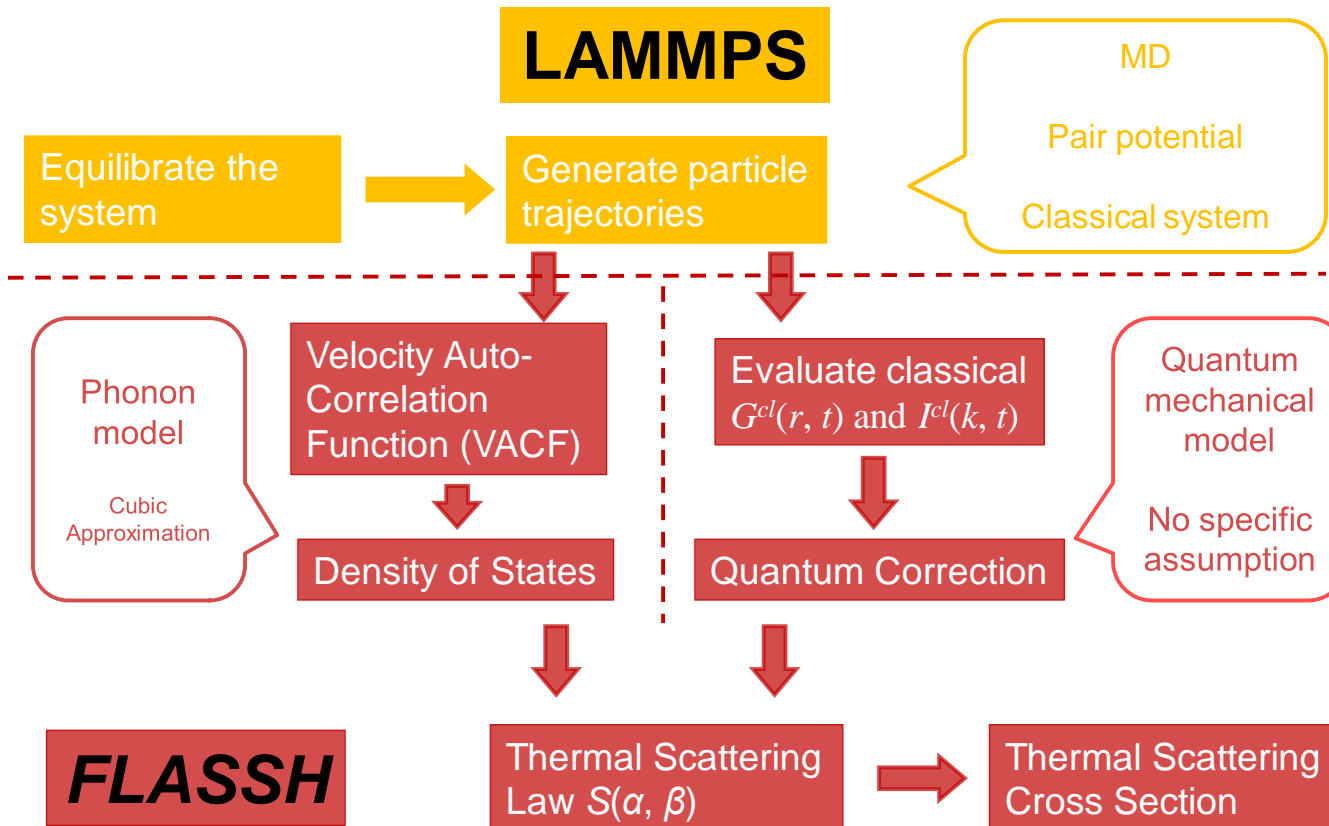
# ENDF/B-VIII TSL Evaluations

Material	ENDF Library Name	Evaluation Basis	Institution
<b>Beryllium metal</b>	tsl-Be-metal.endf	DFT/LD	NCSU
<b>Beryllium oxide (beryllium)</b>	tsl-BeinBeO.endf	DFT/LD	NCSU
<b>Beryllium oxide (oxygen)</b>	tsl-OinBeO.endf	DFT/LD	NCSU
<b>Light water (hydrogen)</b>	tsl-HinH2O.endf	MD	CAB
<b>Light water ice (hydrogen)</b>	tsl-HinIceIh.endf	DFT/LD	BAPL
<b>Light water ice (oxygen)</b>	tsl-OinIceIh.endf	DFT/LD	BAPL
<b>Heavy water (deuterium)</b>	tsl-DinD2O.endf	MD	CAB
<b>Heavy water (oxygen)</b>	tsl-OinD2O.endf	MD	CAB
<b>Polymethyl Methacrylate (Lucite)</b>	tsl-HinC5O2H8.endf	MD	NCSU
<b>Polyethylene</b>	tsl-HinCH2.endf	MD	NCSU
<b>Crystalline graphite</b>	tsl-graphite.endf	MD	NCSU
<b>Reactor graphite (10% porosity)</b>	tsl-reactor-graphite-10P.endf	MD	NCSU
<b>Reactor graphite (30% porosity)</b>	tsl-reactor-graphite-30P.endf	MD	NCSU
<b>Silicon carbide (silicon)</b>	tsl-CinSiC.endf	DFT/LD	NCSU
<b>Silicon carbide (carbon)</b>	tsl-SiinSiC.endf	DFT/LD	NCSU
<b>Silicon dioxide (alpha phase)</b>	tsl-SiO2-alpha.endf	DFT/LD	NCSU
<b>Silicon dioxide (beta phase)</b>	tsl-SiO2-beta.endf	DFT/LD	NCSU
<b>Yttrium hydride (hydrogen)</b>	tsl-HinYH2.endf	DFT/LD	BAPL
<b>Yttrium hydride (yttrium)</b>	tsl-YinYH2.endf	DFT/LD	BAPL
<b>Uranium dioxide (oxygen)</b>	tsl-OinUO2.endf	DFT/LD	NCSU
<b>Uranium dioxide (uranium)</b>	tsl-UinUO2.endf	DFT/LD	NCSU
<b>Uranium nitride (nitrogen)</b>	tsl-NinUN.endf	DFT/LD	NCSU
<b>Uranium nitride (uranium)</b>	tsl-UinUN.endf	DFT/LD	NCSU

