

**US DOE
Nuclear Criticality
Safety Program**

**Technical Program
Review**

**Pantex, Amarillo TX
26-27 March, 2019**

LA-UR-19-20984



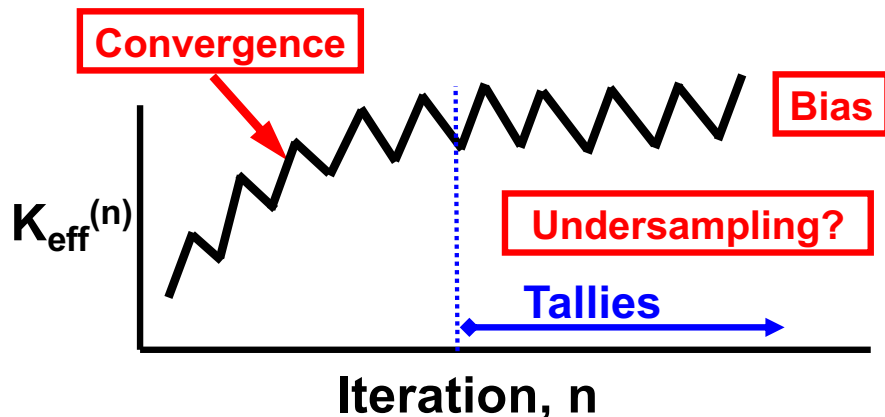
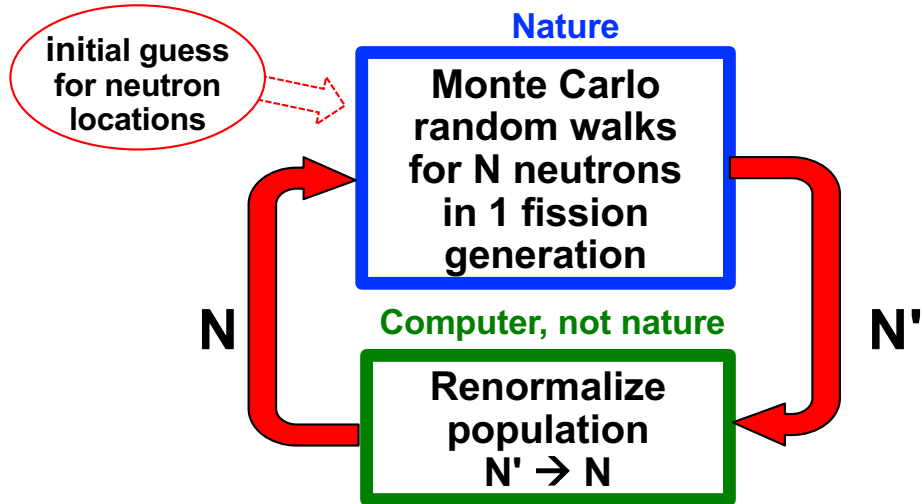
Automated Acceleration & Convergence Testing for Monte Carlo NCS Calculations

Forrest B. Brown

**Senior R&D Scientist
Monte Carlo Methods, Codes, & Applications, LANL**

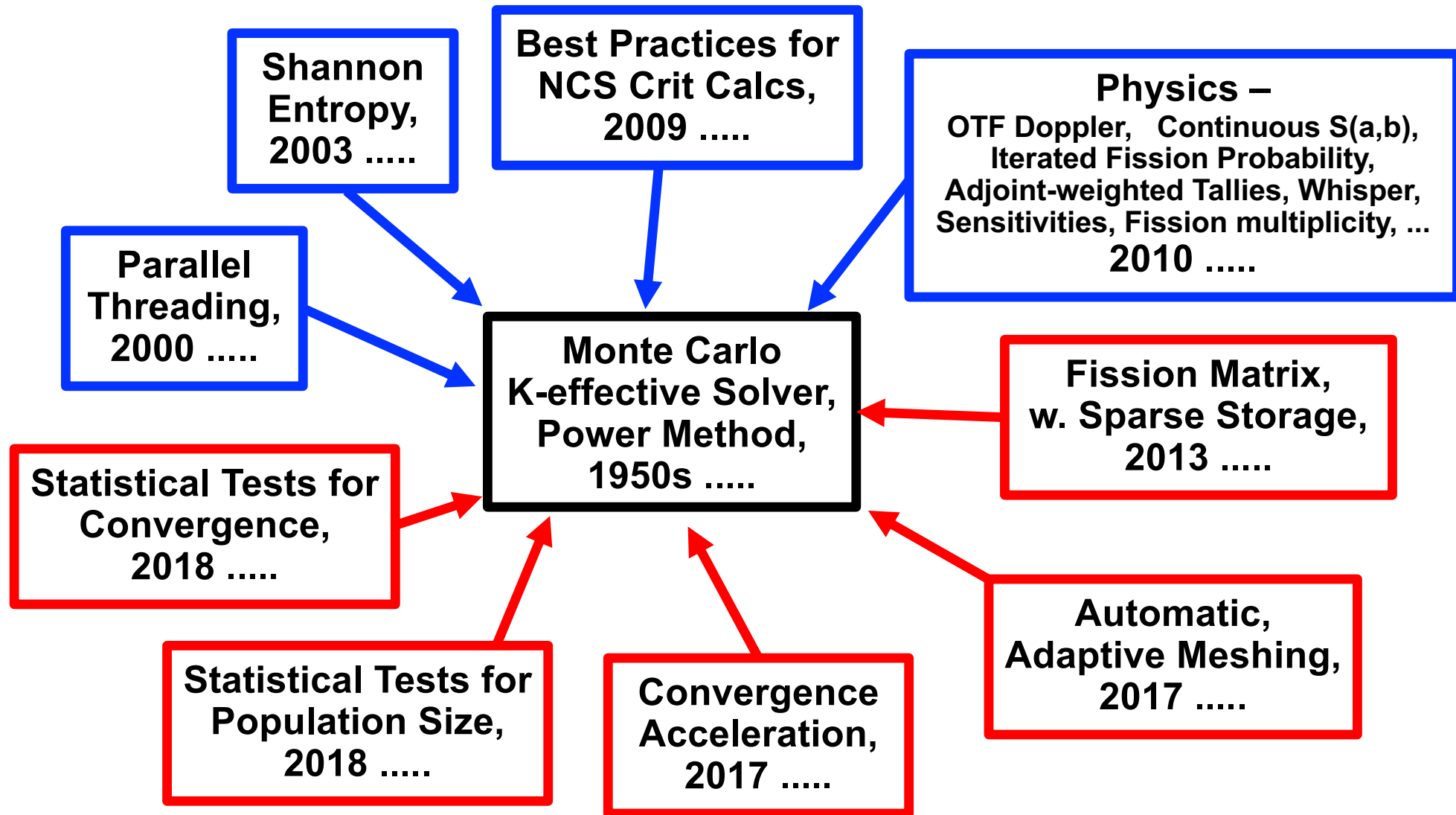
**National Laboratory Professor
Nuclear Engineering Dept., Univ. of New Mexico**

MC Criticality Calculations - Concerns



- **Bias in K_{eff}**
 $\sim -1 / (\text{neutrons/cycle})$
- **Bias in source shape**
 Too low in high-importance regions,
 Too high in low-importance regions
- **Undersampling/clustering**
 Not enough neutrons/cycle to cover space
- **Convergence**
 source shape takes longer than k_{eff}
- **Best Practices**
 Source in all fissile regions.
 Examine H_{src} plot for convergence.
 $>10\text{k}$ neut/cycle ($>100\text{k}$ big probs).
 A few 100 cycles.

LANL R&D for MC Criticality Calculations



This work: Combine & automate the red boxes

Automated acceleration & convergence testing for MC criticality

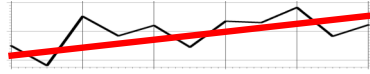
- **Enabling technology, automate & combine new methods**
 - **Automated, adaptive meshing**
 - Basis for Shannon entropy & fission matrix
 - **Fission-matrix with adaptive sparse storage**
 - Reference solution for fission neutron distribution
 - **Accelerate convergence of neutron distribution**
 - **Statistical tests for convergence**
 - 8 tests on metrics, 3 tests on distributions
 - Automatically begin active cycles & tallies
 - **Population size tests**
- **Eliminates the need to run trial calculations, examine Shannon entropy plots, set parameters on KCODE card, & then rerun**
 - **Provides quantitative evidence of convergence**
 - **Enables parameter studies & coupled TH feedback**
 - **Saves significant computer time & people time**

Automated Methods

- **Meshing for convergence tests**
 - **Automatically created & extended if needed, no user input required**
 - Physics basis = L_{fiss} = RMS-distance-to-fission
 - Used for sources, entropy, fission matrix: S_{neut} , H_{src} , $F(I \leftarrow J)$, S_{FM}
- **Cycle 1**
 - Estimate L_{fiss} & set initial mesh
- **Initial cycles**
 - Iterate until S_{neut} & F tallies stabilize
 - Automated, test that (Δ nonzero tallies) < 2%, 5%
- **For blocks of cycles** (default = 10)
 - **Solve F matrix equations** for S_{FM}
 - **Convergence tests**
 - 9 statistical tests must all pass for convergence (also 2 other tests)
 - If not converged, **accelerate source convergence** by importance sampling, weights: $S_{\text{FM}}(m) / S_{\text{neut}}(m)$, $m = \text{bin}$
 - If converged, set active cycles to begin with next cycle, **population size tests**

Statistical tests for convergence

• Slope test



- For a block of cycles (default = 10)
- For result x from each cycle in block, compute least-squares **slope** & σ_{slope}
 - | slope(x) | < 0.0001 → pass, slope ~ 0
 - | slope(x) | < $t_{0.05} \sigma_{\text{slope}}$ → pass, slope ~ 0 within statistics

• Metric tests, at end of block for convergence testing

1. Slope K_{tracklen}
2. Slope K_{collide}
3. Slope K_{absorb}
4. Slope H , Shannon entropy
5. Slope H_X , entropy X marginal
6. Slope H_Y , entropy Y marginal
7. Slope H_Z , entropy Z marginal
8. **H_{block} within 1% of H_{FM}**

If Test 8 passes, strong evidence of convergence
 If Test 8 fails, ignore it – might be low popsize

• Distribution tests, at end of block for convergence testing

9. Kolmogorov-Smirnov test at 95% level, S_{block} & S_{FM} have same distrib.

For multi-D distributions, KS statistic depends on ordering. Take worst case KS statistic for many random permutations.

10. Chi-square 2-point test at 95% level, S_{block} & S_{FM} have same distrib.

11. **Relative entropy (Kullback-Liebler discrepancy) test at 95% level for S_{block} & S_{FM}**

If Test 11 passes, strong evidence of convergence
 If Test 11 fails, ignore it – might be low popsize

• If convergence tests all pass, convergence is locked-in

- Tests continue for each block
- Some tests may fail (due to statistics), but convergence not rescinded

Accelerating Source Convergence

- **At the end of each cycle**
 - S_{FM} is available – source from fission matrix at end-of-block
 - S_{neuts} is available – actual neutron source at end-of-cycle
- **During inactive cycles, can optionally use (S_{FM} / S_{neuts}) for importance sampling of the fission source**
 - Pushes neutron distribution toward F-matrix reference
 - Recomputed each cycle using S_{FM} from previous end-of-block, and S_{neuts} for current end-of-cycle
 - Works – typically reduces inactive cycles by 2-20 X
 - Further development under consideration:
 - Investigate using $S_{FM}^{adjoint}$ for source importance sampling
 - Maybe coarsen the fission matrix, to reduce statistical noise

Statistical tests for Population Size

Performed after convergence, at end of each block of cycles

1. Relative entropy < 0.05 for S_{block} vs S_{FM}
2. $\langle H_{\text{cycle}} \rangle$ within 1% of H_{FM}

If both tests pass, population size is adequate

If either test fails, it is likely that larger neutrons/cycle should be used. A warning message is printed.

For future work, if the popsize tests fail, neutrons/cycle could be automatically increased. That could create resource issues – memory size, run time, etc.

