Lessons Learned from Direct Calculations of Measured Observables in Multiplying Sub-Critical Systems

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Abstract

- **This presentation briefly describes the following topics:**

  - Review of sub-critical multiplication simulations and measurements,
  - Summary of recent sub-critical measurement campaigns,
  - Outstanding issues with measurements, simulations, and inferred values
  - Work in progress / future collaboration

Why pay attention to sub-critical, multiplying systems, Aren't critical systems more valuable and interesting?

- **Critical experiments have been used to benchmark codes, measure nuclear data, well known, easy to measure, …**
- **Sub-critical, multiplying systems are providing invaluable information select few have examined:**
  - Standard comparison of simulated and measurements (fixed source, eigenvalue)
    - Design of future measurements, approach to critical, POI, …
    - Validation of MCNP, nuclear data, in other regimes.
  - New methods for interpreting measured data (keff, M)
  - Quantification of uncertainties in inferred values
  - Impact on nuclear data (ν, σ_i, □)
  - Nuclear safeguards
  - Threats from unknown systems…
  - New radiation detector technologies
- **NCSP funding for sub-critical measurements/simulations has benefited many programs and vice versa!**
  - Revived a decaying capability critical to missions of DP, NP, ER, NCT

NNSS: Access to variety of SNM
Possible measurements are endless!

- Critical Assemblies:
  - Godiva, Flat top, Comet, Planet
- WGPu
- HEU
- Np
- Others
- Moderators/Reflectors

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Nuclear Materials at NNSS DAF

- **Large quantities of nuclear material**
  - U, Pu, Np
- **Multiple pieces of reflector materials**
  - Polyethylene, steel, DU, etc
- **Applications:**
  - Neutron/gamma diagnostic measurements
  - Benchmark measurements
  - Validation of instrument response, analysis/algorithm/simulation capabilities
  - Hands on training
- **Submit an experiment request: http://ncsp.llnl.gov/**

Key Objectives for our Measurements…

- **Benchmark Quality**
  - Material characterization
  - Assembly details
  - Experimental environment
- **Recent Focus on Uncertainties**
  - While planning, performing, and documenting the experiment
  - Uncertainties are as important as the mean!
- **Multiple Critical Configurations**
  - To determine reproducibility
  - With various reflectors to modify neutron spectrum
- **Examine More than Critical Configuration**
  - Sub-critical and super-critical
  - External neutron and gamma spectra
  - Internal foils and detectors to measure reaction rates and reaction products

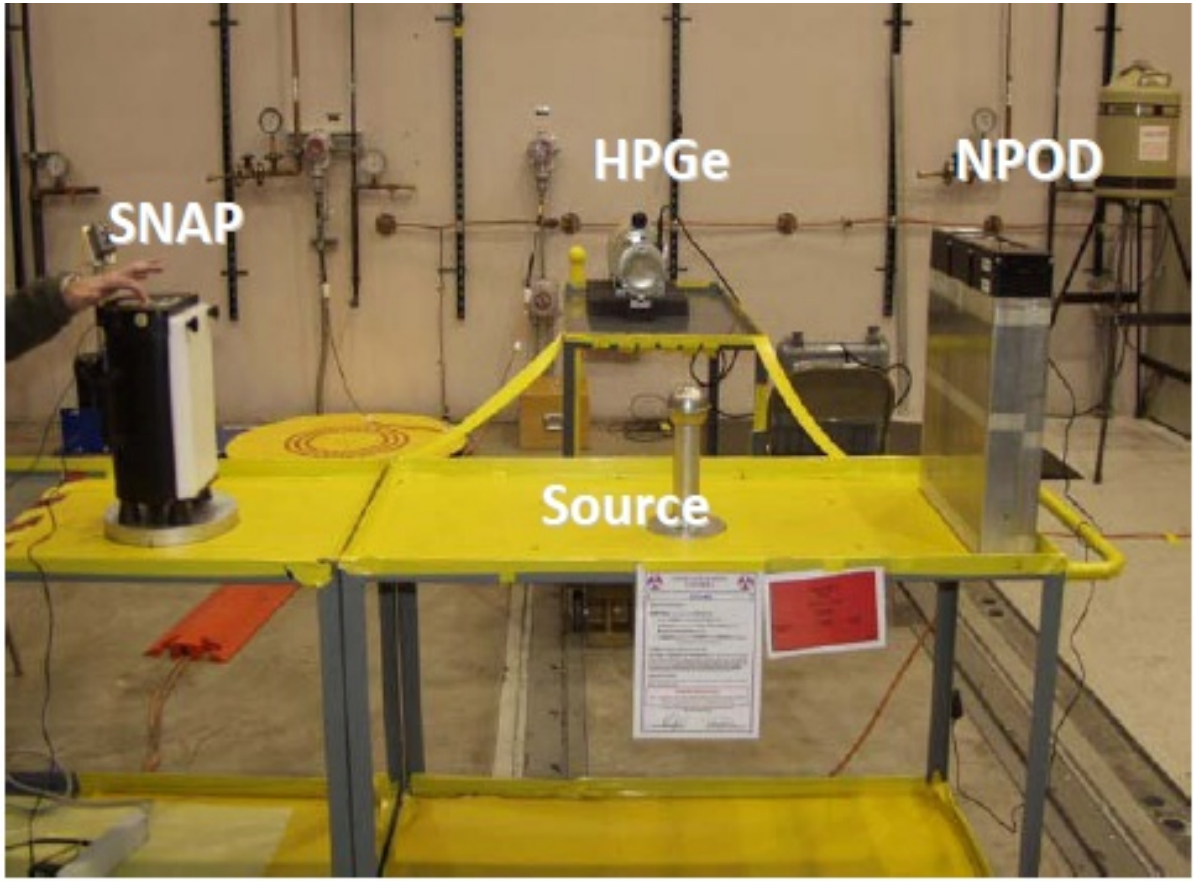
The goal of our simulations is to be predictive!

- **MCNP – well suited for comparing results of sub-critical measurements**
  - Calculates relevant quantities for fixed-source, eigenvalue problems
    - How faithfully is MCNP simulating fission process and what we measure?
    - Correct for averaged values but **not microscopically correct**. Energy is not conserved per collision
- **Other MCNP-like capabilities exist: MCNP-PoliMi, MCNP-DSP**
- **MCNP6 will replace these existing capabilities**
  - Key new capabilities: correlated neutron emission and list-mode data
  - NCSP has helped us revive this capability and validate our simulations
  - NA-22: CGMF (LANL) / Freya(LLNL) multiplicity packages are currently being integrated into MCNP6
    - Ongoing multiplicity measurements occurring at DANCE

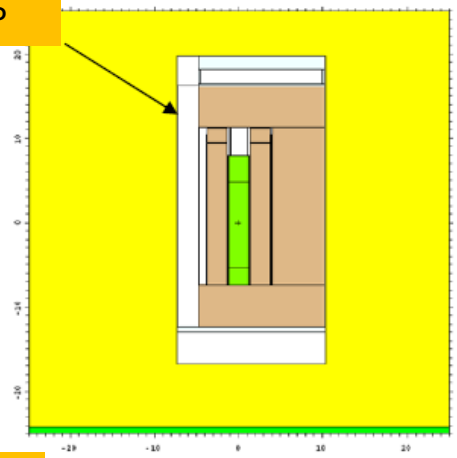
Our Recent Work…

1. Impact on nuclear data (ν, σ_ι, □)
2. Quantification of uncertainties/sensitivities in inferred values (M)

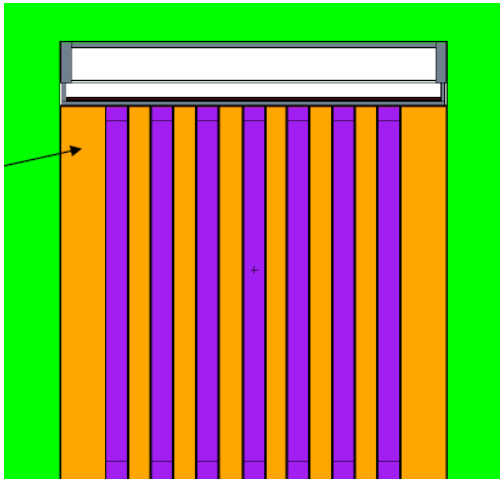
1st "Benchmark" Comparison: SNL - SINBAD