# Thermal Scattering Cross-Sections Evaluation DFT/LD



# Thermal Scattering Cross-Sections Evaluation MD/QM

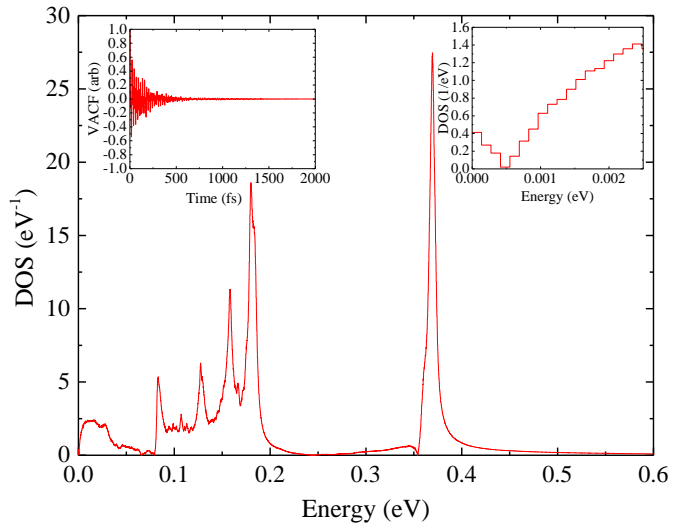
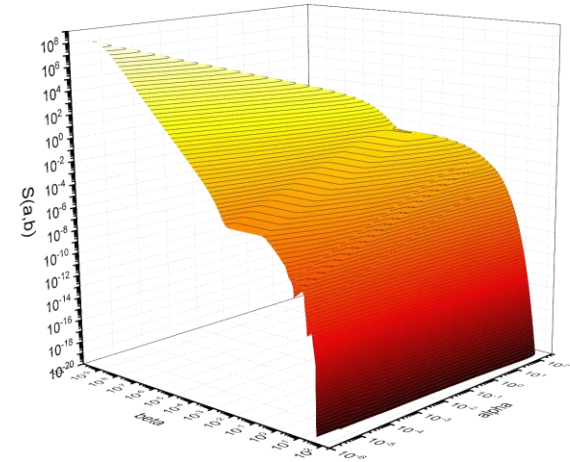
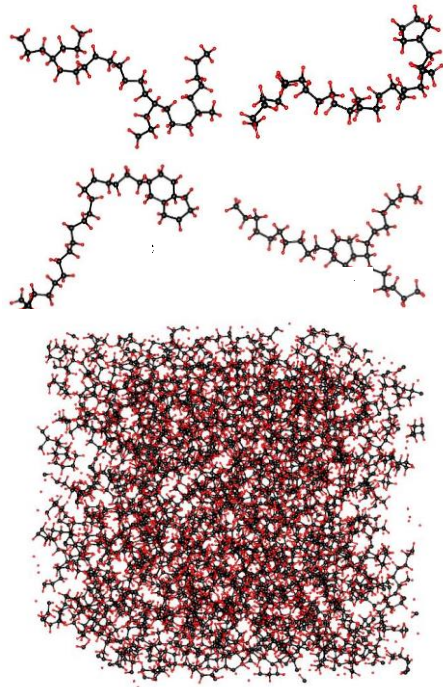


# Computational Capabilities

- ❑ Hybrid mini cluster - 17 nodes
  - 324 CPU cores
  - 22 Nvidia GPUs
  - Expanding.....
  
- ❑ Parallel computations
  - Atomistic simulations
  - TSL analysis
  - Neutronic simulations
  - System design
  