MCNP6 example

```

comment. The MESH (adaptive, axis-aligned, cartesian) to be used for computing
comment. Shannon entropy, fission-matrix tallies (if used), and source
comment. convergence checking is initially defined by:
comment.   max mesh spacing for automesh = 1.0052E+01
comment.
comment.   total mesh cells = 3675
comment.
comment.   Xbins= 35   Xmin=-1.6861E+02   Xmax= 1.6856E+02   dx= 9.6334E+00
comment.   Ybins= 35   Ymin=-1.6856E+02   Ymax= 1.6857E+02   dy= 9.6323E+00
comment.   Zbins= 3    Zmin=-9.6460E+00   Zmax= 9.9571E+00   dz= 6.5344E+00
comment.
comment. the mesh will be automatically extended if necessary,
comment. preserving the original mesh cells and spacing.
comment. -----
comment.
comment. -----
comment. FISSION MATRIX WILL BE COMPUTED to estimate dominance ratio,
comment. based on fission sites only - not flights or collisions'
comment.
comment. The mesh for the fission matrix is the same as the entropy mesh,
comment. using 3675 mesh bins for tallying fission neutrons
comment.
comment. Fission matrix mesh will be extended if
comment. any fission sites are found outside this mesh.
comment.
comment. Fission matrix tallies will be reset after cycle 1
comment. Fission matrix eigenfunction will be found every 10 cycles.
comment.
comment. Fission matrix dimensions: 3675 x 3675
comment.
comment. Compressed-row-storage is used for the fission matrix.
comment. max number of nonzero entries: 13505625
comment.
comment. FMATCONVRG option is being used.
comment. Statistical tests on the neutron & fission-matrix distributions
comment. will be used to determine convergence & begin active cycles.
comment. The 3rd entry on the KCODE card may be ignored.
comment.
comment. Targets for statistical tests:
comment.   h_slope: < 0.95 conf level, or < 0.0001
comment.   k_slope: < 0.95 conf level, or < 0.0001
comment.   distribs: < 0.95 conf level, h_diff: < 0.01
comment.
comment. FMATACCEL option is being used.
comment. Fission matrix will be used to ACCELERATE source convergence
comment. of the neutron distribution during inactive cycles.
comment. Importance-factor-limits: min= 0.20, max= 5.00
comment. -----

```

MCNP6 example

cycle	k(col)	ctm	entropy	active	k(col)	std dev	chains
1	1.35733	0.04	0.60521				35416
2	1.16857	0.10	0.62080	extend H-mesh to:	36 x 35 x 4		22433
3	1.08223	0.13	0.63109	extend H-mesh to:	37 x 35 x 4		17100
4	1.05100	0.17	0.63410	dS= 3%, dF= 34%, shift window extend H-mesh to:	37 x 36 x 4		13800
5	1.02827	0.21	0.63348	dS= 2%, dF= 19%, shift window extend H-mesh to:	37 x 37 x 4		11529
6	1.02118	0.25	0.61732	dS= 1%, dF= 14%, shift window extend H-mesh to:	37 x 37 x 5		9997
7	1.02018	0.29	0.61762	dS= 0%, dF= 10%, shift window			8746
8	1.02413	0.32	0.61845	dS= 1%, dF= 9%, shift window			7790
9	1.01974	0.37	0.61766	dS= 0%, dF= 7%, shift window			6974
10	1.01709	0.43	0.61656	dS= 0%, dF= 7%, shift window			6313
11	1.02129	0.48	0.61606	dS= 1%, dF= 5%, shift window			5815
12	1.01705	0.53	0.61452	dS= 1%, dF= 5%, shift window			5351
13	1.02459	0.58	0.61263				4975
14	1.02193	0.65	0.61214				4640
15	1.02741	0.70	0.60894				4372
16	1.03005	0.73	0.60600				4091
17	1.03266	0.78	0.60435				3852
18	1.03369	0.83	0.60065				3628
19	1.03485	0.87	0.59622				3426
20	1.03631	0.91	0.59177				3245
21	1.04159	0.96	0.58774				3074

Source,
fission matrix,
& mesh
stabilization

Block
of
cycles

fmatrix keff= 1.12401, DR= 0.91098, iters= 199

MCNP6 example

fmatrix keff= 1.12400, DR= 0.91098, iters= 199

CONVERGENCE INFO & CHECKS: (based on last 10 cycles)

entropy for fmatrix eigenvector = 0.35378
 entropy for neutron last cycle = 0.58774 dif= 66.13%
 relative entropy for last cycle = 2.06900

slope of keff (tracklen)	= 2.0E-03,	target: < 5.3E-04	FAIL
slope of keff (collide)	= 2.1E-03,	target: < 5.3E-04	FAIL
slope of keff (absorb)	= 2.0E-03,	target: < 5.8E-04	FAIL
slope of entropy	= -2.6E-03,	target: < 4.3E-04	FAIL
slope of entropy X marginal	= -2.1E-03,	target: < 5.1E-04	FAIL
slope of entropy Y marginal	= -2.1E-03,	target: < 4.2E-04	FAIL
slope of entropy Z marginal	= 8.7E-04,	target: < 3.3E-04	FAIL
entropy dif, neut vs fmat	= 7.1E-01,	target: < 1.0E-02	n/a
Kolmo-Smirnov, distrib, stat	= 6.8E-01,	target: < 9.1E-02	FAIL
Chi-square, distrib, stat	= 5.0E+04,	target: < 5.1E+02	FAIL
rel-h-block, distrib, stat	= 2.5E+00,	target: < 5.1E-03	n/a

***** convergence tests were NOT passed *****

MISCELLANEOUS INFO & CHECKS:

rmse = 1.16 %
 fmat nnz= 11884, 0.09 %

22	1.10782	0.81	0.38309	accelerate: Imin= 0.2, Imax= 4.7	2134
23	1.11376	0.85	0.35605	accelerate: Imin= 0.2, Imax= 3.8	1499
24	1.11583	0.88	0.35129	accelerate: Imin= 0.2, Imax= 3.2	1233
25	1.11726	0.92	0.35104	accelerate: Imin= 0.2, Imax= 5.0	1077
				accelerate: Imin= 0.2, Imax= 3.4	

MCNP6 example

31 1.11257 1.12 0.35069 680

fmatrix keff= 1.11187, DR= 0.91653, iters= 138

CONVERGENCE INFO & CHECKS: (based on last 10 cycles)

entropy for fmatrix eigenvector = 0.35656
 entropy for neutron last cycle = 0.35069 dif= -1.65%
 relative entropy for last cycle = 0.00972

slope of keff (tracklen)	=	4.2E-03,	target:	< 5.1E-03	PASS
slope of keff (collide)	=	4.6E-03,	target:	< 4.9E-03	PASS
slope of keff (absorb)	=	4.6E-03,	target:	< 4.9E-03	PASS
slope of entropy	=	-1.4E-02,	target:	< 1.6E-02	PASS
slope of entropy X marginal	=	-1.8E-02,	target:	< 1.9E-02	PASS
slope of entropy Y marginal	=	-1.8E-02,	target:	< 1.9E-02	PASS
slope of entropy Z marginal	=	1.3E-03,	target:	< 1.6E-03	PASS
entropy dif, neut vs fmat	=	-9.1E-04,	target:	< 1.0E-02	PASS
Kolmo-Smirnov, distrib, stat	=	2.5E-03,	target:	< 9.1E-02	PASS
Chi-square, distrib, stat	=	9.0E+01,	target:	< 5.1E+02	PASS
rel-h-block, distrib, stat	=	2.8E-03,	target:	< 5.1E-03	PASS

**Quantitative
Evidence
For
Convergence**

```

*****
*****
** FISSON SOURCE HAS CONVERGED, based on last 10 cycles **
** Metrics: **
** slope of keff (tracklen) is 0 (within uncert) **
** slope of keff (collide) is 0 (within uncert) **
** slope of keff (absorb) is 0 (within uncert) **
** slope of entropy is 0 (within uncert) **
** slope of entropy X marginal is 0 (within uncert) **
** slope of entropy Y marginal is 0 (within uncert) **
** slope of entropy Z marginal is 0 (within uncert) **
** entropy dif, neut vs fmat is 0 (within uncert) **
** Distribution checks: **
** Kolmo-Smirnov, distrib, stat, neut vs fmat (within conf) **
** Chi-square, distrib, stat, neut vs fmat (within conf) **
** rel-h-block, distrib, stat, neut vs fmat (within conf) **
*****
*****

```

**Quantitative
Evidence
For
Convergence**

Convergence is locked-in, even if some tests fail in future cycles ←

Active cycles will begin with cycle = 32 ←
 Active cycles will end with cycle = 131
 Total active cycles to be run = 100

MCNP6 example

```

40  1.11130      1.43  3.45E-01      9  1.11344      0.00061      499
41  1.12440      1.47  3.44E-01     10  1.11454      0.00122      487

```

```
fmatrix keff= 1.11470, DR= 0.91540, iters= 126
```

CONVERGENCE INFO & CHECKS: (based on last 10 cycles)

```

entropy for fmatrix eigenvector = 0.35367
entropy for neutron last cycle  = 0.34421   dif=  -2.67%
relative entropy for last cycle  = 0.01140

```

```

slope of keff (tracklen)      = -2.3E-04, target: < 3.8E-04  PASS
slope of keff (collide)      =  9.9E-06, target: < 4.6E-04  PASS
slope of keff (absorb)       = -3.7E-05, target: < 4.9E-04  PASS
slope of entropy              = -9.2E-04, target: < 4.7E-04  FAIL
slope of entropy X marginal   = -1.1E-03, target: < 8.0E-04  FAIL
slope of entropy Y marginal   = -1.4E-03, target: < 6.8E-04  FAIL
slope of entropy Z marginal   =  9.4E-05, target: < 3.9E-04  PASS
entropy dif, neutrs vs fmat   = -9.0E-03, target: < 1.0E-02  PASS
Kolmo-Smirnov, distrib, stat  =  5.3E-03, target: < 9.0E-02  PASS
Chi-square, distrib, stat     =  8.8E+01, target: < 5.1E+02  PASS
rel-h-block, distrib, stat    =  2.5E-03, target: < 5.1E-03  PASS

```

```

convergence checks passed      at cycle = 31
active cycles based on fmatconvr begin at cycle = 32

```

```

entropy for fmatrix eigenvector = 0.35367
entropy for neutron active cycles = 0.35111   dif=  -0.72%
relative entropy for active cycles = 0.00249

```

POPULATION SIZE INFO & CHECKS: (based on last 10 cycles)

```
population check using relative entropy PASS
```

```

warning: The average entropy for the last      cycles
differs from the entropy for the fission matrix
fundamental mode by -1.1%. This indicates
undersampling or possible clustering.

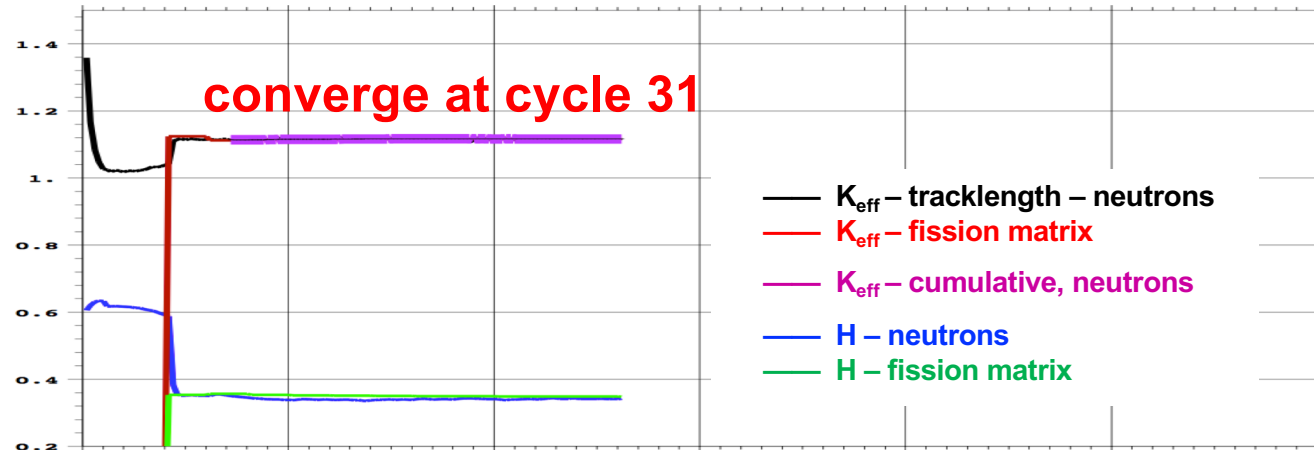
```

CONSIDER USING MORE NEUTRONS/CYCLE.