SNAP



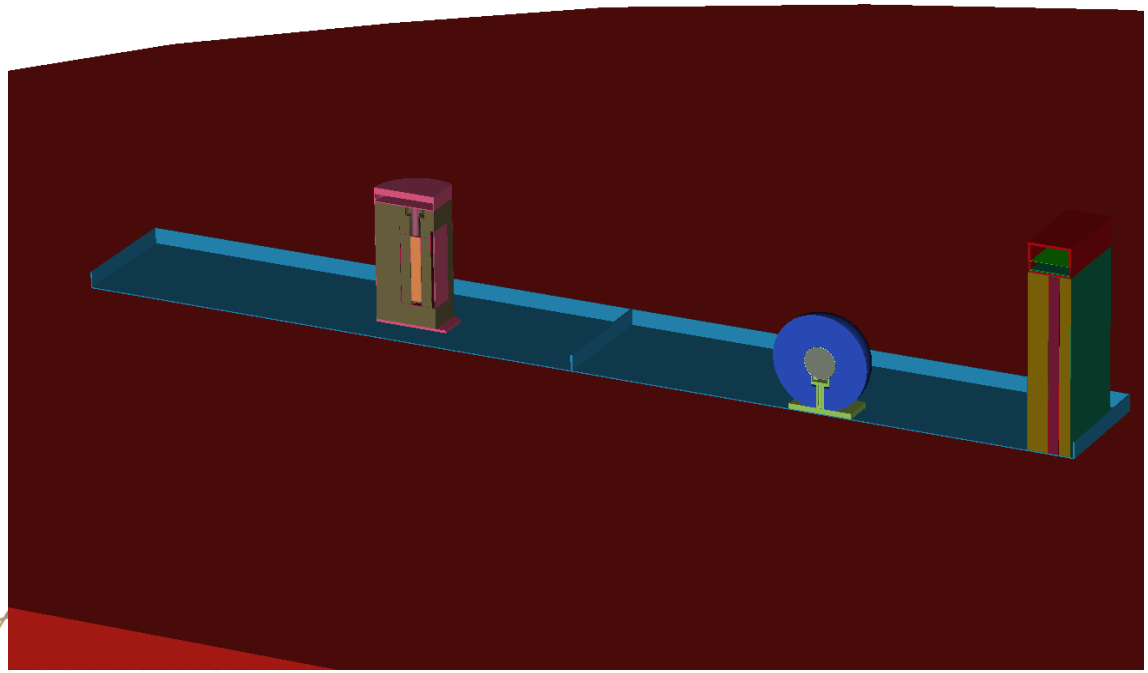
NPOD-3

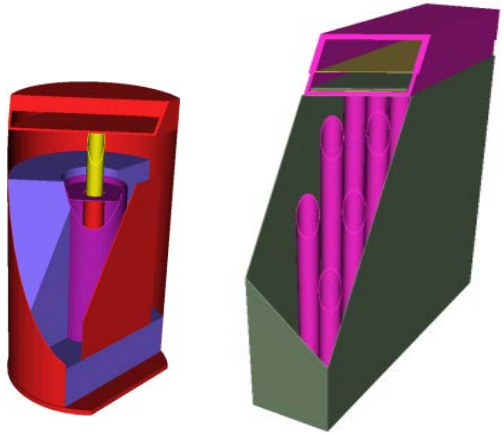


- J. Mattingly, "Polyethylene-Reflected Pu metal Sphere," SAND2009-5804

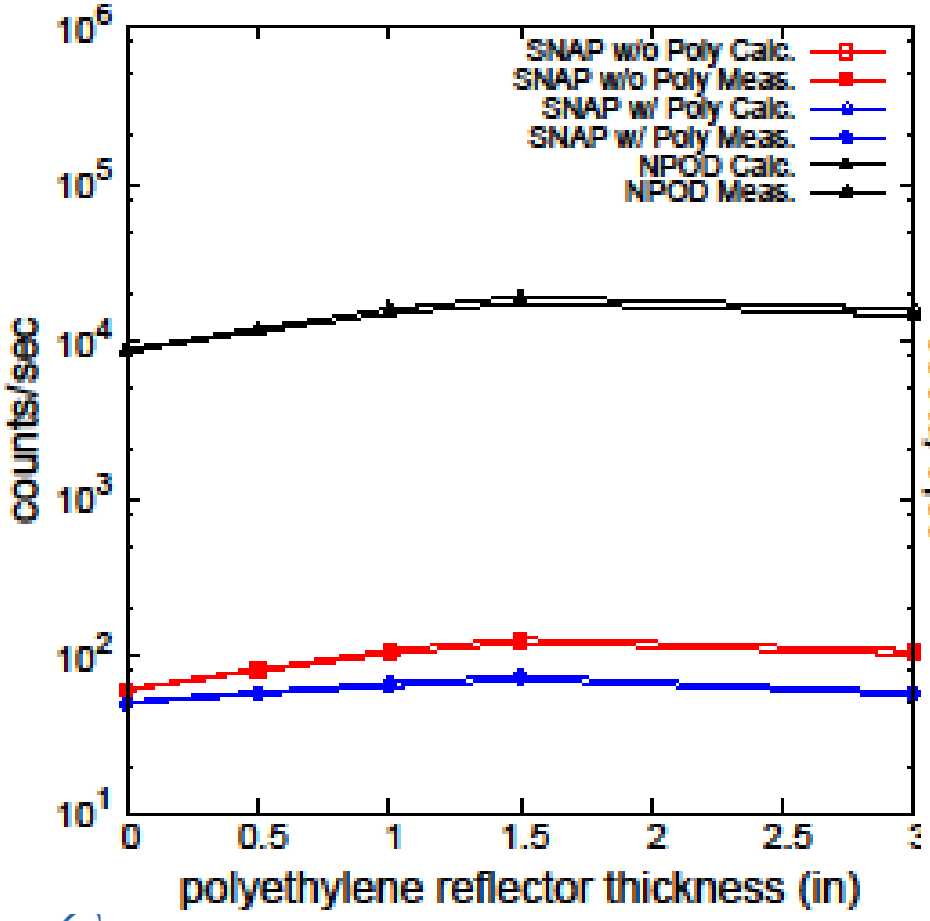
Measurement Configurations

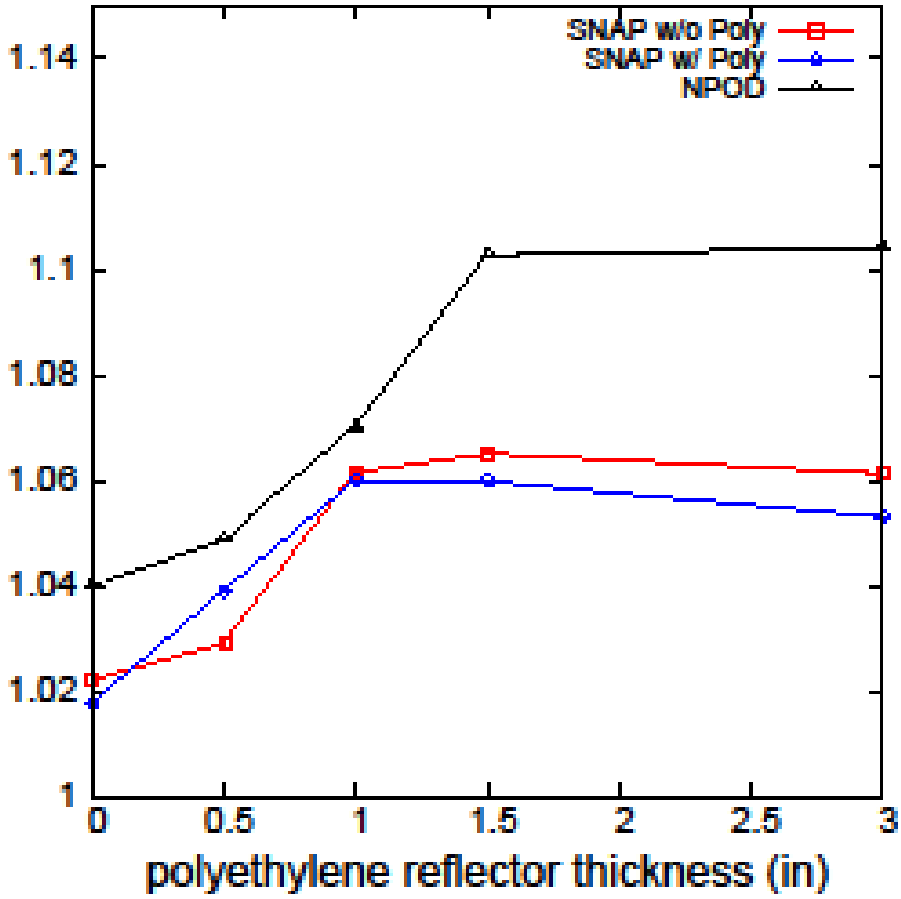
- **Solid Sphere of Plutonium (BeRP Ball)**
- **Different thicknesses of polyethylene shells are placed around the BeRP ball (0-6” of radial reflection)**
- **Neutron measurements are made with SNAP and NPOD detectors**