- ❑ VASP, PHONON, LAMMPS  
PREPRO, NJOY, FUDGE,  
SAMMY, MCNP, Serpent,  
GEANT4, McStas, PARET,  
RELAP, COMSOL



# Heavy Paraffinic Oil



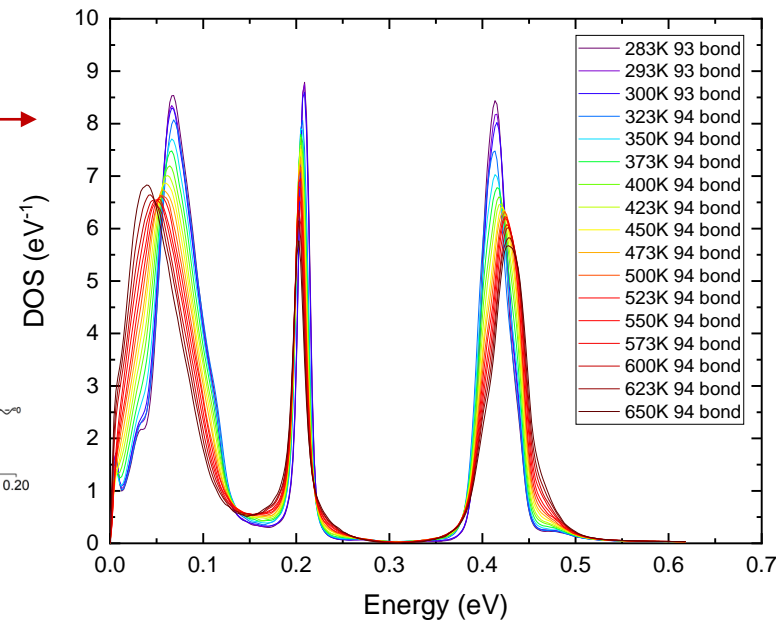
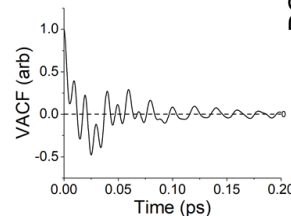
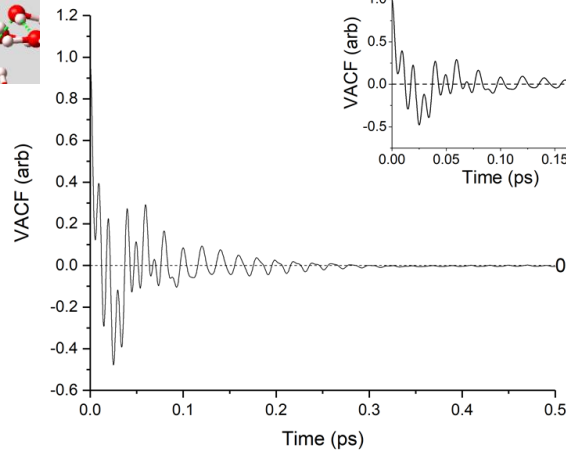
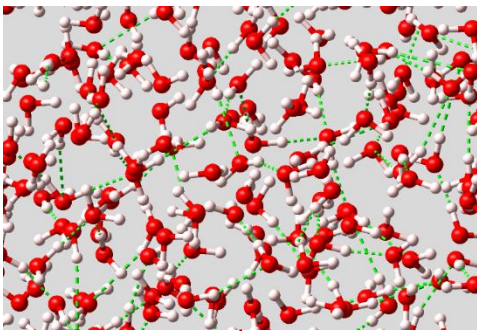
```

0.990000+2 3.491673-1 -1 0 0 0 0 38 1451 1
0.000000+0 0.000000+0 0 0 0 0 6 38 1451 2
3.000000+0 5.000000+0 0 0 12 8 38 1451 3
0.000000+0 0.000000+0 0 0 34 2 38 1451 4
R(Paraffinic Oil) LEIP 138 EVAL-SEP17 C.A. Manning, A.I. Hawari 38 1451 5
-----DIET----- 38 1451 6
---ENDF/B-VIII.1 MATERIAL 38 38 1451 7
----THERMAL NEUTRON SCATTERING DATA 38 1451 8
----ENDF-6 FORMAT 38 1451 9
Temperatures = 300 K 38 1451 10
Background 38 1451 11
38 1451 12
38 1451 13
38 1451 14
38 1451 15
This library was produced by the Low Energy Interactions Physics 38 1451 16
(LEIP) group at North Carolina State University. A single 38 1451 17
temperature is available in this library. The inelastic 38 1451 18
scattering cross section libraries for H in paraffinic oil 38 1451 19
were developed using molecular dynamics methods [1]. This 38 1451 20
material is a viscous liquid polymer, such that the diffusion 38 1451 21
contribution to the TSH is not expected to be large. The 38 1451 22
diffusive behavior of the TSH for the current evaluation was 38 1451 23
approximated as a langvin fluid of oil molecules with an average 38 1451 24
molecular mass of 421 amu. Due to the fact that the quasi- 38 1451 25
elastic scattering peak is included in the TSH, only MT = 4 is 38 1451 26
tabulated in this file. The LEIP module of the NJOY2016 code 38 1451 27
system was used to produce the MT = 4 data. To avoid confusion 38 1451 28
with other materials, MAT 38 and ZA 138 were used for 38 1451 29
R(Paraffinic Oil). 38 1451 30
References 38 1451 31
----- 38 1451 32
38 1451 33
38 1451 34
1. A.I. Hawari, "Modern Techniques in Inelastic Thermal Neutron 38 1451 35
Scattering Analysis," Nuclear Data Sheets 110 (2014) 172. 38 1451 36
38 1451 37
    
```

# Light Water

## Flexible TIP4P/2005 potential

$$U^{tot} = U^{inter} + U^{intra} = \sum_{i \neq j} 4\epsilon \left[ \left( \frac{\sigma}{r_{O,O_j}} \right)^{12} - \left( \frac{\sigma}{r_{O,O_j}} \right)^6 \right] + \sum_{i \neq j} \frac{q_i q_j}{r_{ij}} + D_r \left\{ 1 - \exp[-\beta(r_{OH} - r_{eq})] \right\}^2 + \frac{1}{2} K_\theta (\theta - \theta_{eq})^2$$