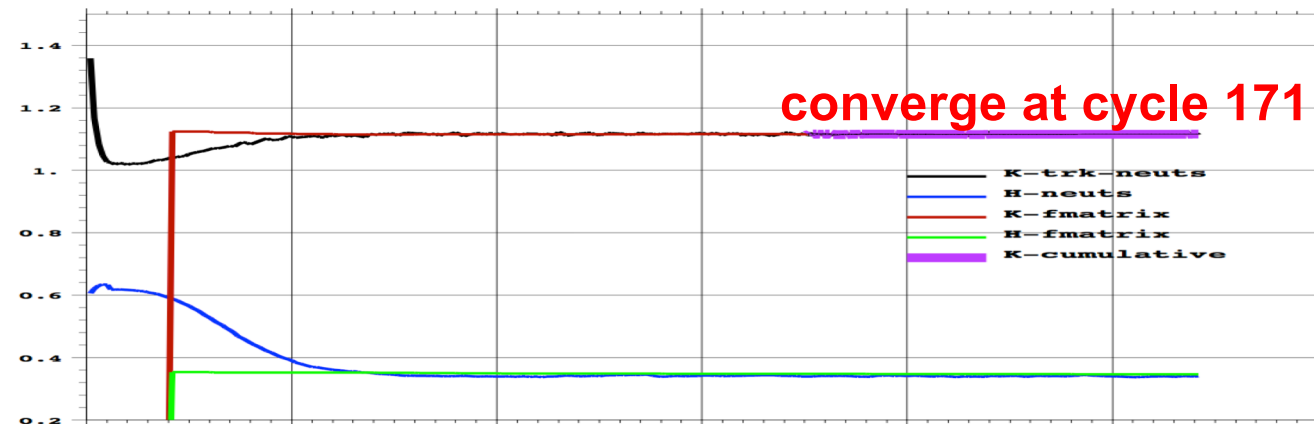


OECD-NEA Source Convergence Problem TEST4S

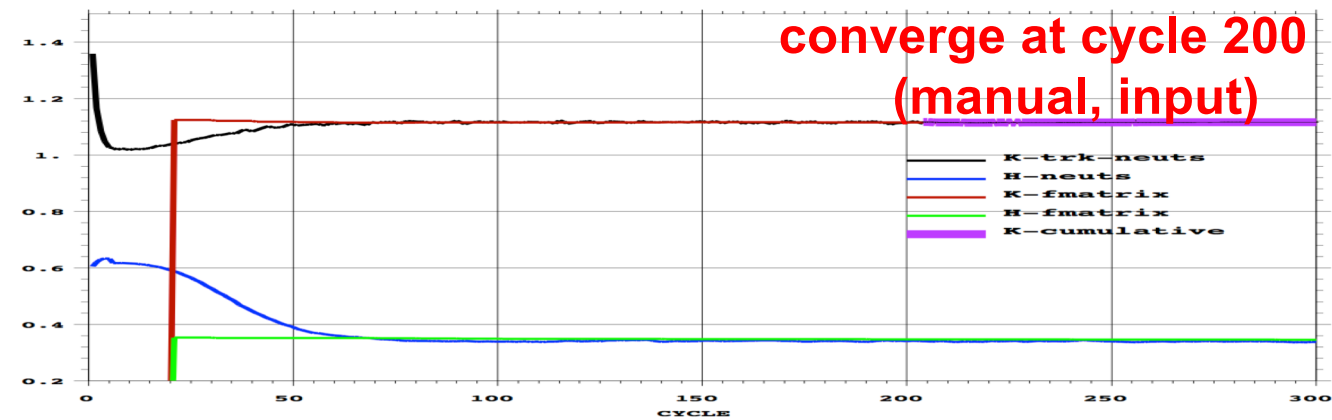
50,000 neutrs/cycle
acceleration
auto-converge
 $k = 1.1165 (2)$



50,000 neutrs/cycle
no acceleration
auto-converge
 $k=1.1161 (2)$

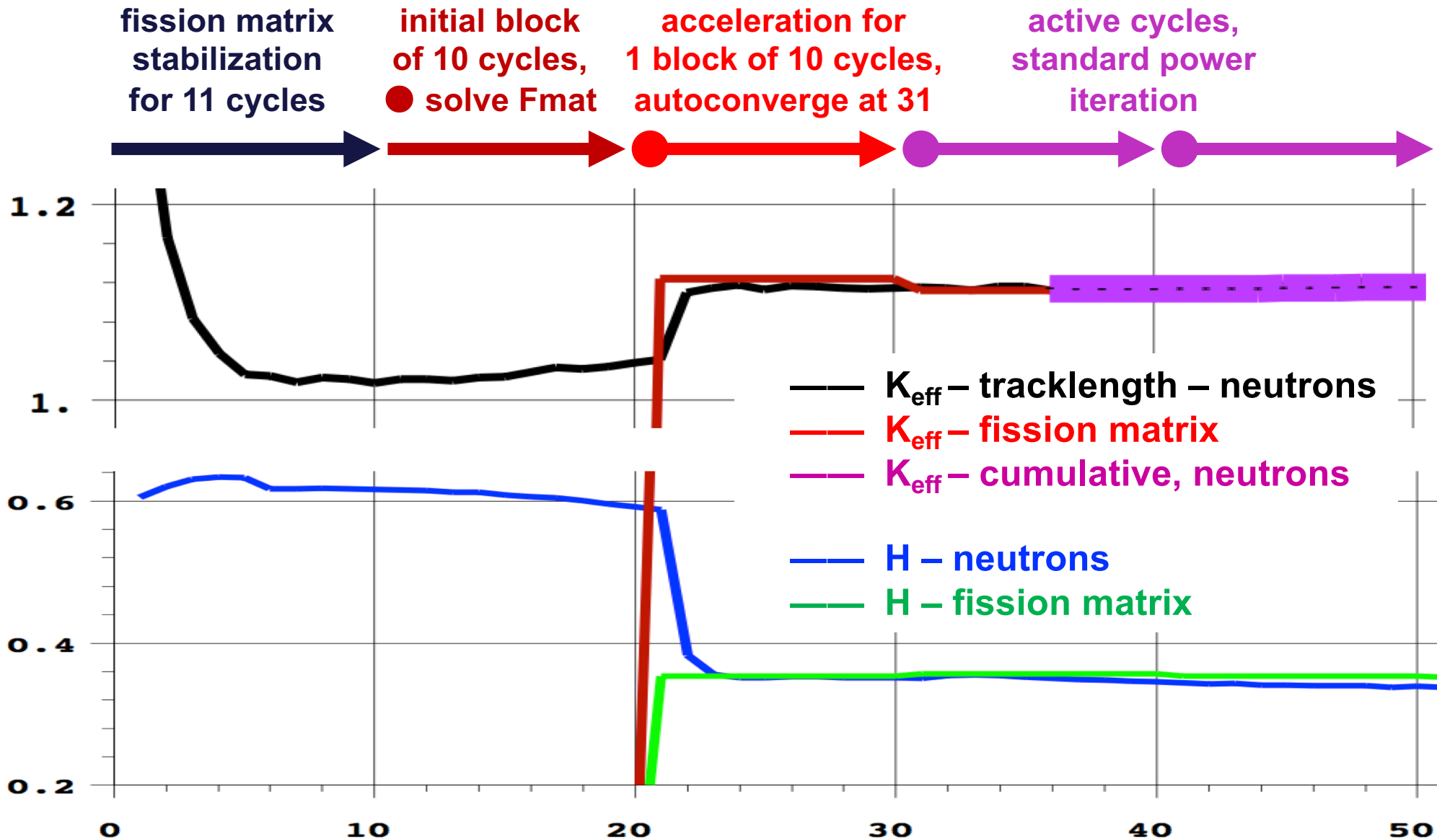


50,000 neutrs/cycle
no acceleration
no auto-converge
 $k = 1.1164 (2)$



OECD-NEA Source Convergence Problem TEST4S

50,000 neutrs/cycle, **acceleration, auto-converge**, $k = 1.1165$ (2)



MCNP6 Test Problems for Fission Matrix Based Automated Convergence & Acceleration of K-eigenvalue Problems

- **VALIDATION_CRITICALITY benchmark suite**
- **Godiva – bare HEU sphere**
- **PWR2d – commercial PWR**
- **ATR – advanced test reactor**
- **C5G7 3D U-Mox benchmark, OECD-NEA**
- **Triga reactor**
- **ACRR burst reactor, with FREC**
- **LCT-078-001 - Sandia critical experiment**
- **3D PWR – Hoogenboom-Martin benchmark, OECD-NEA**
- **Whitesides problem – K-effective of the world model**
- **TEST4S – simplified Whitesides, OECD-NEA**
- **FPOOL – OECD-NEA source convergence benchmark 1**

VALIDATION_CRITICALITY benchmark suite

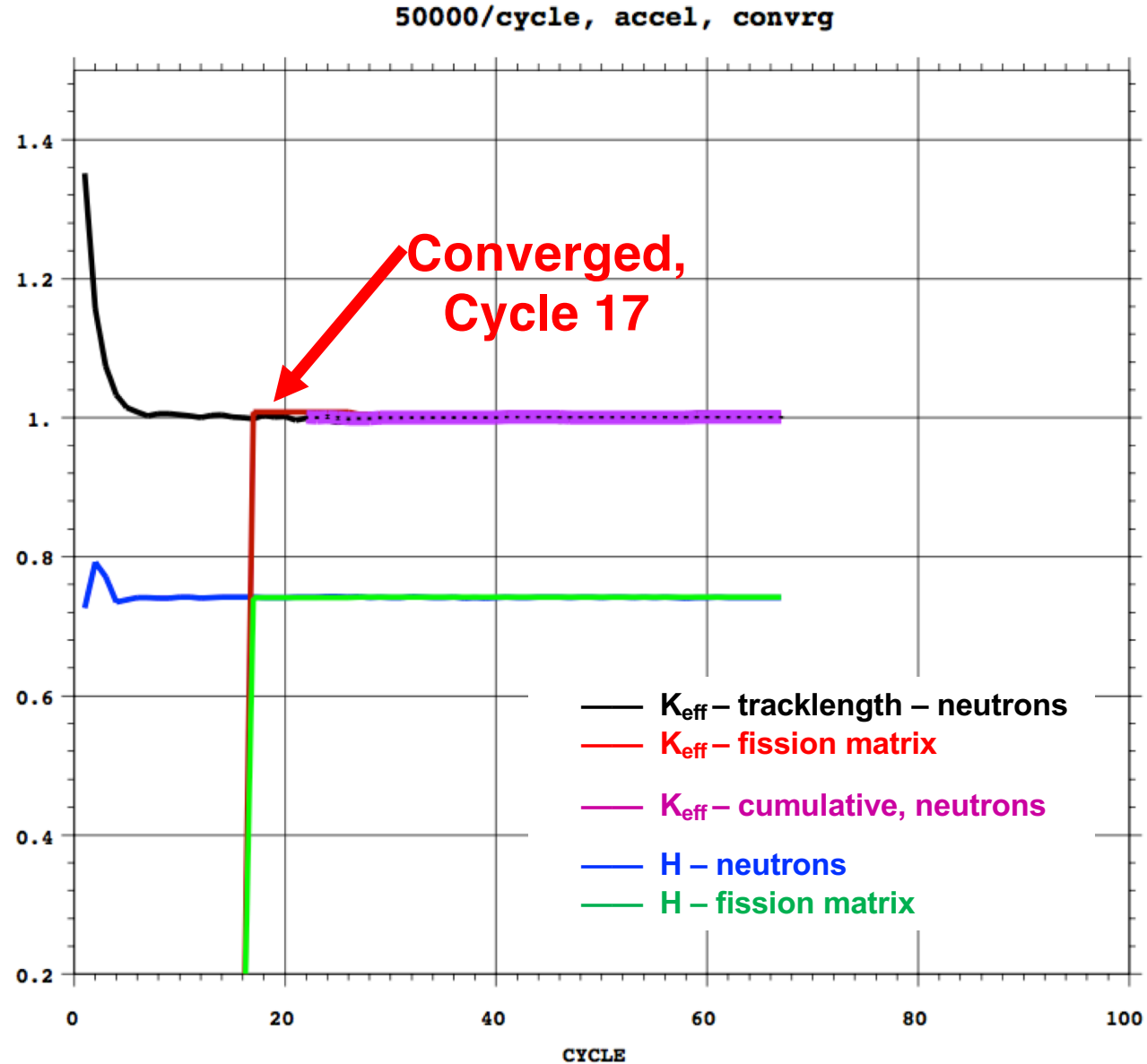
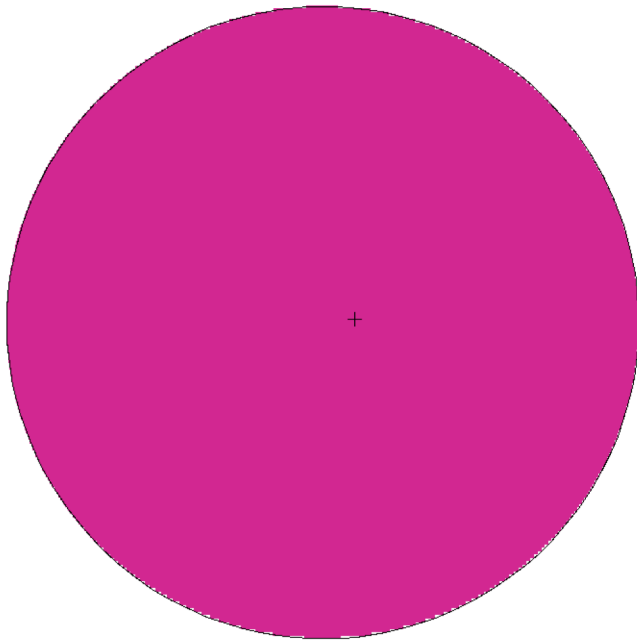
- **Standard MCNP validation suite since 2002 (Mosteller)**
 - 31 ICSBEP Handbook problems, critical experiments
 - Run using ENDF/B-VII.1 nuclear data
 - Timing results include all I/O, input & xsec file processing, Monte Carlo random walks, printing results, etc. for all 31 problems
- **Timing tests**
 - 50,000 neutrons/cycle for all runs
 - For standard runs, 100 inactive cycles, 100 active cycles
 - For auto accelerate & converge, 100 active cycles