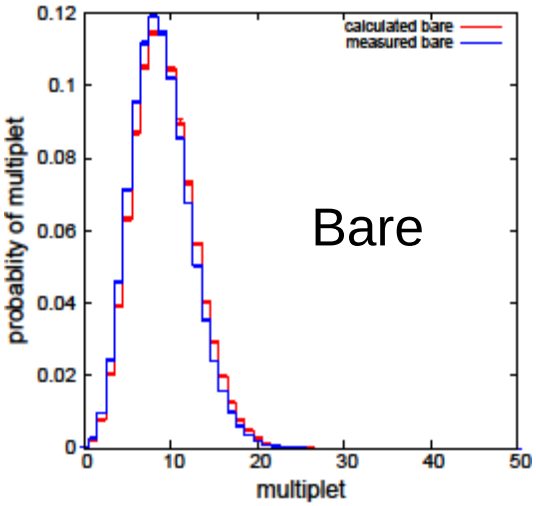


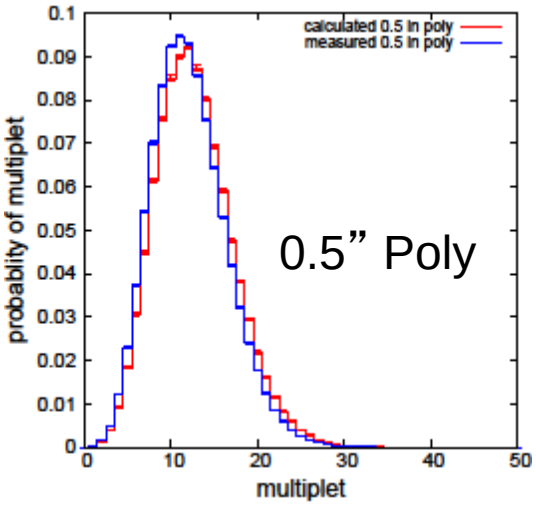
LANL Count Rate Comparisons

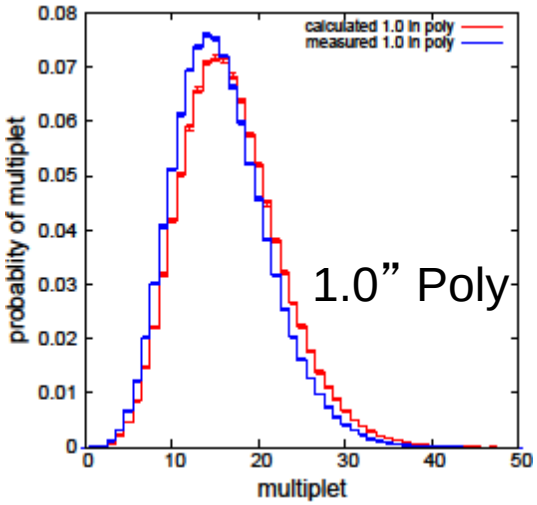


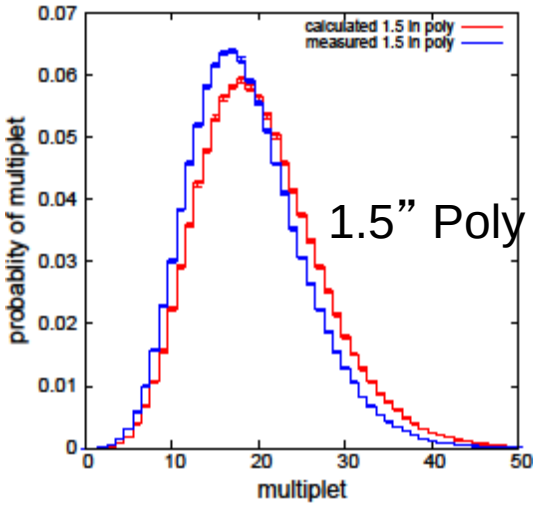


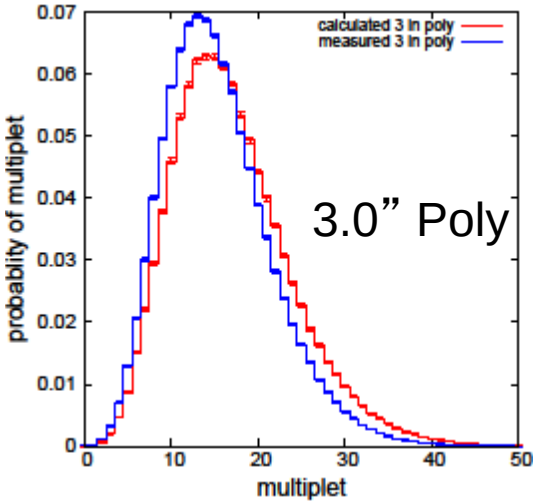
Multiplicity Distributions (1060 μs gate)









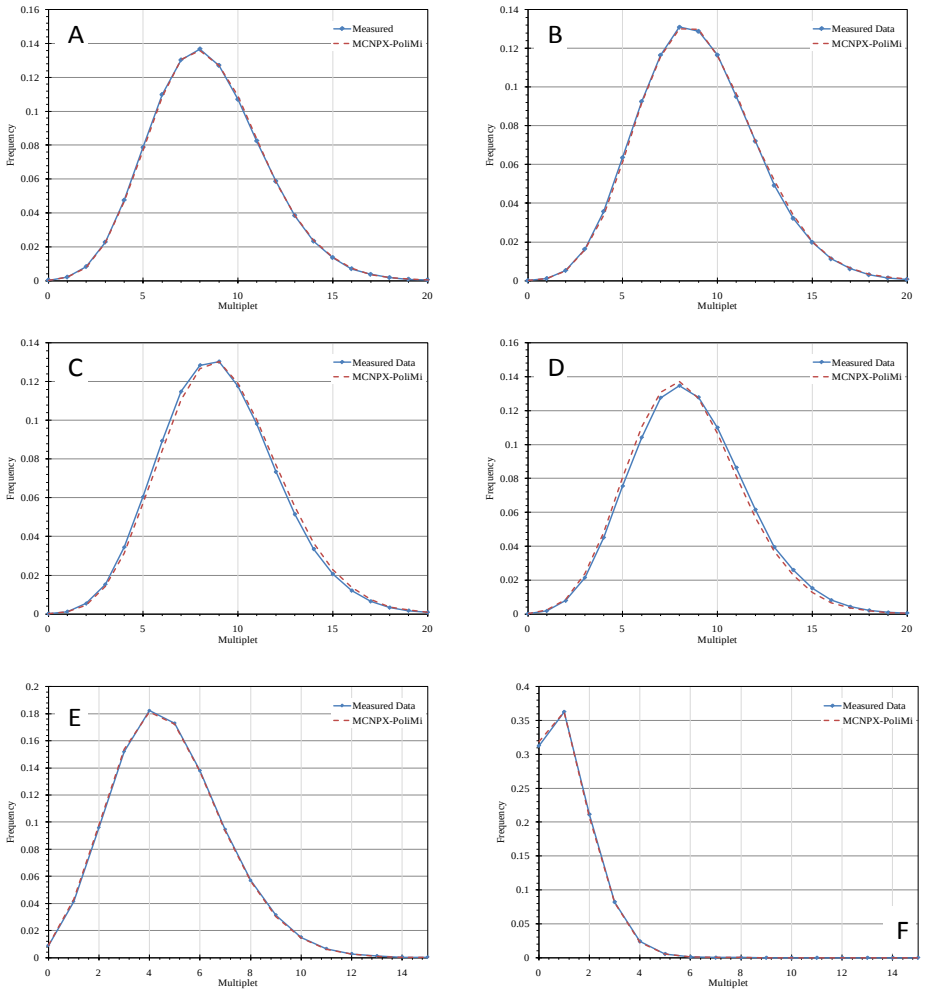


Multiplicity Distribution Moment Comparisons

Poly. Thick.	First Moment					Second Moment				
	Calc.	Rel Err.	Meas.	Rel. Err	C/M	Calc.	Rel. Err.	Meas.	Rel. Err.	C/M
Bare	9.31	0.0020	8.95	0.0010	1.04	99.13	0.0023	91.94	0.0012	1.08
0.5"	12.74	0.0020	12.15	0.0011	1.05	182.40	0.0023	166.24	0.0013	1.10
1.0"	16.84	0.0020	15.73	0.0013	1.07	317.08	0.0023	277.38	0.0015	1.14
1.5"	20.24	0.0020	18.63	0.0014	1.09	461.07	0.0023	390.29	0.0016	1.18
3.0"	16.97	0.0020	15.37	0.0013	1.10	335.87	0.0025	274.78	0.0016	1.22

Initial Conclusions

- Seem to be consistently over calculating the count rates and moments of the Feynman histograms
- Similar measurements and analysis made by E. Miller and J. Mattingly (collaborations with LANL)
- No dead time issues, source-detector uncertainty effects quantified
- Measurements were also performed using a Cf-252 source inside each reflector
- This test validated the geometry and material models of the poly reflectors and the NPOD



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Summary from Moderated point & WGPu

- **Simulation of two different polyethylene reflected plutonium experiments exhibit similar systematic trends**
- **Two independent modifications to MCNP produce the same systematic trend**
  - most modeling errors have been investigated (Mattingly et. al.)
  - 239Pu ν , σ_f , and possibly χ may be the source of the error
- **these subcritical differential experiments are highly sensitive**
  - (correlated counts on 100s–1000s μs) to input data
- **subcritical experiment and critical experiment data, including but not limited to keff, should be used for data regression**

Investigation into the nuclear data

- **Nubar is adjusted by CSWG to make simulations match**
  - In 239Pu artificially high below ~1.5 MeV to match JEZEBEL
  - Lies about 2 std. dev above covariance data
- **Let's use covariance data to generate random samples of energy dependent nubar**
- **Input the new samples of nubar into MCNP**
- **Run simulations and compare to experimental data**
- **Repeat until a set of nubar that matches multiplicity distribution without significantly altering keff is found**