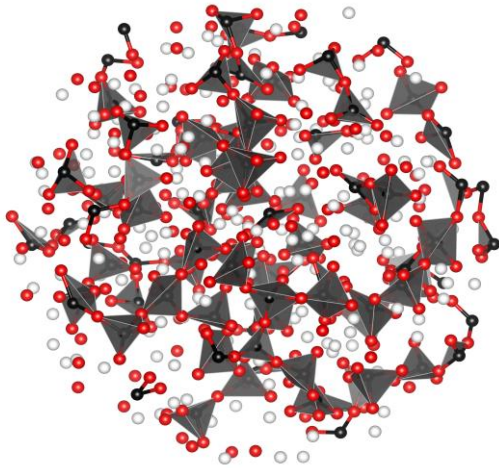








# Liquid FLiBe – FY 2019

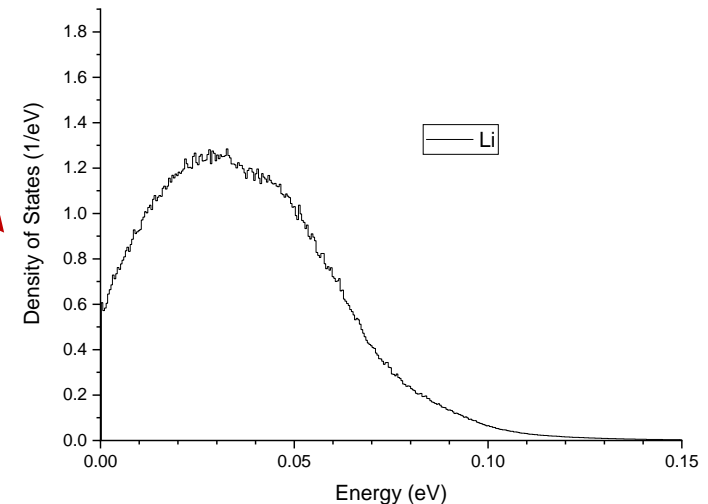


Born-Mayer potential

$$U_{i,j}(r_{i,j}) = U_1 + U_2 + U_3 + U_4$$

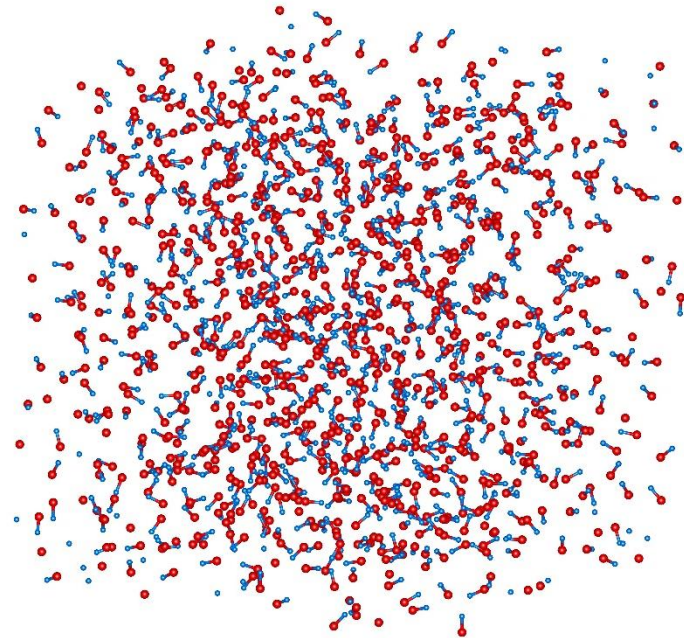
$$U_1 = \frac{Z_i Z_j e^2}{r_{i,j}}, U_2 = -\frac{c_i c_j}{r_{i,j}^6}, U_3 = \frac{D}{r_{i,j}^8}, U_4 = b_{i,j} \left( 1 + \frac{Z_i}{N_i} + \frac{Z_j}{N_j} \right) \exp\left( \frac{r_i + r_j - r_{i,j}}{\rho} \right)$$

- ❑ Eutectic with a mixture of 2:1 ratio of LiF and BeF<sub>2</sub>
- ❑ Melting Point: 732K Boiling Point: 1703K
- ❑ DFT and MD analysis (with QM corrections)
- ❑ TSL evaluation between 750K and 1500K



# Hydrofluoric Acid

- ❑ Strong hydrogen bonding
- ❑ Unique molecular structure dynamics



## Example CMD Potential:

Inter-

Intra-

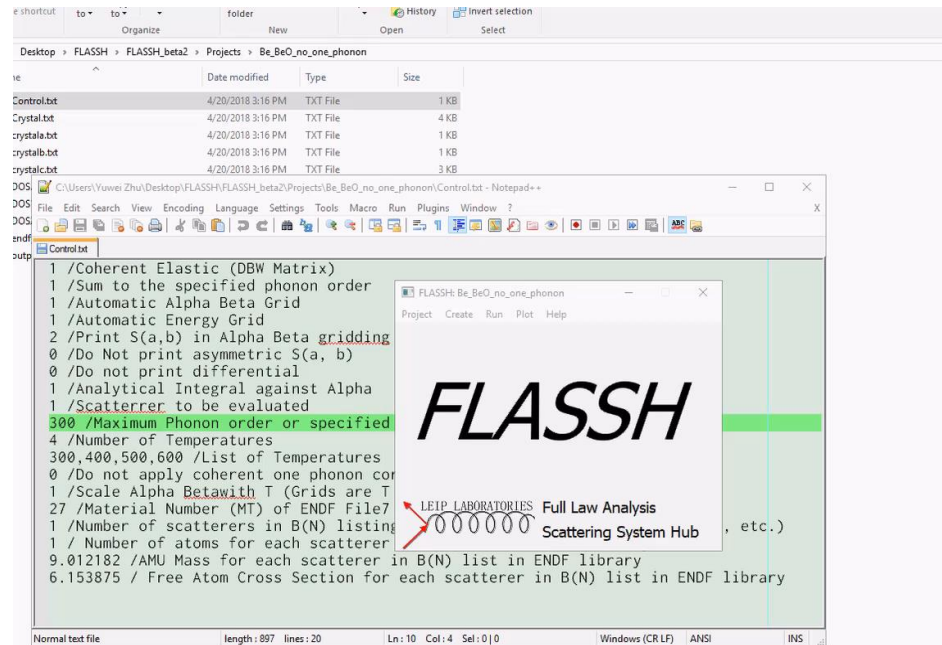
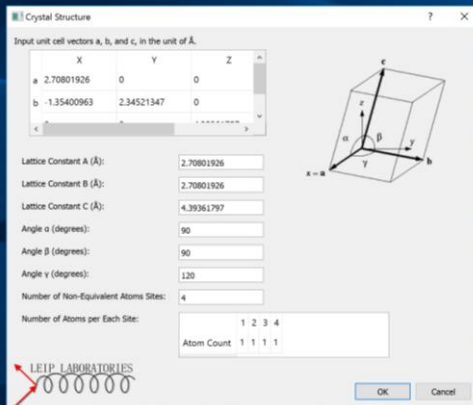
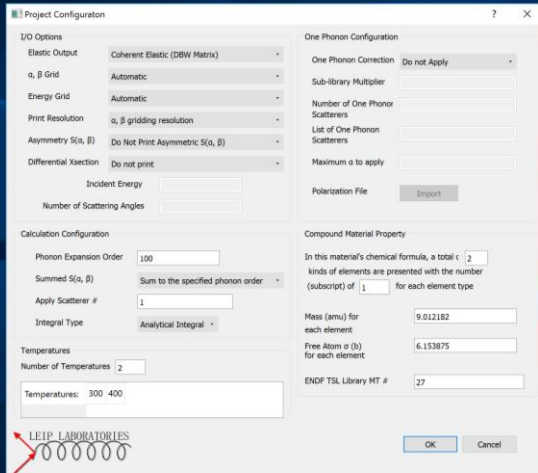
LJ + 3 pt. chg.