Standard run: 106 minutes

Auto accel & converge: 70 minutes

Godiva Problem

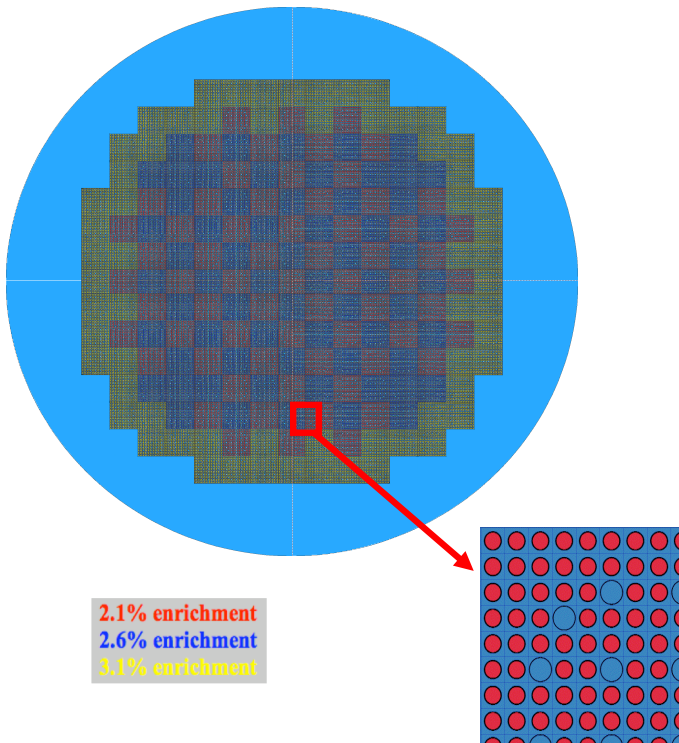
Bare HEU sphere



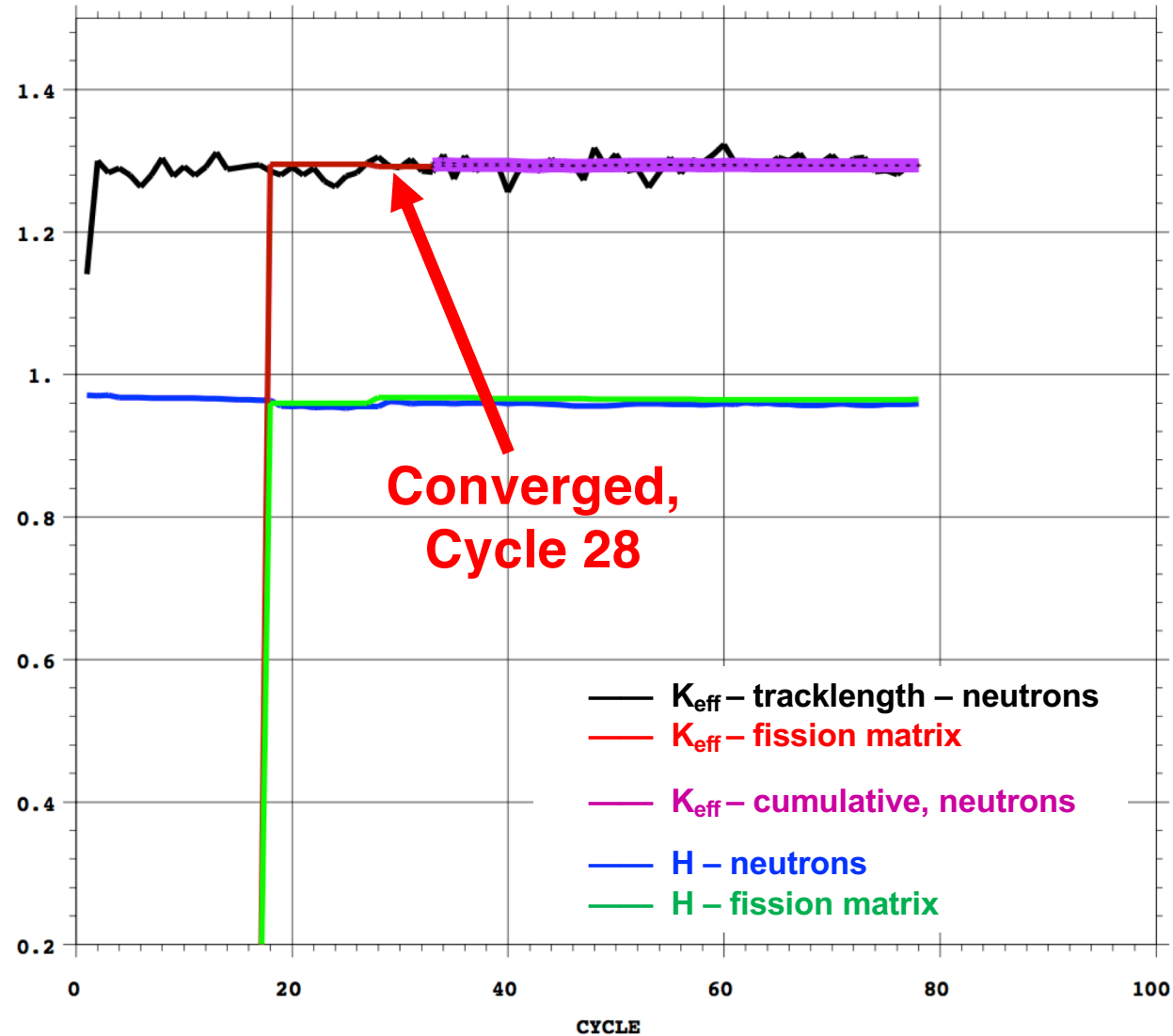
Whole-core 2D PWR Model

2D PWR (Nakagawa & Mori model)

- 193 fuel assemblies:
 - 50,952 fuel pins with cladding
 - 4825 water tubes
- Each assembly:
 - Explicit fuel pins & rod channels
 - 17x17 lattice
 - Enrichments: 2.1%, 2.6%, 3.1%
- Calculations used whole-core model

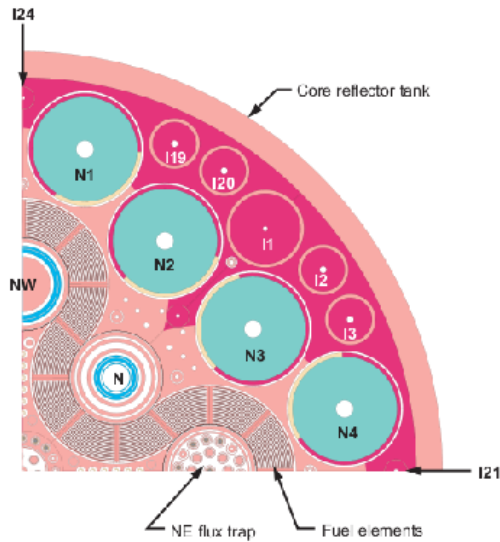
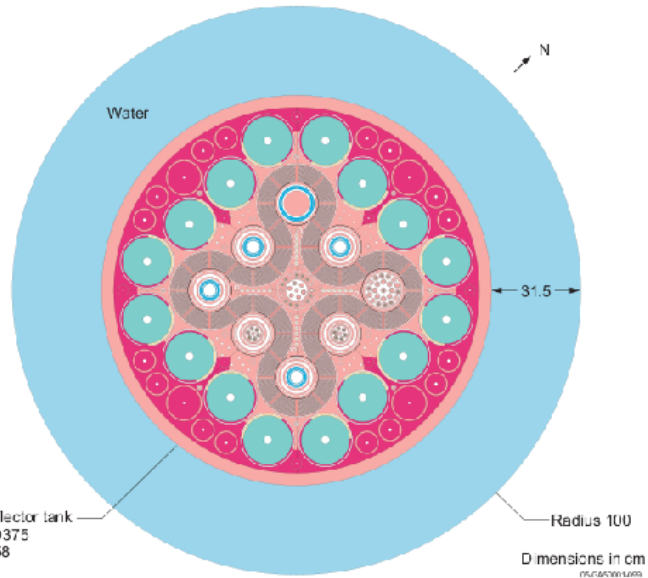


10000/cycle, accel, convrg

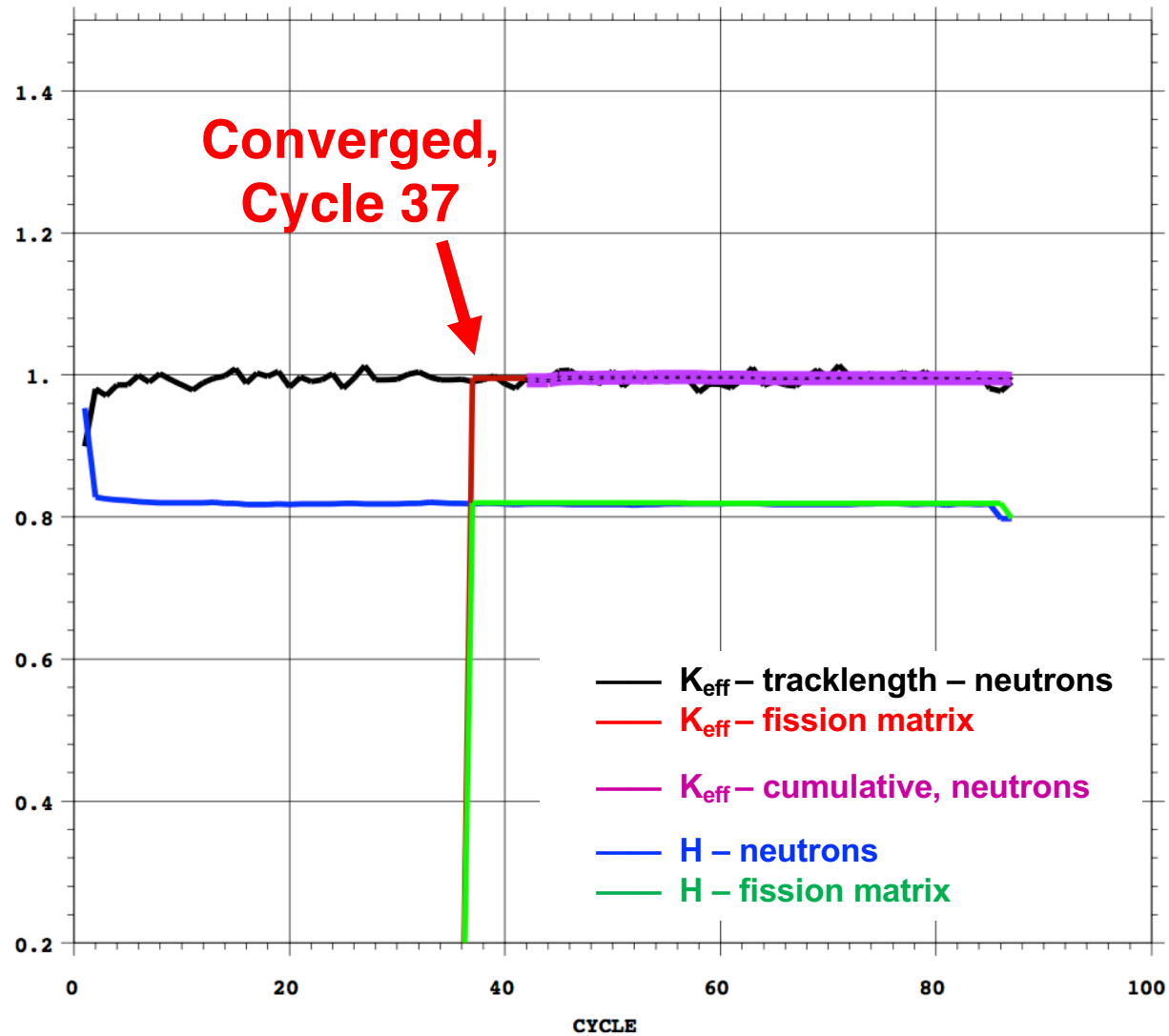


Advanced Test Reactor

“Serpentine Arrangement of Highly Enrichment Water-Moderated Uranium-Aluminide Fuel Plates Reflected by Beryllium”