Methodology: Use covariance data to vary nu-bar

- **Read in original nubar from ACE: $\nu\ (E)$**
- **Read in covariance data from ENDF/B-VII.0**
- **Generate a set of correlated random numbers: $\mathbf{R}$**
  - Each element of $\mathbf{R}$ corresponds to an energy group Eg
  - Each element $\mathbf{R}$ Eg is the number of std. dev. to shift ν
- **Generate a new a new set of nubar:**
  - Need to map ν energies to groups Eg:
  - E in Eg: $\nu'E=\sigma E$ $\mathbf{R}$ $Eg+\nu(E)$
- **Rewrite the new data $\nu'(E)$ to a unique ACE File**
- **Run 6 different MCNP simulations**
  - 5 different BeRP ball experiments
  - JEZEBEL fast critical experiment
- **Generate multiplicity distributions and use FOM to evaluate best data set**

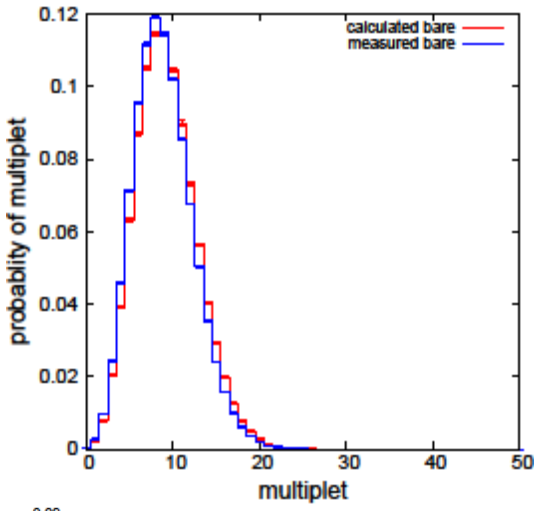
Does Nu-bar change impact critical experiments?

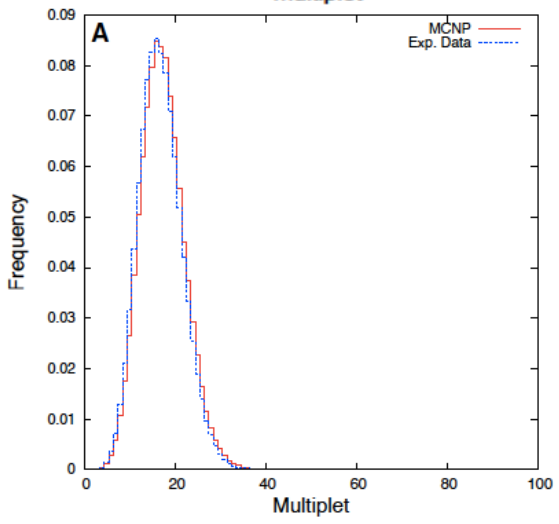
k_{eff} comparison between Trial with lowest FOM and original ENDF7 Data

	Benchmark		Original ACE Data			Lowest FOM		
Benchmark	k-eff	Rel Error	k-eff	Rel Error	# Sigma	k-eff	Rel Error	# Sigma
pu-met-fast-021-case-1	1.0000	0.0026	1.0021	0.0003		1.0001	0.0031	
pu-met-fast-003-case-103	1.0000	0.0030	0.9981	0.0003		0.9958	0.0031	**
pu-comp-inter-001	1.0000	0.0110	1.0121	0.0002	*	1.0099	0.0029	*
mix-comp-therm-002-case-pnl30	1.0024	0.0060	1.0011	0.0003		0.9983	0.0043	
mix-comp-therm-002-case-pnl31	1.0009	0.0047	1.0025	0.0003		1.0004	0.0044	*
mix-comp-therm-002-case-pnl33	1.0024	0.0021	1.0079	0.0003	**	1.0046	0.0042	*
mix-comp-therm-002-case-pnl34	1.0038	0.0025	1.0042	0.0003		1.0017	0.0029	*
mix-comp-therm-002-case-pnl35	1.0029	0.0027	1.0066	0.0003	*	1.0036	0.0033	*
	TOTAL RMS DIFFERENCE:		0.49%			0.51%		

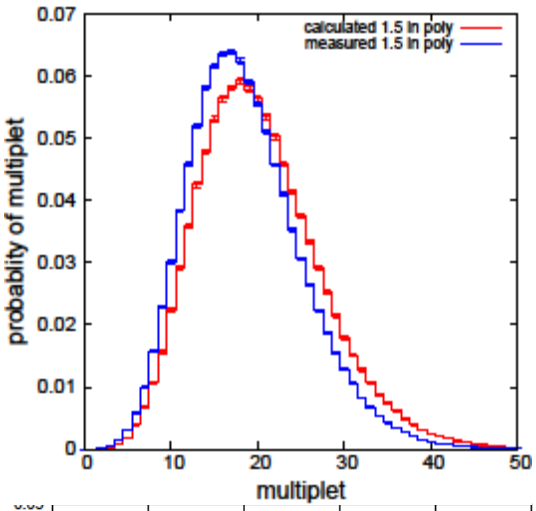
Effects of nu-bar change on moderated WGPu