Perturbed Morse Osc.

$$V(r) = A \left[ \left( \frac{B}{r} \right)^{12} - \left( \frac{B}{r} \right)^6 \right] + \dots$$

$$V(r) = C \{ \exp[-D(r - r_0)] - 1 \}^2 + E_r$$

# FLASSH Code

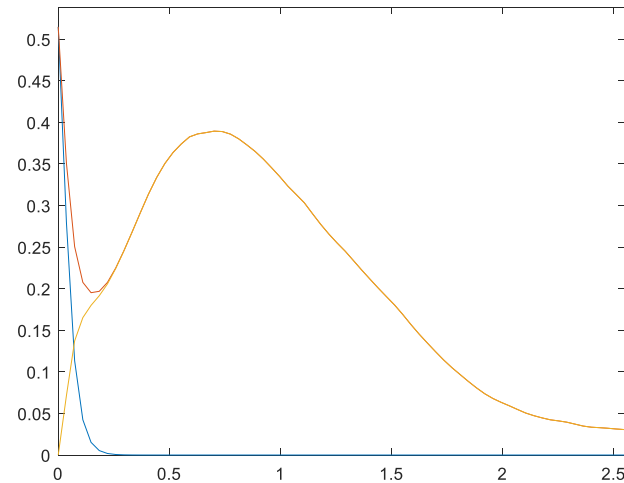


# *FLASH* Code Features

	<b>NJOY</b>	<b>FLASH</b>
Coherent Inelastic	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Remove Incoherent Approximation	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Remove Short Collision Time (SCT) Approximation	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Integral against alpha differential cross section	Numerical	Default: Analytical Optional: Numerical
a, B Gridding	User input	Default: Automatic grid Optional: User input
Parallel Computing	<input type="checkbox"/>	<input checked="" type="checkbox"/> Using OpenMP
Graphite User Interface	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Syntax and Error Checking	<input type="checkbox"/>	<input checked="" type="checkbox"/>
<b>Coherent Elastic Calculation</b>		
Supported Structure	Hexagonal, FCC, BCC	Any crystal structure
Supported Materials	Graphite, Beryllium, Beryllium Oxide, Aluminum, Lead, Iron	Any material
Compound Materials	2 elements with ratio 1:1	Any number of elements with any ratio
Remove Cubic Approximation	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Remove Atom Site Approximation	<input type="checkbox"/>	<input checked="" type="checkbox"/>
Coherent Elastic Scattering Cross Section	Over Ewald Sphere	On every reciprocal lattice point
Need to modify source code if calculating other materials	Yes	No

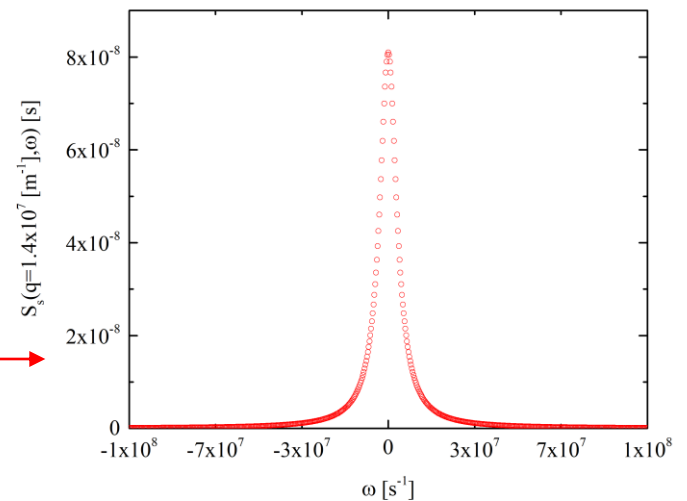
# Liquid Physics in *FLASSH*

□ **Separation** of the diffusive DOS from the continuous (solid) DOS in LEAPR



□ **Convolution** of the solid and liquid TSL components

$$S_{total}(\alpha, \beta) = \left( S_{diff.}(\alpha, \beta) * S_{cont.}(\alpha, \beta) \right) (\beta)$$



# Liquid Physics in *FLASSH*

## Construct fine beta grid (for convolution):

- ❑ Call **convolve\_grid** subroutine to determine appropriate resolution and lower/upper beta limits



## Build liquid TSL model over the new beta grid:

- ❑ Call liquid model function (e.g., **lang** for Langevin)
  - Call **besk1** (Bessel) function if necessary



## Interpolate solid TSL onto new beta grid:

- ❑ Call **interp\_grid** subroutine to interpolate values for every convolution 'window'

## Convolve the liquid and solid components:

- ❑ Call **convolve** subroutine to perform the convolution



## Construct total TSL:

- ❑ Add in extra DW term



## Output results:

- ❑ Write TSL components to various files

# Doppler Broadening

## ▶ Free Gas

- Assumes a Maxwellian velocity distribution

$$\sigma^{FG}(E) = \int_0^{\infty} dE' S^{FG}(E', E) \sigma(E'), \quad S^{FG}(E', E) = \frac{1}{\Delta \sqrt{\pi}} \sqrt{\frac{E'}{E}} \exp\left[-\frac{(E' - E)^2}{\Delta^2}\right]$$