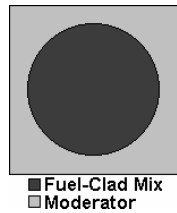


25000/cycle, accel, convrg



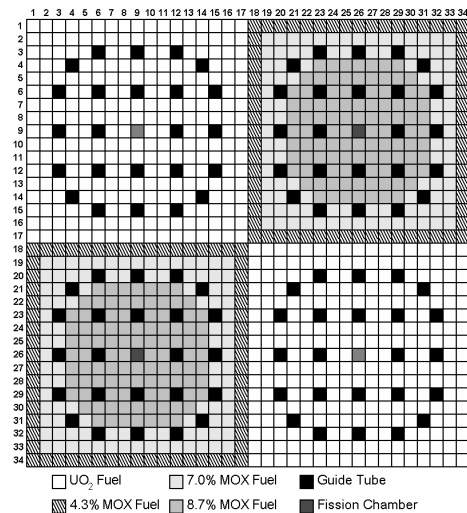
OECD-NEA Benchmark - C5G7

Figure 2. Fuel pin layout

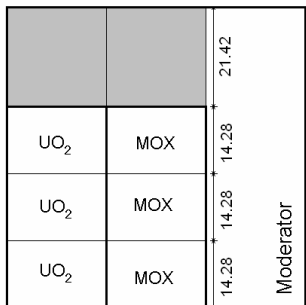


3D model

Figure 3. Benchmark fuel pin compositions and numbering scheme

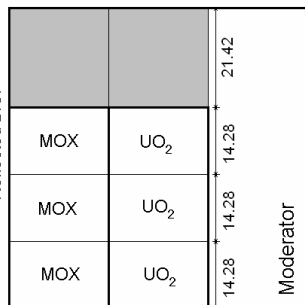


Section A-A
Vacuum B.C.



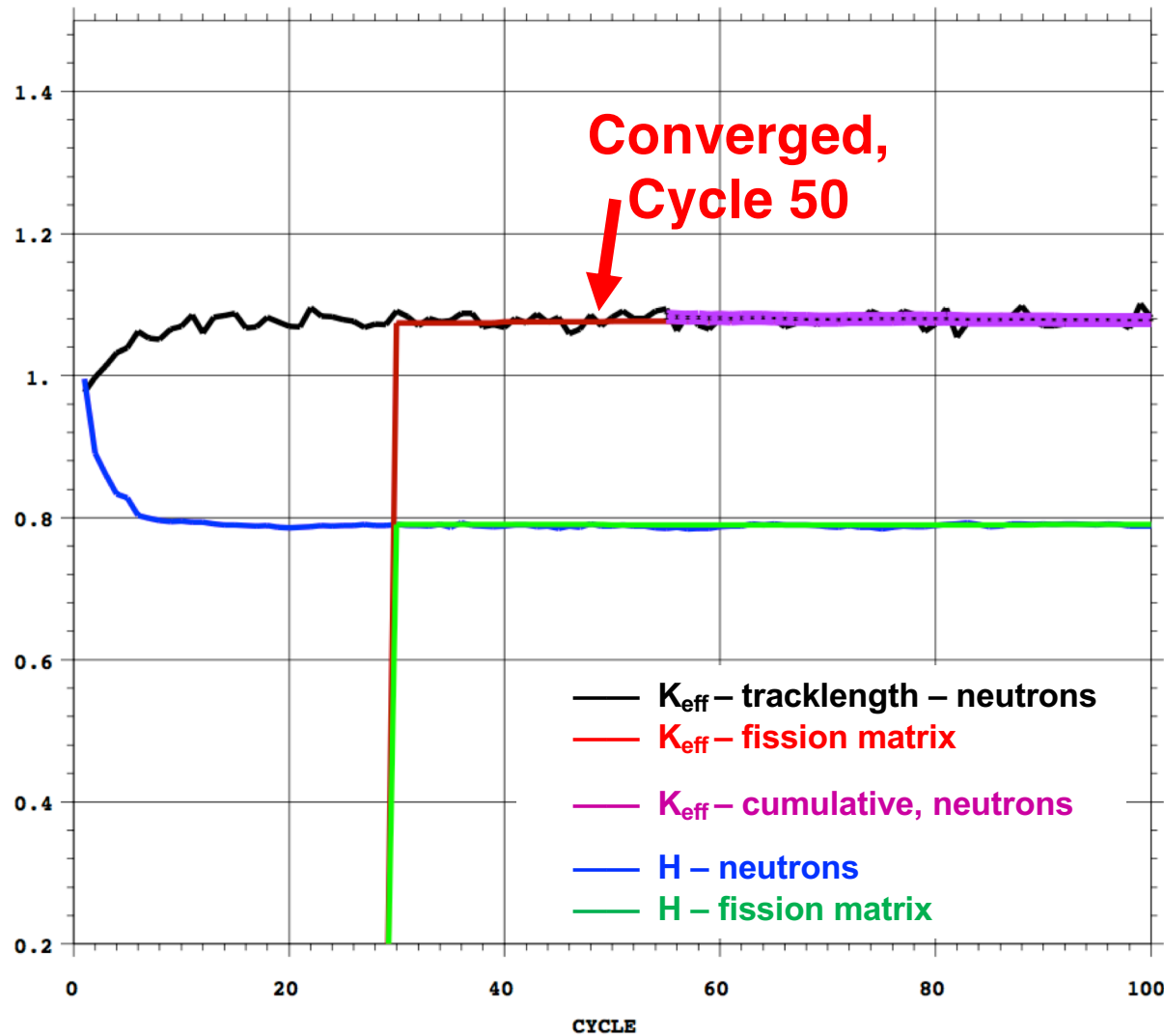
Reflected B.C.

Section B-B
Vacuum B.C.



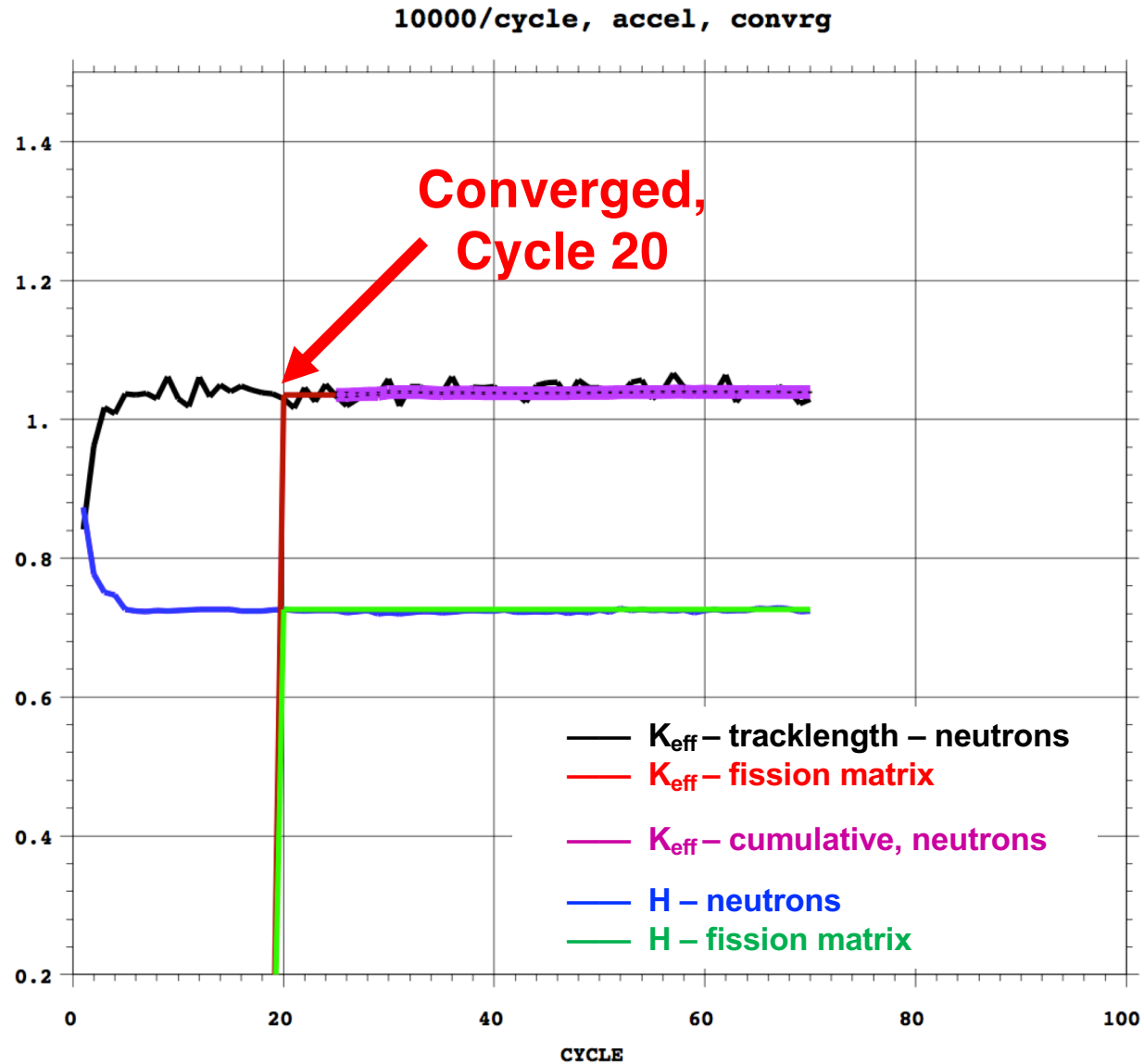
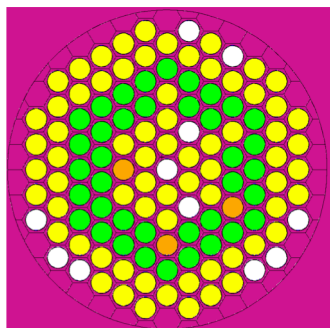
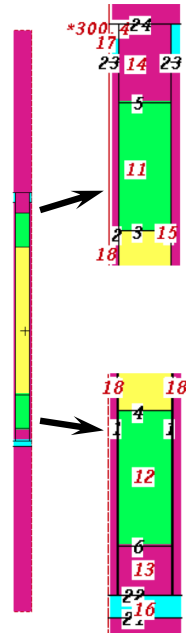
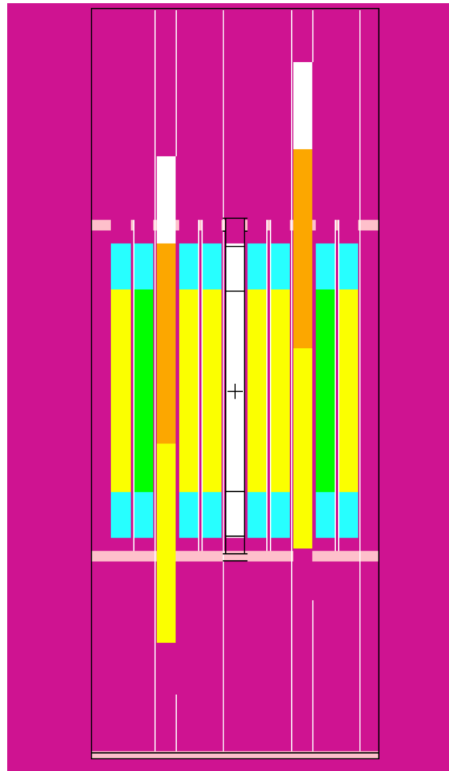
Reflected B.C.