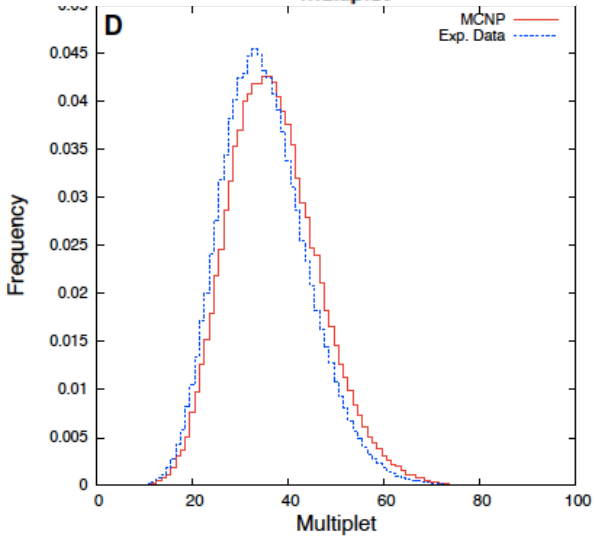
Bare



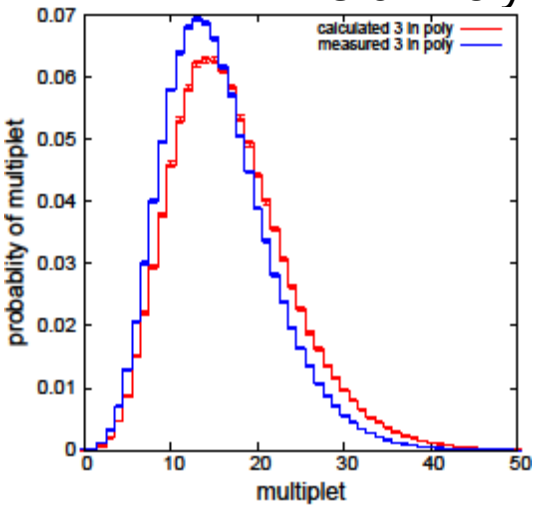


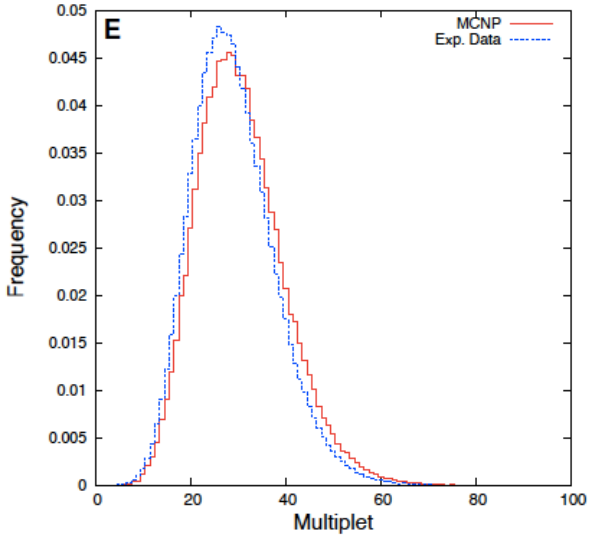
1.5” Poly



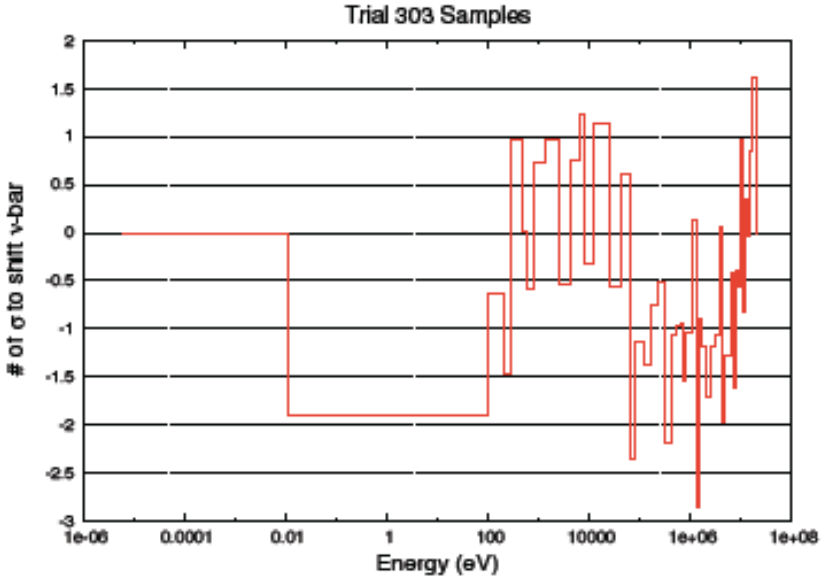


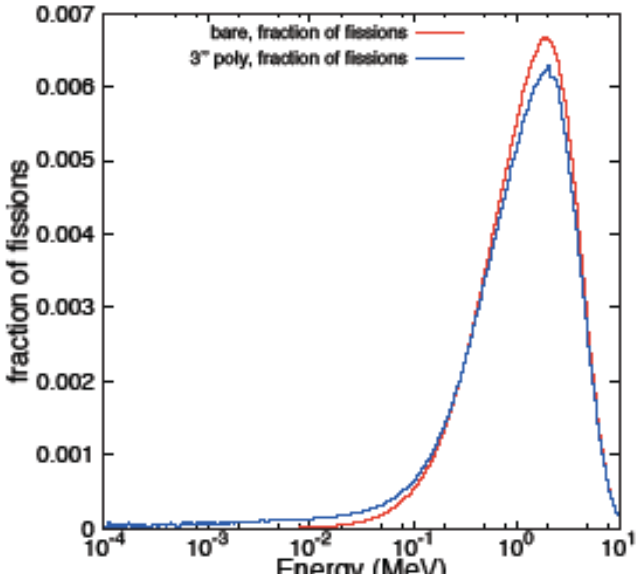
3.0” Poly





Modification to Pu-239 nu-bar





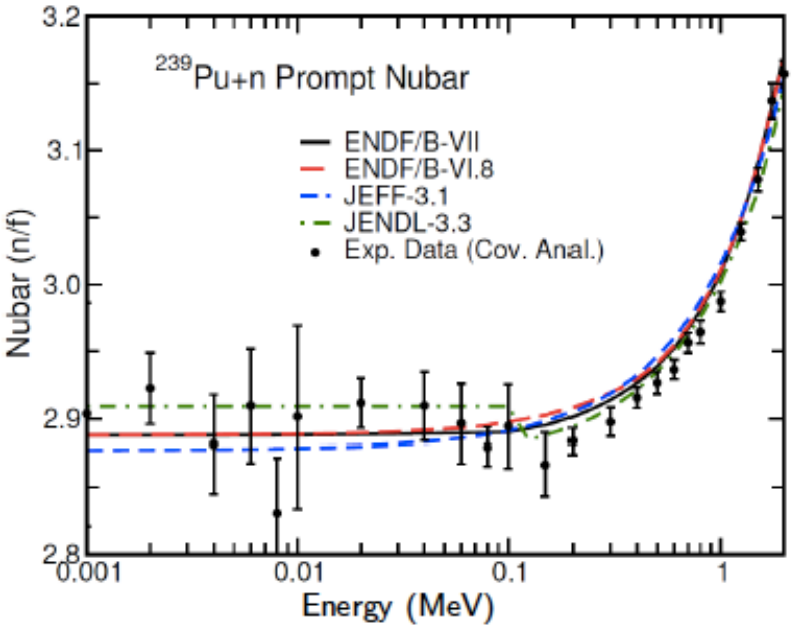
Modified $\overline{\nu}$ data (top-left) suggests:

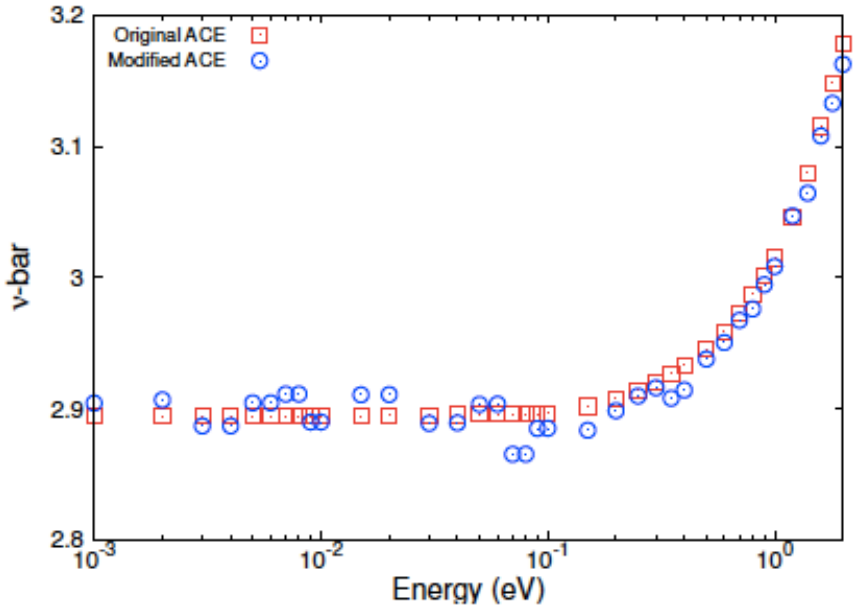
1. a decrease to Pu-239 $\overline{\nu}$ above ≈100 keV
2. an increase to Pu-239 $\overline{\nu}$ below ≈100 keV

The spectrum of neutrons causing fissions (top-right) in this system

1. is highly peaked around fission energies with only 2-5% of the fissions induced by neutrons less than 100 keV
2. indicates that the modifications to $\overline{\nu}$ below ≈10 keV and above ≈10 MeV are not meaningful

Comparison of Modified Data with Young et al.

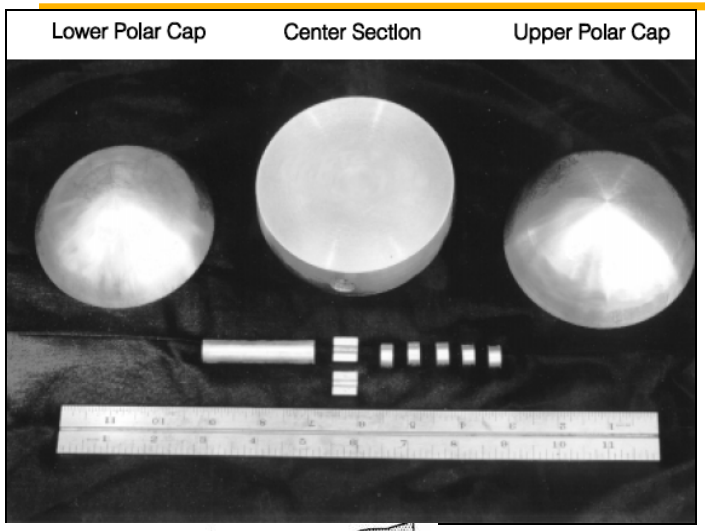


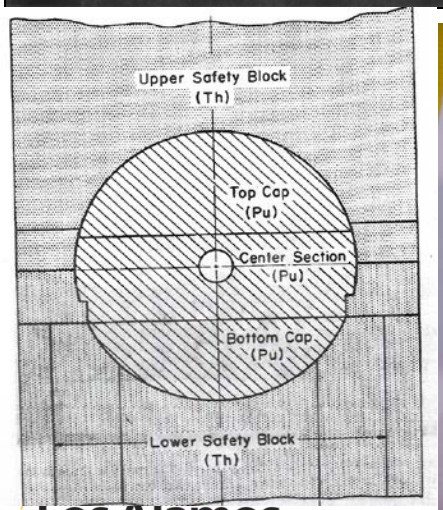


The Young et al. “raw" data was shifted upward to match the integral Jezebel keff

However, the Young et al. “raw" data is seemingly more consistent with the “best" match to the NPOD simulations

Quantifying Uncertainties in Sub-critical Multiplying Systems: Mass and Position Perturbations