## ▶ Crystal Lattice

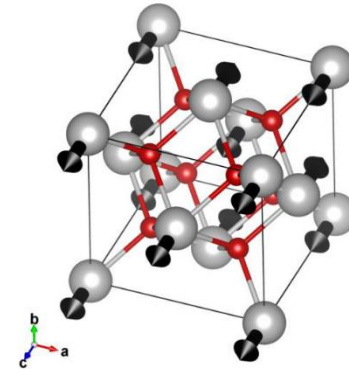
- Compound nucleus effects separated from lattice effects
- Transition probability
- Self Scattering Law
  - Identical to that used in thermal scattering
  - Describes the energy-momentum phase space of a material

$$\sigma(E) = \frac{\sigma_0 \Gamma^2}{4} \int_{-\infty}^{\infty} d\beta \frac{S_s(\alpha, \beta)}{(E - E_0 - \beta k_B T)^2 + (\Gamma/2)^2}$$

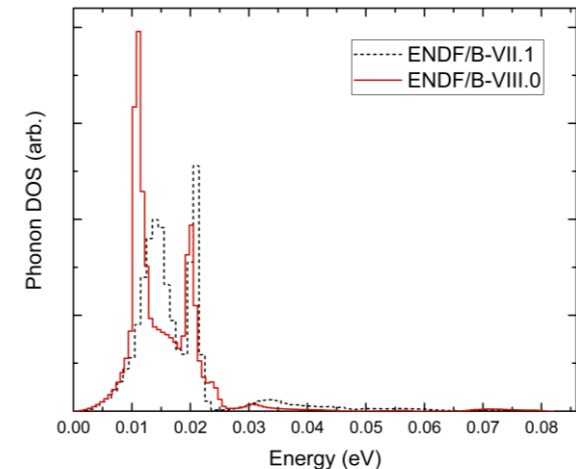
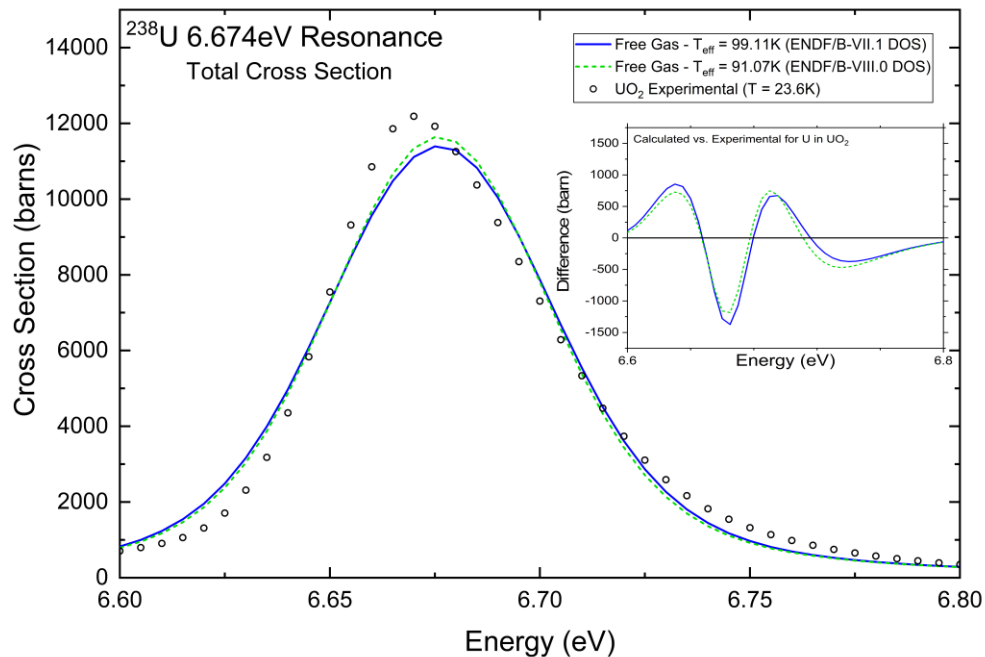
# Doppler Broadening

## ► *Ab initio* lattice dynamics

- Predictive density of states (DOS)
- Current DOS implemented in the ENDF/B-VIII.0 cross section library for U in  $\text{UO}_2$