10000/cycle, accel, convrg



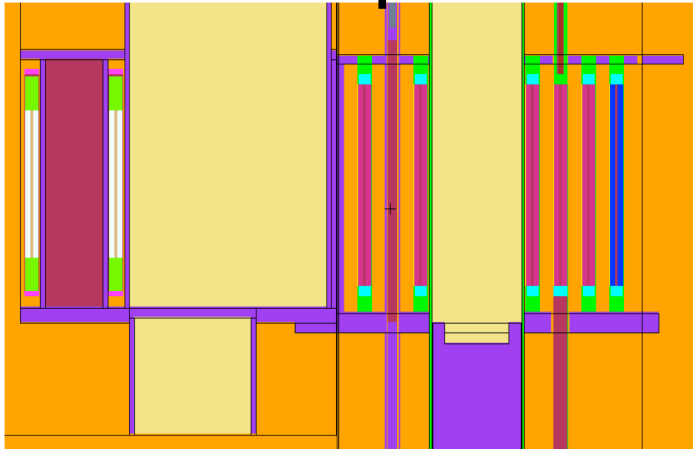
- K_{eff} - tracklength - neutrons
- K_{eff} - fission matrix
- K_{eff} - cumulative, neutrons
- H - neutrons
- H - fission matrix

TRIGA Reactor

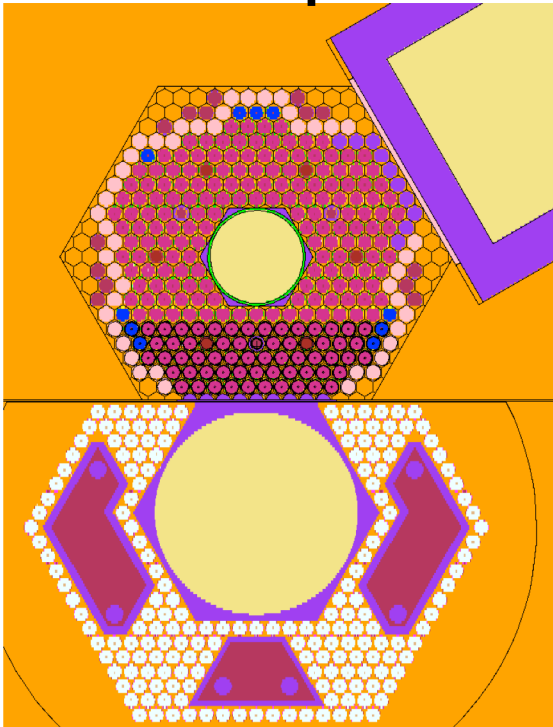


Sandia burst reactor - ACRR, with FREC

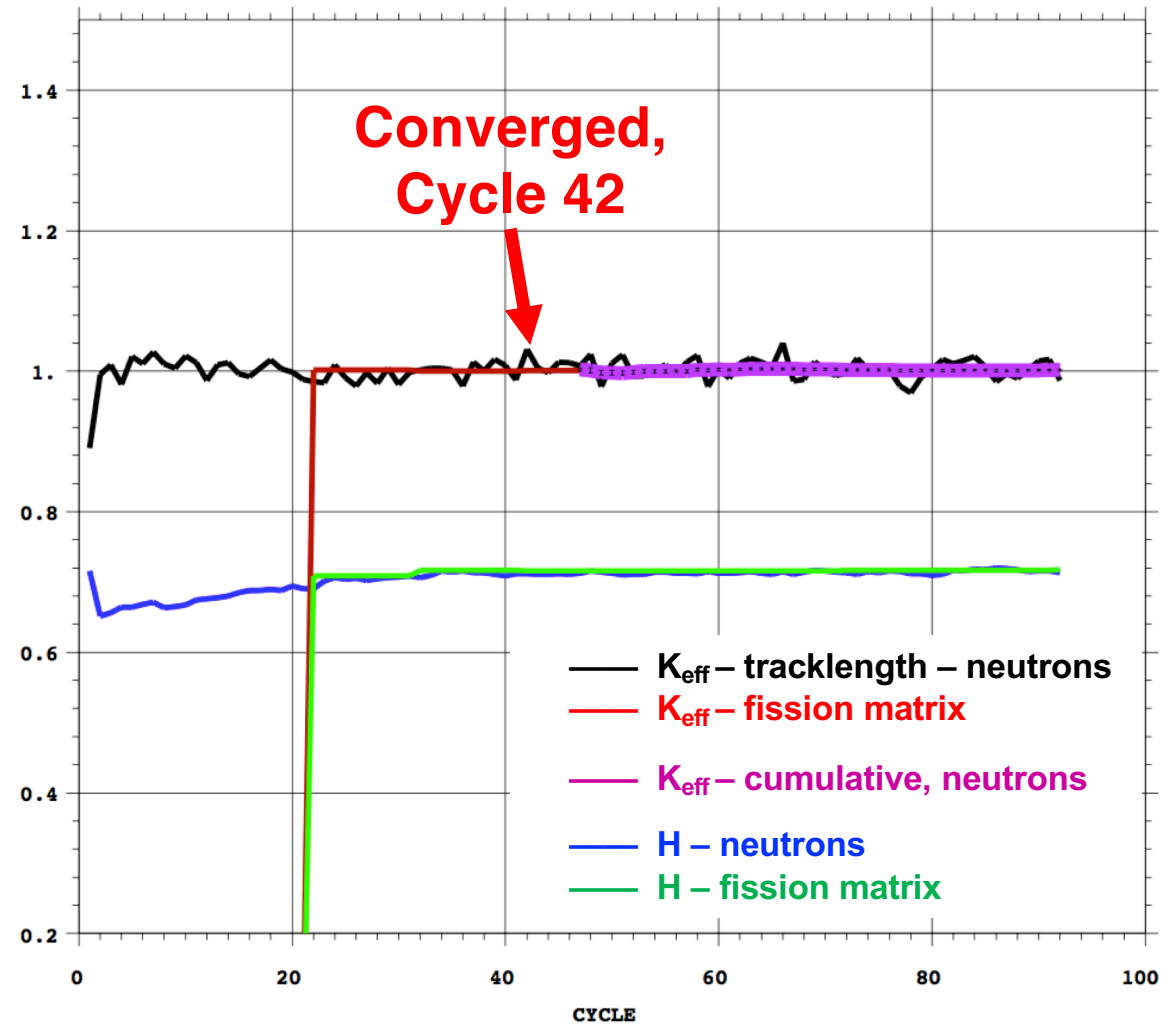
Y-Z plot



X-Y plot

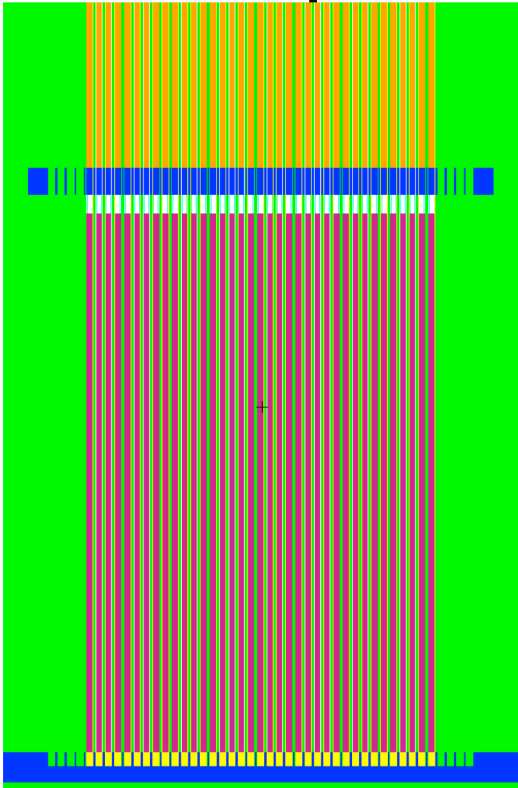


10000/cycle, accel, convrg

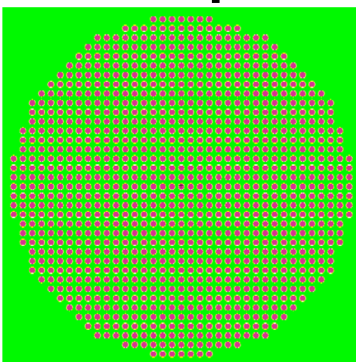


Sandia critical experiment – LCT-078-001, 1,057 rod assembly

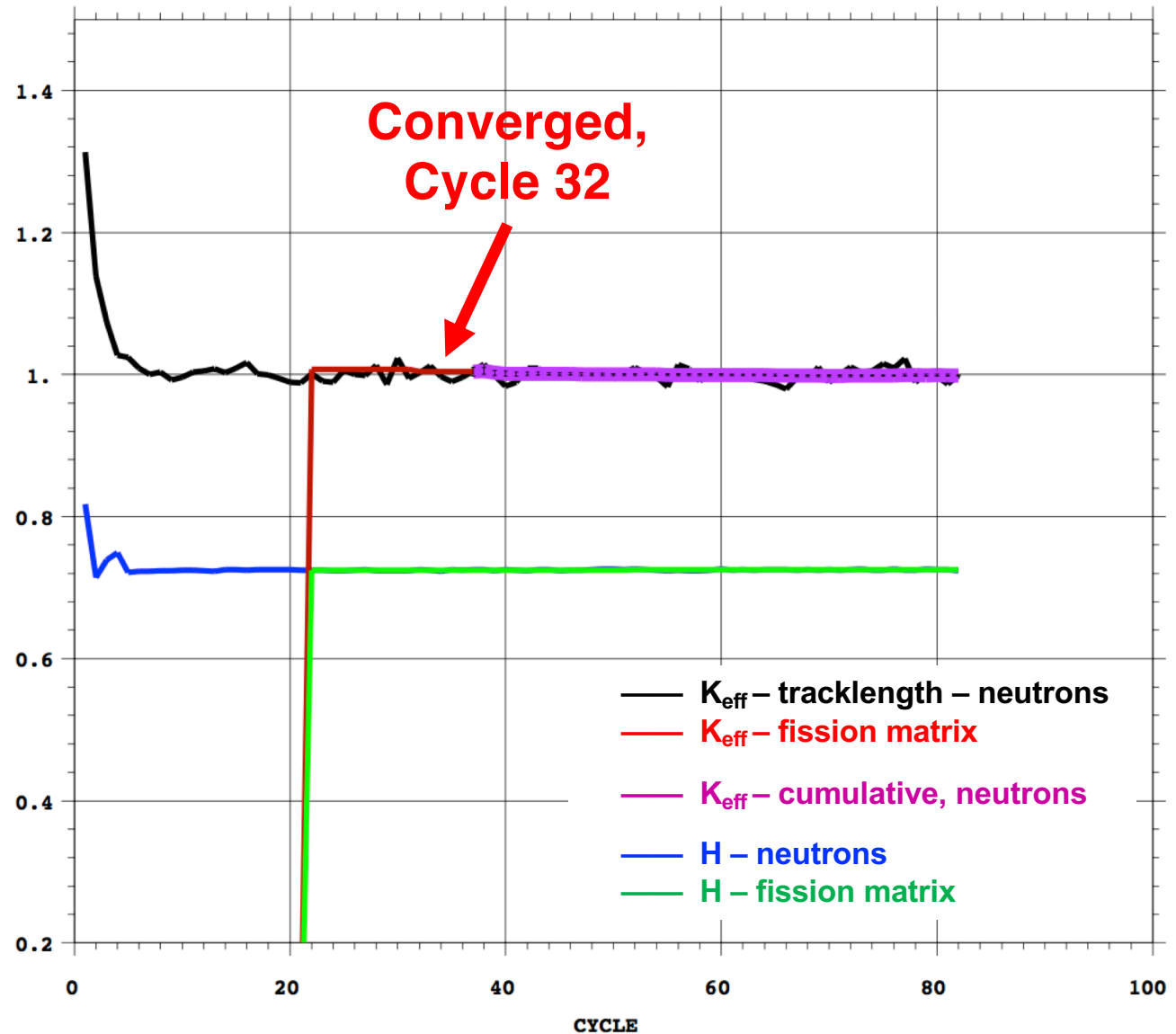
Y-Z plot



X-Y plot



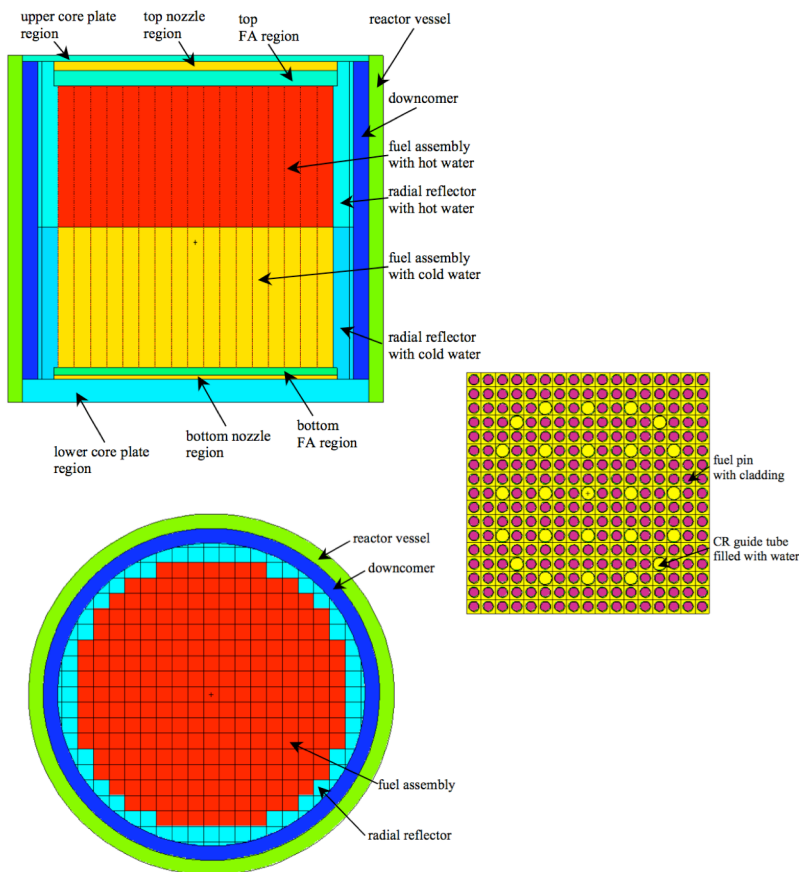
20000/cycle, accel, convrg



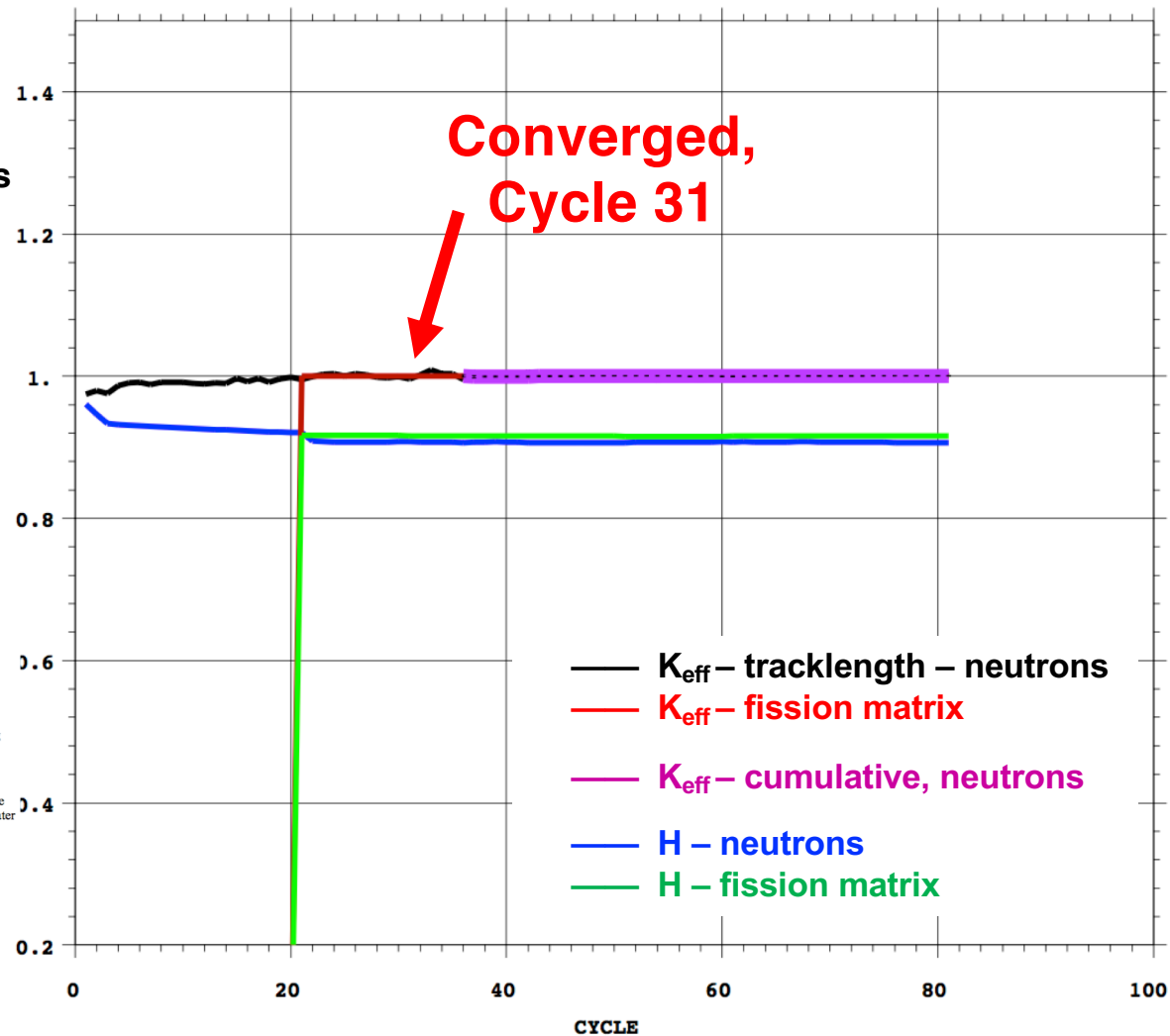