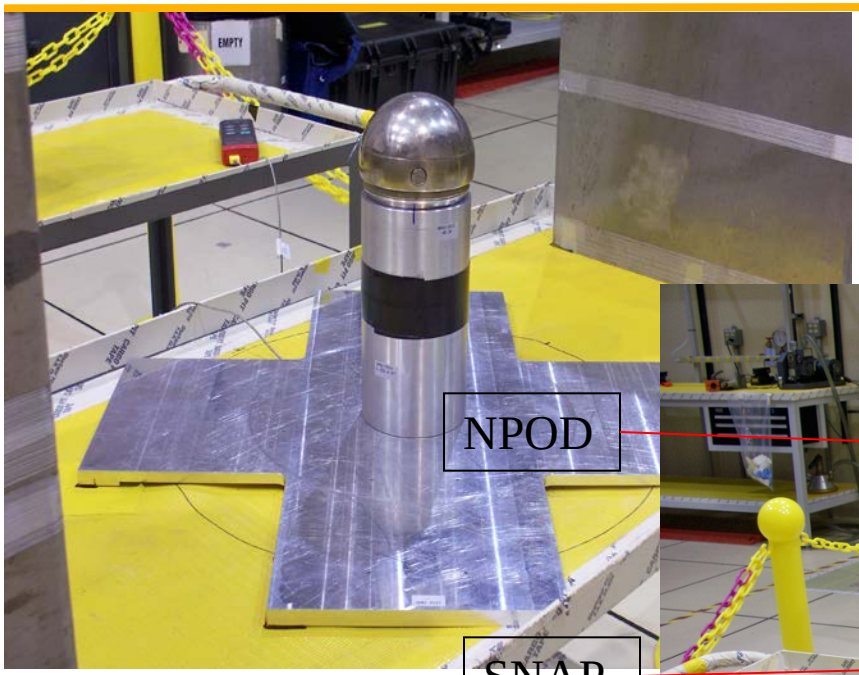


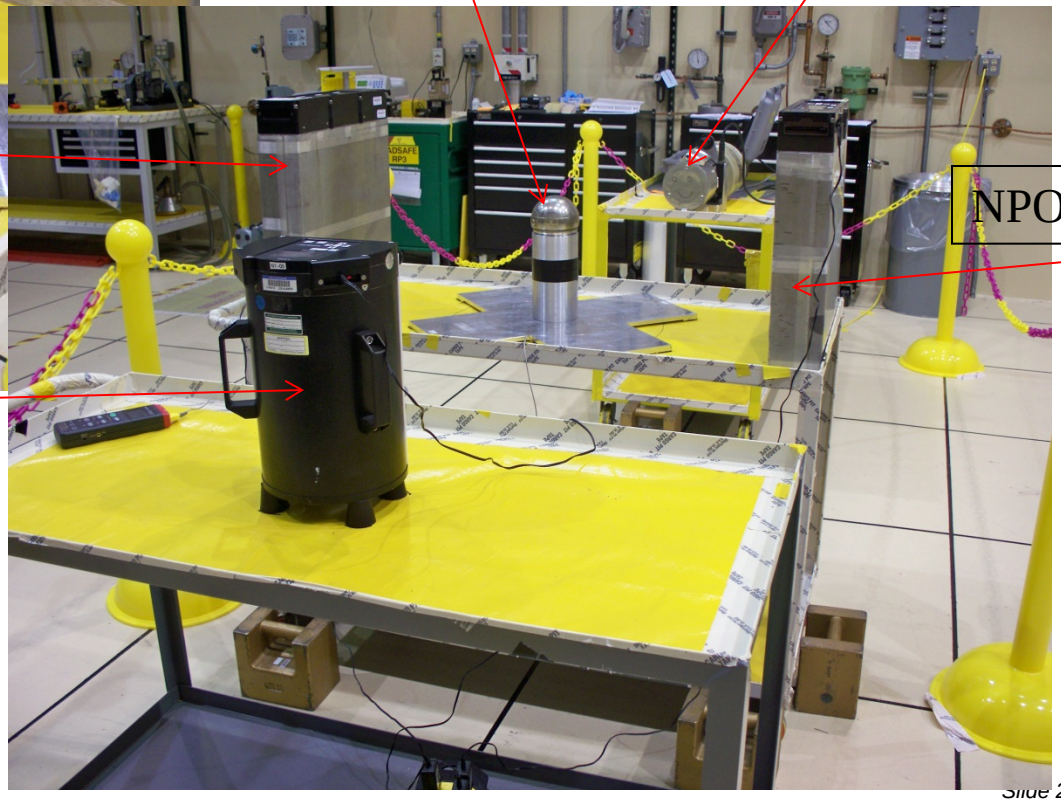


Configuration Number	Thor Core			Glore hole mass (g)
	Lower	Center	Upper	
1	+			0
2		+		0
3		+		206.9
4			+	0
5	+	+		0
6	+	+		206.9
7	+		+	0
8		+	+	0
9		+	+	206.9
10	+	+	+	0
11	+	+	+	206.9
12	+	+	+	109.5
13	+	+	+	49.0
14				206.9

The upper, center, and lower sections have net masses of 3273.9 g, 4158.2 g, and 2216.9 g respectively.

When put together, the three pieces form an approximate sphere with a total net mass of 9649.0 g.





Thor

HPGe

NPOD

NPOD

SNAP

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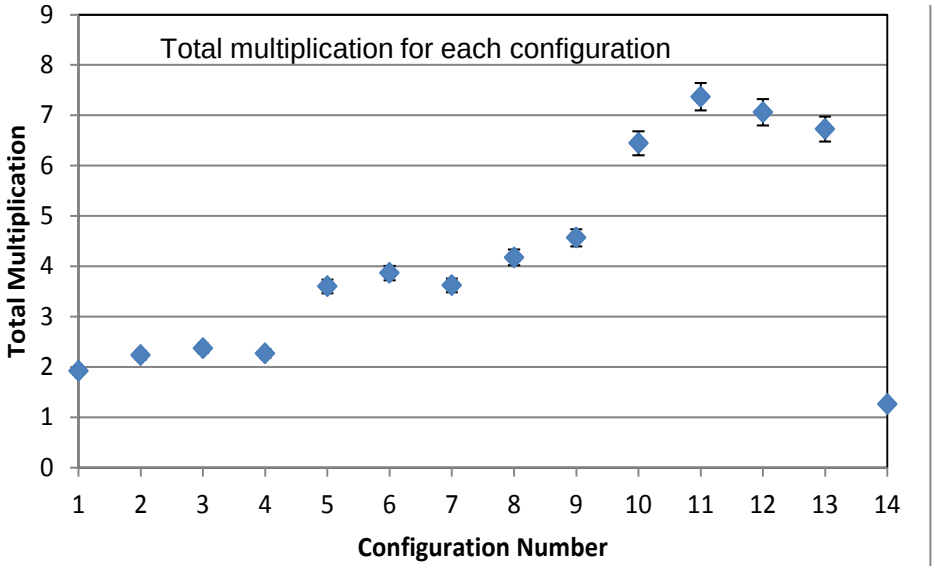
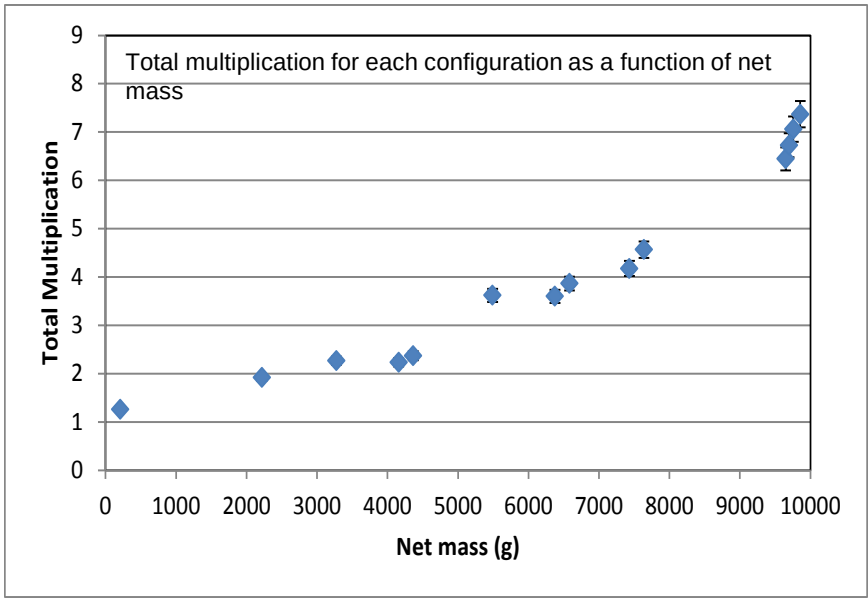
Are these configurations distinguishable?

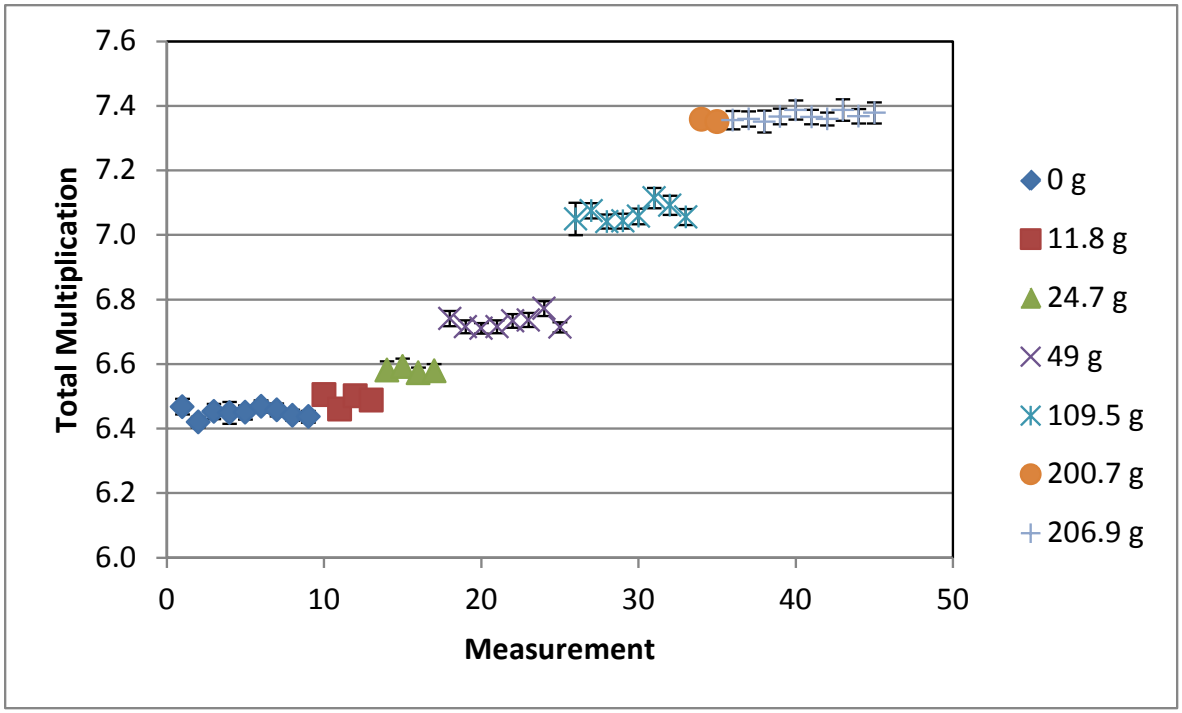




- Error bars are total uncertainty in the measurement (due to measured data, detector efficiency, and nuclear data)