- Fluorite Structure
- 2x2x2 supercell
- GGA-PBE+U

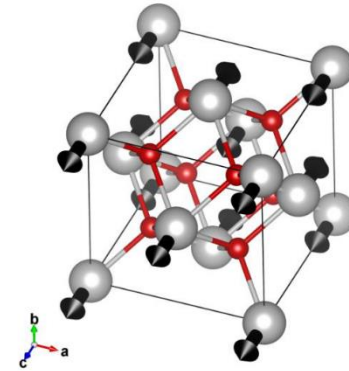




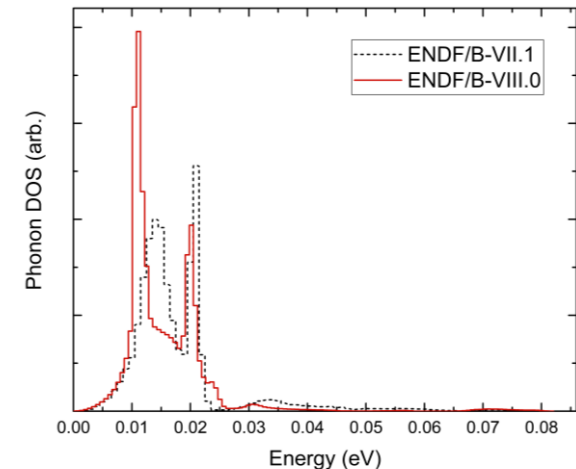
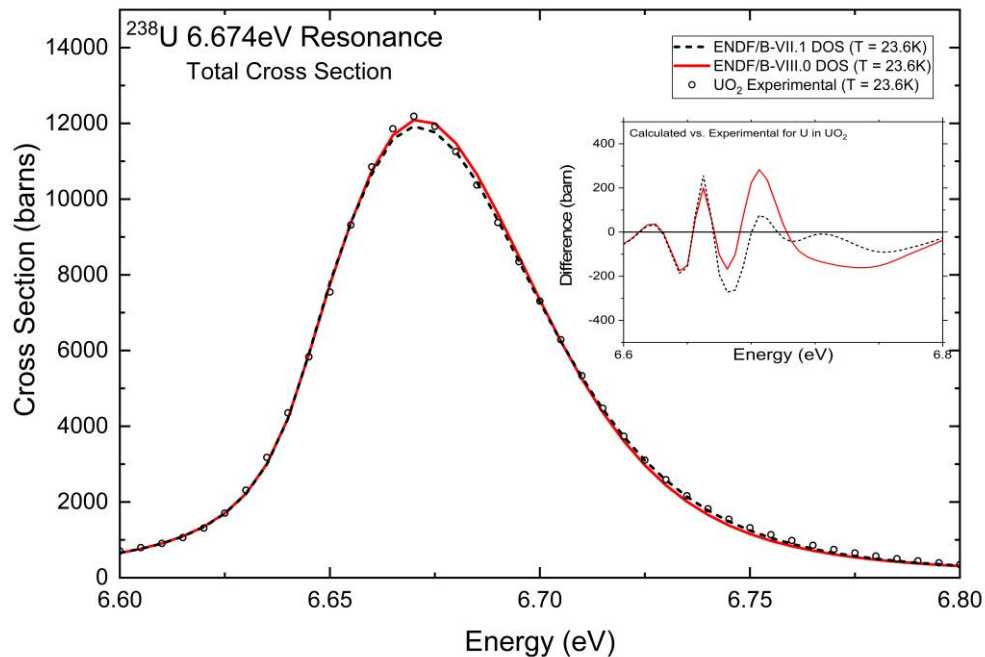
# Doppler Broadening

## ► *Ab initio* lattice dynamics

- Predictive density of states (DOS)
- Current DOS implemented in the ENDF/B-VIII.0 cross section library for U in  $\text{UO}_2$



- Fluorite Structure
- 2x2x2 supercell
- GGA-PBE+U



# FLASSH Generalized TSL

$$S_s(\vec{k}, \omega) = \frac{1}{2\pi\hbar} \int e^{-i\omega t} e^{\langle U^2 \rangle + \langle UV_0 \rangle} dt = e^{-2W} \sum_{n=0}^{\infty} \frac{(2W)^n}{n!} G_n$$

Full Equation

$$\sum_s (\vec{k} \cdot \vec{e}_s)^2 = \frac{1}{3} k^2$$

Cubic Approximation

$$G_1(\omega) = \frac{1}{\gamma(0)} \frac{1}{N} \frac{\hbar}{k_B T} \sum_s \frac{|\vec{e}_k \cdot \vec{e}_s|^2}{2\beta \sinh(\beta/2)} e^{-\beta/2}$$

$$G_1(\omega) = \frac{1}{\lambda} \frac{\rho(\beta)}{2\beta \sinh(\beta/2)} e^{-\beta/2}$$

$$2W = \frac{\hbar}{2MN} \sum_s \frac{|\vec{k} \cdot \vec{e}_s|^2}{\omega_s} \coth\left(\frac{\hbar\omega_s}{2k_B T}\right)$$

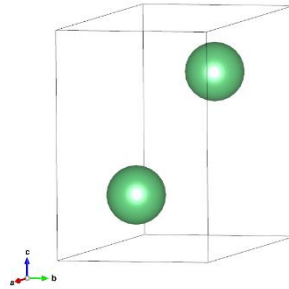
$$2W = \frac{\hbar k^2}{2M} \int_0^{\omega_m} \frac{\rho(\omega)}{\omega} \coth\left(\frac{\hbar\omega_s}{2k_B T}\right) d\omega$$

- ▶ Function of the polarization vector and dispersion relations

- ▶ Function of the density of states  $\rho(\omega)$

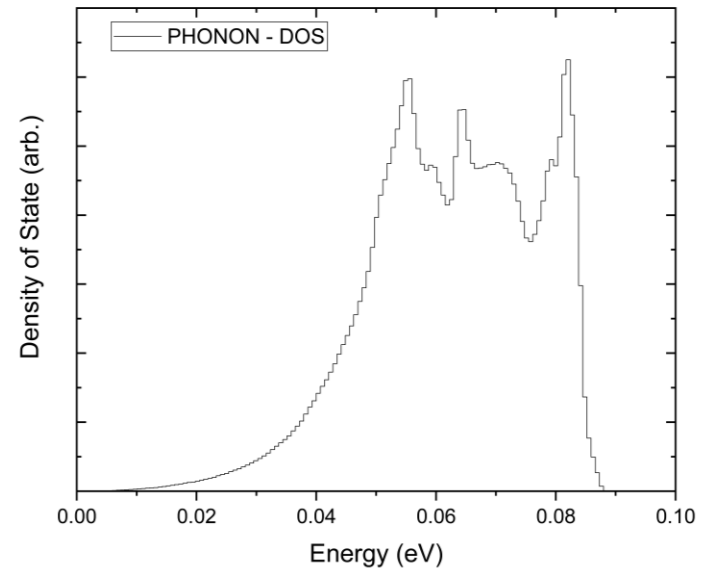
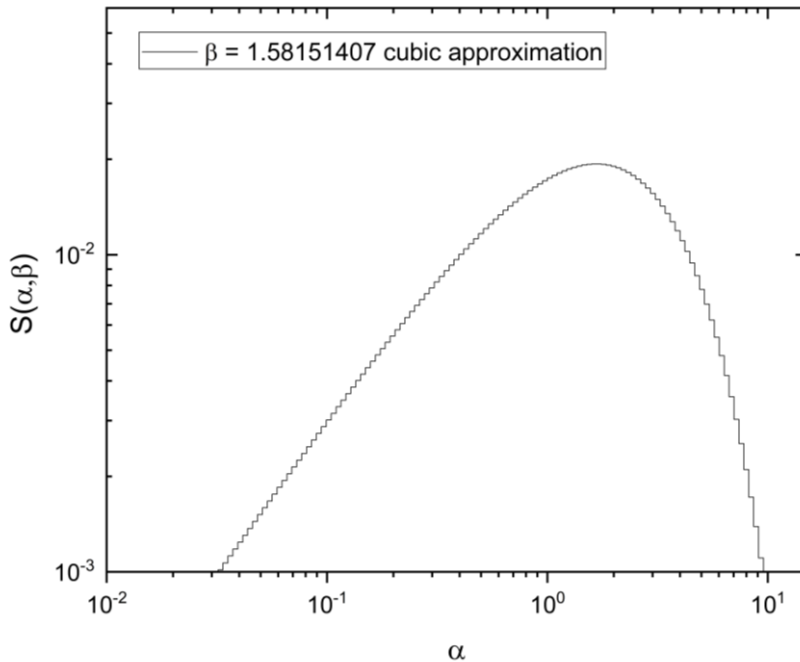
# FLASSH Generalized TSL