OECD-NEA "Hoogenboom-Martin Performance Benchmark"

Reactor – full core, 3D

- LWR model: 241 assemblies, 63,624 pins
- Fuel: 17 actinides + 16 FPs; borated water
- Detailed 3D MCNP model
(63,624 pins) x (100 axial) = 6.3M pin powers
Runs easily on desktop computer



100000/cycle, accel, convrg



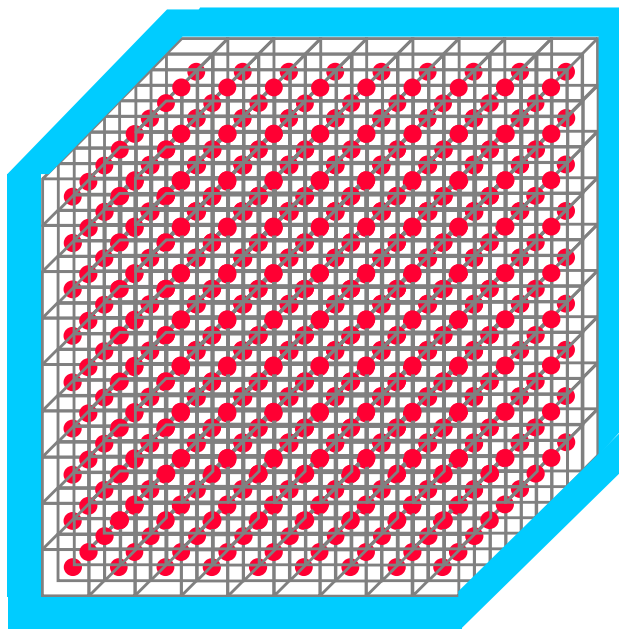
Whitesides' Model Problem – K-eff of the World

9 x 9 x 9 array of Pu-239 spheres

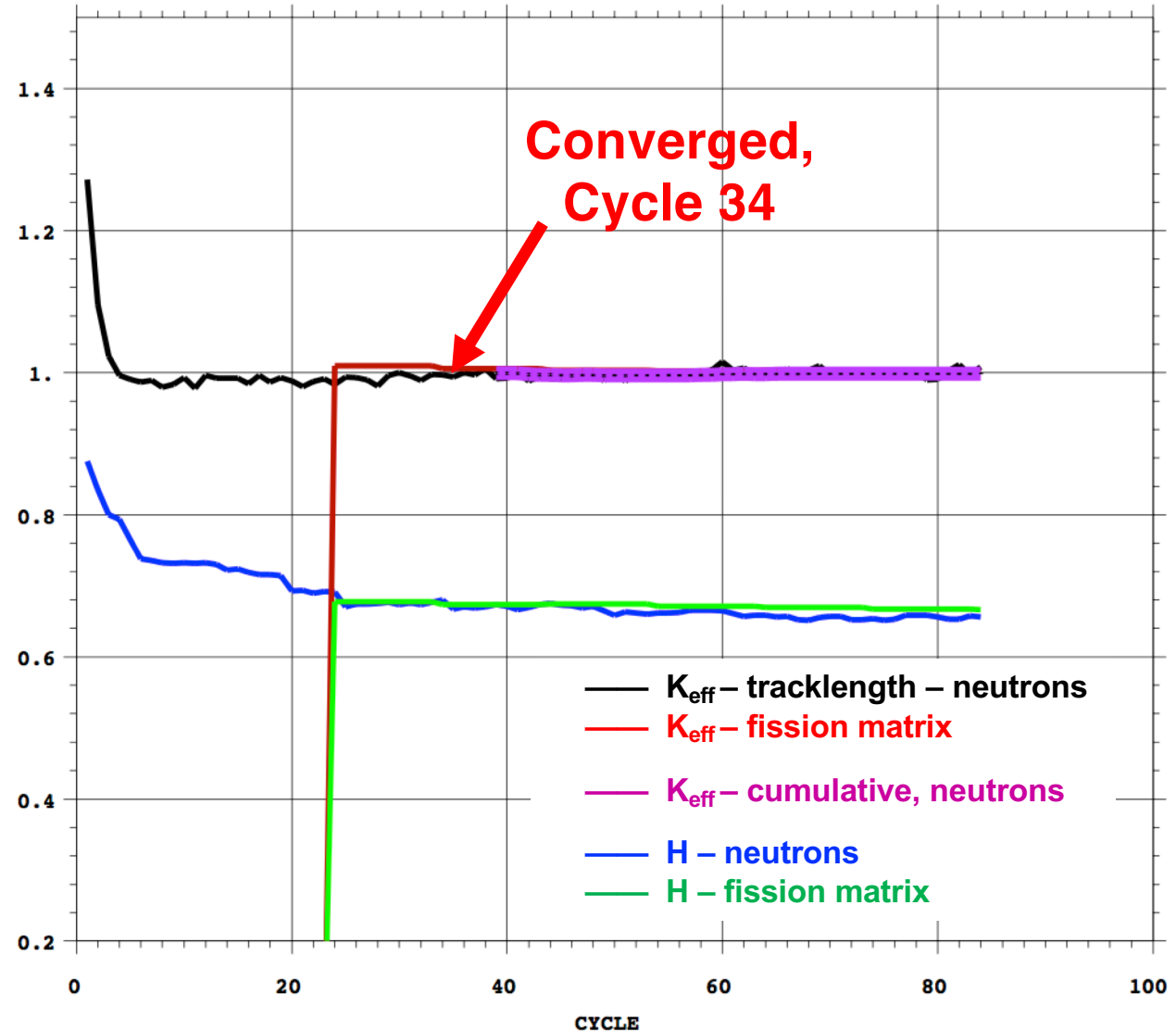
- 729 spheres
- Void between spheres
- Surrounded by 30 cm water
- Sphere radii ~ 4 cm
- Pitch = 60 cm
- $K_{eff} \sim 0.93$

Replace center sphere of array by larger (critical) sphere

Should be supercritical - is it ?