Mass Perturbations

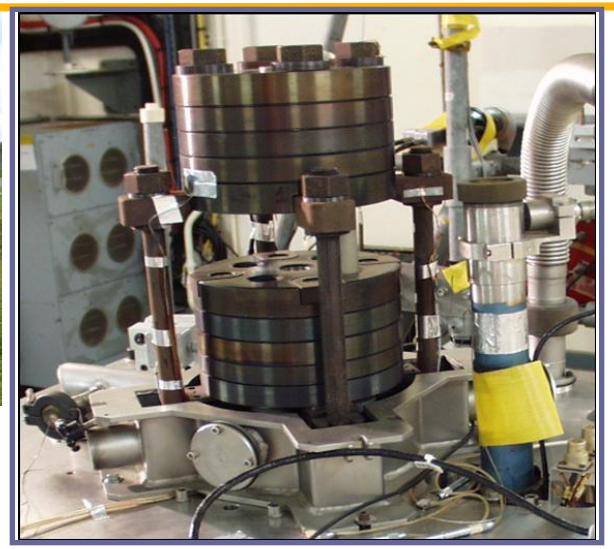
- **Measured total multiplication for the full Thor core.**
- **Error bars are uncertainty due to the measured data only (detector efficiency and nuclear data not included).**



- **Full Thor Core**
  - 6.2 g difference is not distinguishable (200.7 g same as 206.9 g)
  - 11.8 g has a standard deviation between 1 and 3.
  - 12.9 g or greater is easily distinguishable.
- **Thor Core Center:**
  - 24.7 g is equivalent but 151.7 g clearly different.

As mass (and consequently multiplication) increases, the mass threshold decreases.

Sub-critical measurements on critical assemblies: Caliban and Godiva



- **Metallic cylindrical core**
  - ^{235}U and Mo alloy
  - Diameter = 19.5 cm
  - Height = 25 cm
  - 113 kg of HEU
- **2 blocks**
  - A mobile one (below)
  - A fixed one (above)
- **4 control rods**
  - Same composition as the blocks
- **Cylindrical central irradiation channel**

- Cylindrical uranium metal fast burst assembly
- 65 kg, 93% enriched
- 7-inch diameter (17.8 cm), 6-inch tall (15.2 cm)
- Operates at delayed critical or prompt critical
- Maximum burst is approximately 90,000 MW_{th} with a full width half max (FWHM) pulse of 25 μs

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Results: Sub-critical assemblies

- **Joint CEA/LANL measurements performed on four critical assembly machines (this work only focused on Caliban and Godiva-IV results).**
- **Over 50 configurations were measured. Configurations were at various subcritical multiplications as well as at and above delayed critical.**
- **Results were favorable:**
  - The largest Δk_{eff} between the measured data and the control rod worth curves was 0.008.
  - The largest Δk_{eff} between the measured and simulated data was 0.009.
- **CEA**
  - Chapelle to defend thesis in May 2014.
- **US**
  - Apply recent subcritical uncertainty analysis to all measured data.
  - Document list-mode simulations for all configurations.
  - Analyze Planet and Flat-Top results (“best” nubar for Flat-Top?).
  - Investigate SCRAM and other measurements.

Future plans

- **Historical measurements have included:**
  - Point sources (bare, moderated) (SF, (a,n))
  - Spherical sources (WGPu, HEU), point sources (SF, (a,n))
  - Critical Assemblies (Caliban, Godiva)
- **NCSP sponsored simulations/measurements of sub-critical systems have:**
  - Helped to eliminate multiple MCNP codes versions; maintain one modern version
  - Driven us to examine both correlated neutrons and gamma emissions (simulation)
  - Re-examine fundamental nuclear data
  - Quantify and document our uncertainty methodology
  - Examine sensitivity/uncertainty in our measurements to determine M
- **What's next?**
  - University research reactors?
  - Refine our analysis capabilities for unknown, sub-critical, multiplying systems
- **Lots of data has been accumulated**
  - Analysis of this data is needed

Backup Slides

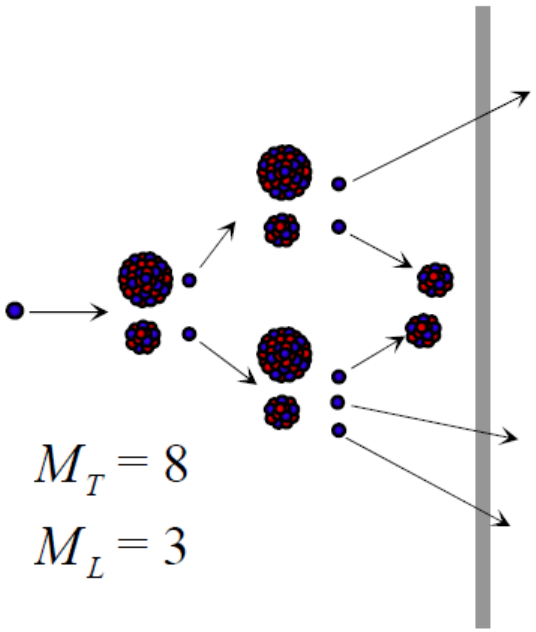
Review: Keff and multiplication

- **k_{eff} is the multiplication factor**
- **k_{eff} is** $$k = \frac{Number\ of\ neutrons\ in\ one\ generation}{Number\ of\ neutrons\ in\ preceeding\ generation}$$
- **k_{eff}<1 (subcritical); k_{eff}=1 (critical), k_{eff}>1, (supercritical)**
- **k_{eff} is a system global neutron balance**
  - It cannot be measured but only inferred from other aspects of a system
  - Depends on
    - Material properties of the system
      - ^{239}Pu, ^{235}U: Non-fissile nuclide such as ^{238}U, ^{240}Pu, etc. can also affect k_{eff}
    - Metal, oxide
  - Energy of neutrons in the system
  - Physical properties of the system: mass, volume, shape
  - Leakage vs. absorption
    - Surface area, scattering, capture, fission

Review: Keff and multiplication

- **Neutron Multiplication is the total number of neutrons that would be generated through fission from a single starter neutron**
- **Not the same as the multiplication factor!**
- **Only for subcritical systems**

$$M_T = \frac{1}{1 - k_{eff}}$$



k_{eff}	M_T
0.5	2.0
0.90	10.0
0.95	20.0
0.99	100.0

Review: Relevant Physics for Sub-Critical Multiplication

- **(Source) Passive neutrons from fissionable material emitted from:**
  - Spontaneous fission neutron sources:
    - Correlated in time and location of fission
    - Examples: Cf-252, Pu-240
  - (α ,n) reactions:
    - Produced when α particle is absorbed and neutron is emitted
    - Not correlated in time and location
    - Examples: Am-Be
- **(Induced) Neutron multiplication dominated by two physical processes:**
  - (n,2n)
    - Occurs mostly above 7 MeV
    - Does not contribute significantly to most scenarios
  - Neutron-induced Fission

Simulations need to be "microscopically" correct for comparisons with measured data.