- ▶ Beryllium Metal
  - HCP (P63/mmc)
  - *Ab initio* lattice dynamics
  - 4x4x3 Supercell



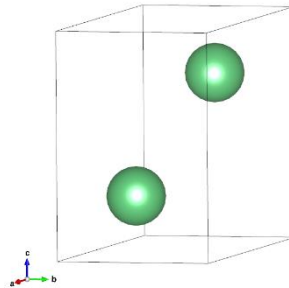
5.49E-3	0	0
0	5.49E-3	0
0	0	5.49E-3

Exact Debye-Waller  
Matrix ( $\text{\AA}^2$ )



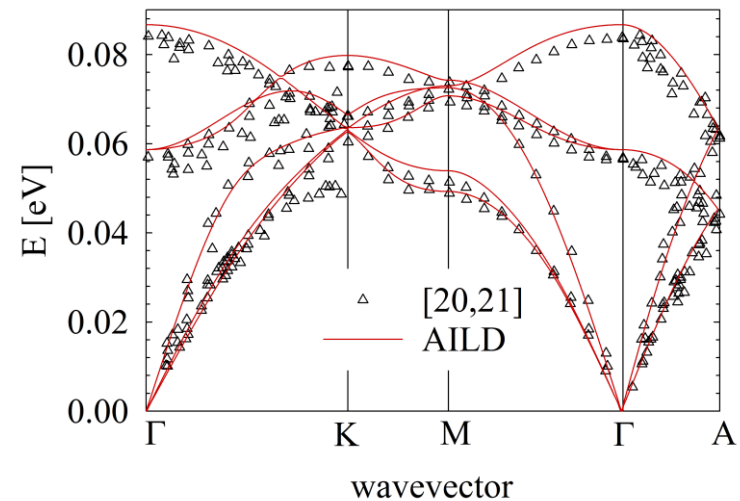
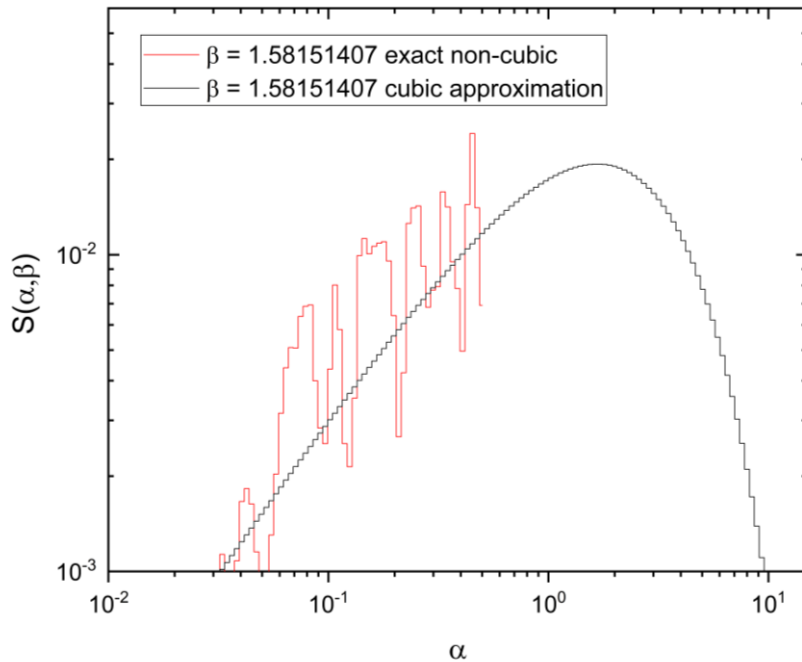
# FLASH Generalized TSL

- ▶ Beryllium Metal
  - HCP (P63/mmc)
  - *Ab initio* lattice dynamics
  - 4x4x3 Supercell



5.76E-3	-1.58E-5	1.01E-6
-1.58E-5	5.74E-3	-5.18E-7
1.01E-6	-5.18E-7	5.02E-3

Exact Debye-Waller  
Matrix ( $\text{\AA}^2$ )





# Summary

---

- ❑ FY 2018
  - 10 new TSL evaluations contributed to ENDF/B-VIII.0
  - 5 first-of-a-kind evaluations
- ❑ FY 2019
  - 4 new TSL evaluations
  - 3 first-of-a-kind evaluations
- ❑ Modern predictive methods for thermal neutron cross section calculations based on the use of atomistic simulations
  - Ab initio lattice dynamics
  - Molecular dynamics (ab initio and classical)
    - ❑ New materials
    - ❑ All states of matter (solid, liquid, gas)
    - ❑ Imperfect structure
- ❑ *FLASSH* is a new thermal scattering analysis platform that uses a generalized theoretical approach for TSL calculations
  - Removed approximations such as incoherent approximation, cubic approximation, atom site approximation and SCT approximation
- ❑ Progress on FY 2019 tasks