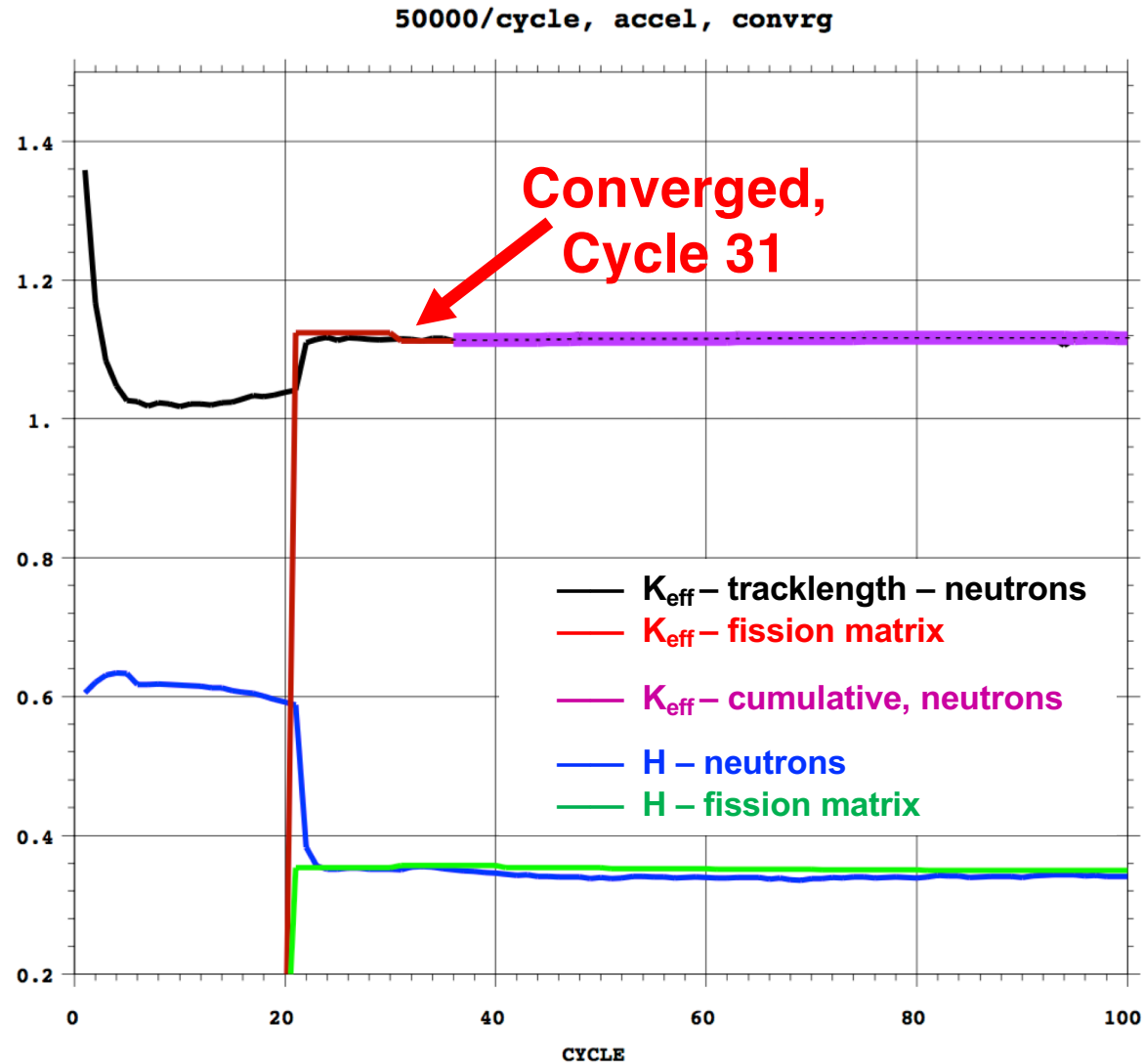
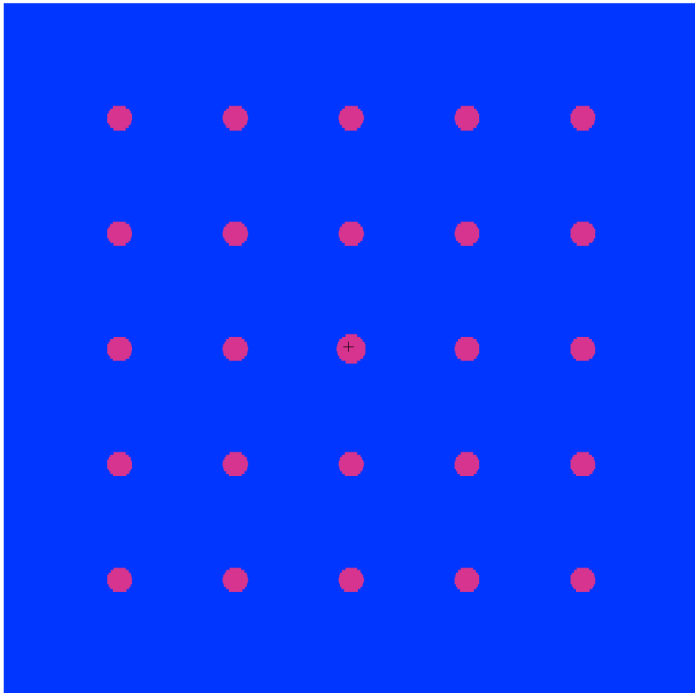
20000/cycle, accel, convrg



OECD-NEA Source Convergence Problem TEST4S

OECD-NEA source convergence benchmark

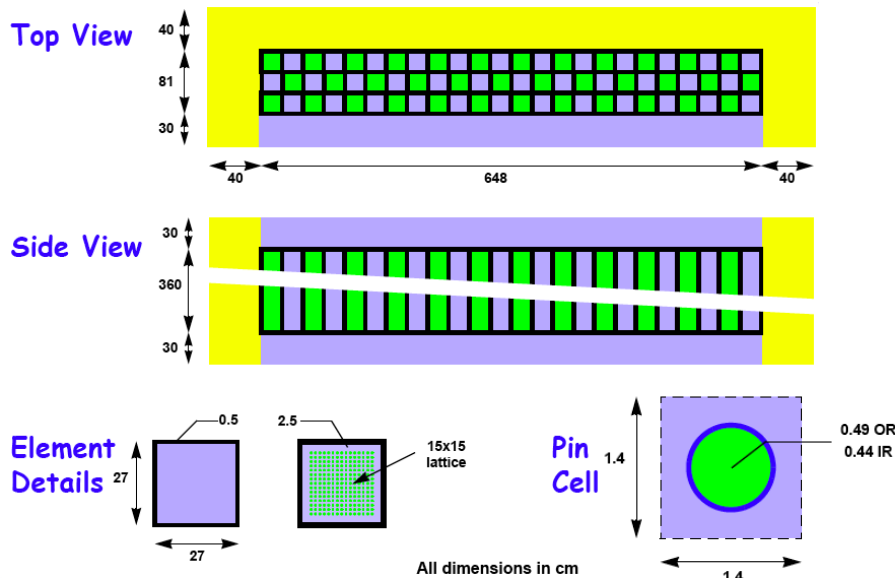
- Simplified version of Whitesides problem
- 5 x 5 array of HEU spheres
 - center sphere, $R = 10$ cm
 - others, $R = 8.71$ cm
 - pitch = 80 cm
 - air in between spheres
 - vacuum boundary conditions



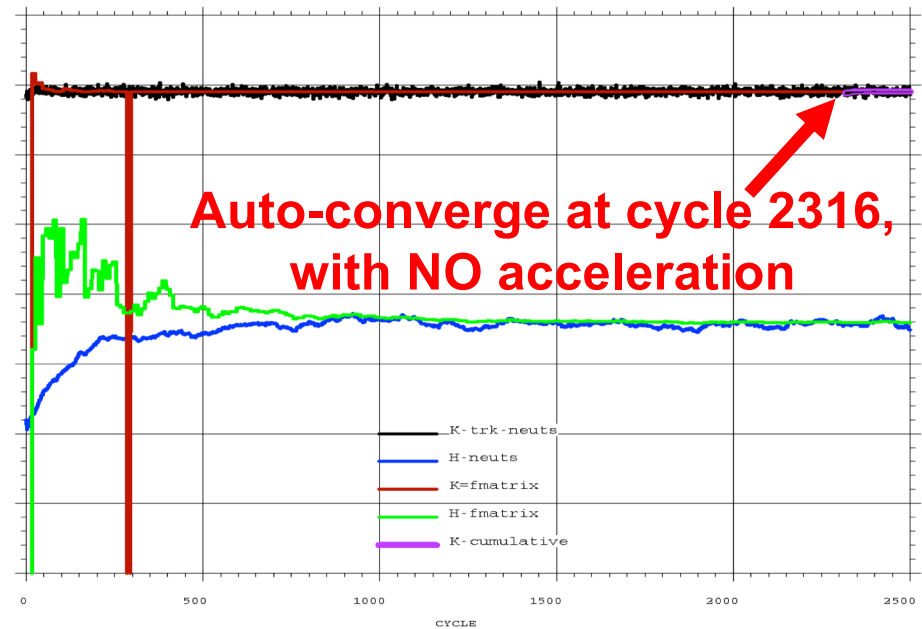
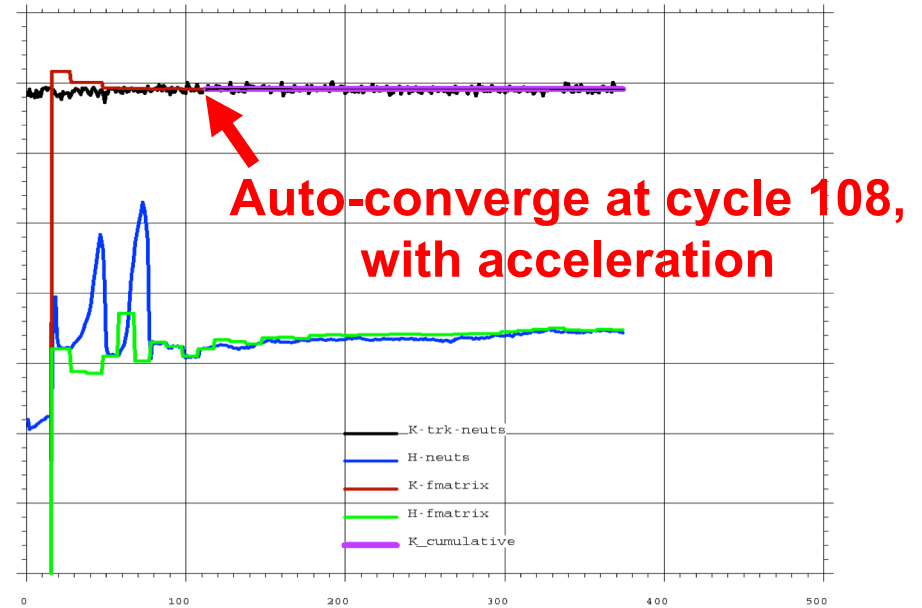
OECD-NEA Fuel Storage Pool

OECD-NEA-WPNCS Expert Group on Source Convergence

Benchmark 1



100k neutrs/cycle



Current Work

- **Summer 2019**

- Limited release to NCS early adopters, more testing & feedback

- **Near-term R&D Work**

- **Source guess**

- Handle a list of axis-oriented bounding boxes (AABB)
- For 1 large bounding box, handle source overruns
- Should be possible to completely automate

- **Fission matrix**

- Better eigensolver ?
- Investigate matrix size vs neutrons/cycle
 - Statistical noise on matrix elements – effect on solution & stability
 - Kord-Smith problem, fuel storage pool problem

- **Convergence tests**

- Add more ?
- Determine precise confidence level for passing all tests

- **Acceleration**

- Possibly find more robust, stable method

- **Population size tests**

- Scheme for predicting adequate size

- **More examples & tests**

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