Sub-critical Measurements

- **Neutron multiplication measurements (passive and active) designed to separate the correlated emission events, e.g., SF, from the uncorrelated events, e.g. , (α,n),**
  - Record time of neutron capture in a neutron detector over a large collection time (e.g. 300 sec)
  - Group these capture times in a large number (e.g. 1 M) smaller time sub-intervals (e.g. 250 μsec)
    - These time sub-intervals are larger than typical neutron detection and lifetimes (~50 μsec)
  - Multiplicity histogram is constructed
    - Obeys Poisson statistics if system is non-multiplying (i.e. neutrons are emitted randomly in time)
    - Data analysis begins…Feynman Variance-to-Mean, CSDNA, etc

Data recorded is neutron detector location and time: list-mode data

Simulations need to produce list-mode data for comparisons

MCNP Simulations of Sub-Critical Measurement

- **MCNP simulation of neutron sources and detection**
  - User can define location, direction, energy, time, and intensity of SF, (α,n) neutron sources
  - User **cannot** define fission events
    - e.g. sample number of neutrons emitted from ν_{bar}
  - User **cannot** define correlated (time, location) neutron sources
    - MCNP samples these values from user's input
  - User cannot (easily) record location and time of detection.
    - Possible using MCNP's PTRAC capability and a user-created external script to extract this information

Standard MCNP is not microscopically correct enough to compare with current sub-critical measurements:

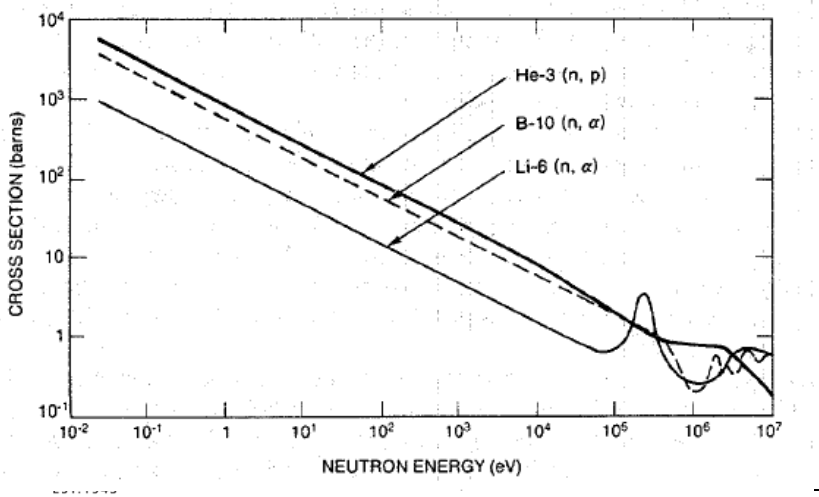
MCNP Modifications

- **SOURCE/TALLYX subroutines written**
  - Subroutines intended for user-defined source and tally
  - Independent of MCNP – no modification of MCNP is required
  - Works with MCNP5.1.60 (current release)
- **SOURCE—modifies definition of starting particle to be a starting reaction, e.g., spontaneous fission, samples starting particles from that reaction**
- **TALLYX—modifies a type 4 tally to produce a file of absorption times in tally cell, a.k.a. list-mode data**
- **Tally output can be processed to analyze inferred multiplication**

Adaptation into MCNP6 with documentation, validation benchmarks, etc in progress

Review: Sub-critical Measurements - Radiation Detection

- **Nearly all neutron detectors actually detect secondary radiation (protons, alpha particles, fission products, etc).**
  - Capture, scattering, and fission based detectors exist.
- **Some have threshold energies but most take place at thermal energies (moderating materials may be used to slow down the neutrons which will increase the detector efficiency) due to the cross-section of the detection media.**
- **Neutron detectors generally give a signal when a neutron is absorbed but most other information is lost (such as energy).**



- **Many data acquisition systems exist:**
  - Scalers/counters
  - Coincidence circuits
    - Shift-register
  - List-mode

Operated by Los Alamos National Security, LLC for NNSA

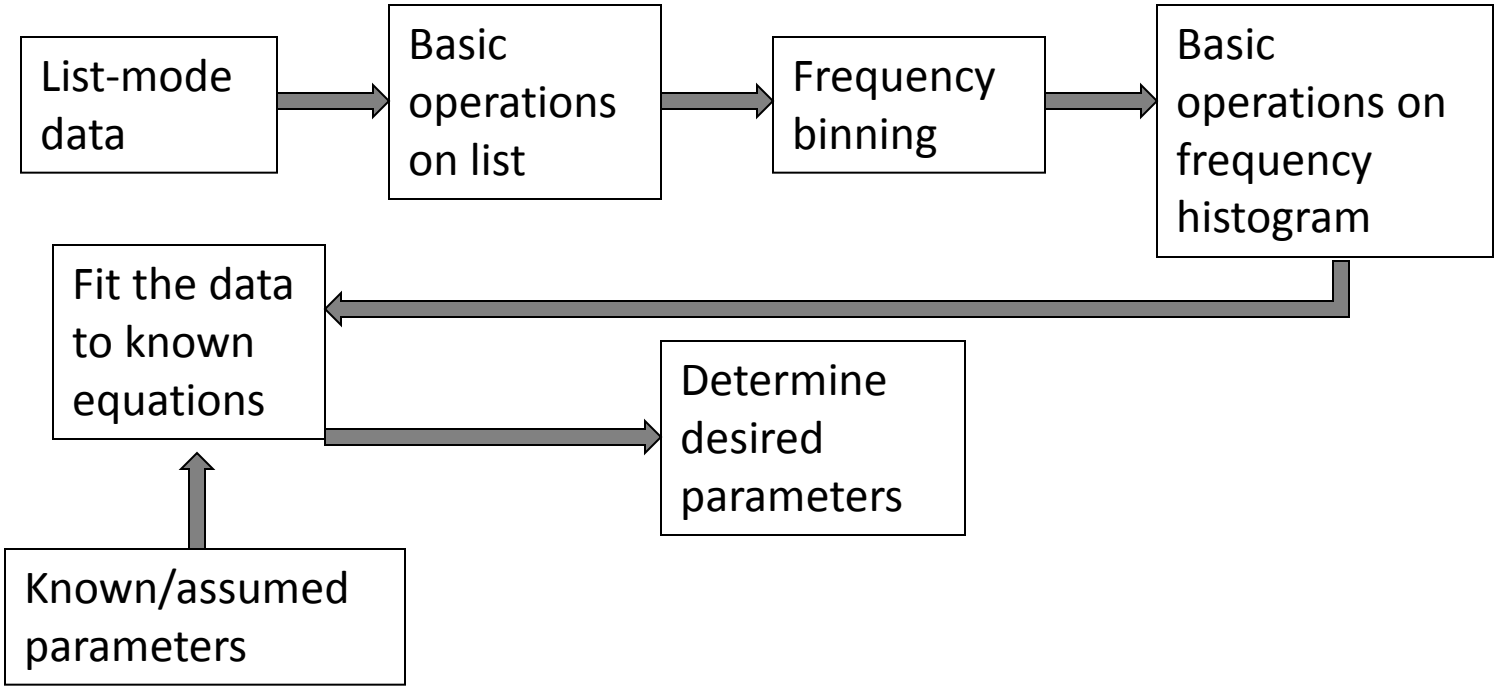
Review: Sub-critical Measurements-Radiation Detection – Gross Counting -

- **Things we can learn from gross neutron counting:**
  - The location of sources of neutrons/confirmatory measurements.
  - If the efficiency of the detector system is well known, one can estimate the neutron emission rate of a system.
  - If multiple measurements are performed it might be possible to learn the following (note that here the efficiency does not need to be known):
    - Relative multiplication of a system (comparison of the multiplication of two similar systems).
    - Multiplication of a system (if the multiplication from one of the measurements is known from other means or if replacement measurements are performed).
    - Absorption properties of materials.
    - Room return information.

Review: Sub-critical Measurements-Radiation Detection – Correlated Neutron Analysis-

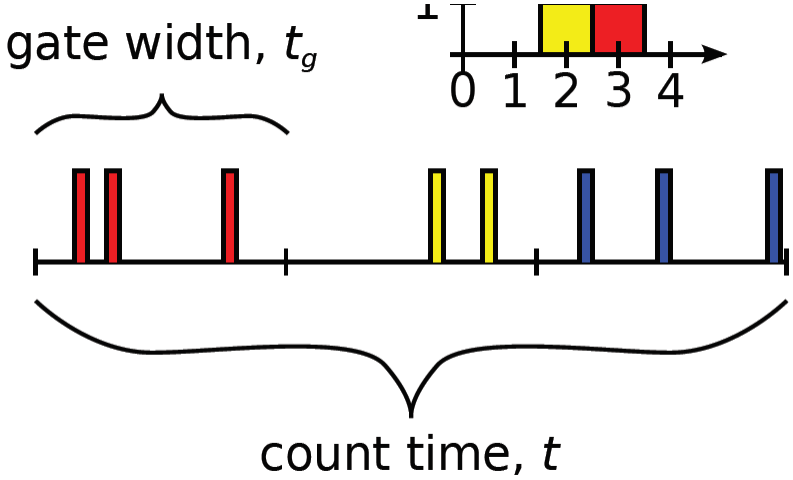
- **Many methods exist. Most take advantage of the physics properties shown in the previous slides (particularly the fact that multiple prompt neutrons are emitted "instantly" after fission).**
- **Most methods were developed for zero power reactors and later applied to subcritical systems or power reactors.**
  - Most of the work took place from 1940s-1960s.
- **Differ in energy response, timing required, detection system required, multiplication over which the method is valid, etc…**
- **Time-Correlation Analysis**
  - Rossi-Alpha, Interval distribution (Babala), Correlation analysis
- **Moments Analysis**
  - Feynman variance (Hansen-Dowdy or Hage-Cifarelli), Bennett Variance
- **Probability of Neutron Detection**
  - Zero-Count probability (Mogilner)

Review: Sub-critical Measurements - Data Analysis

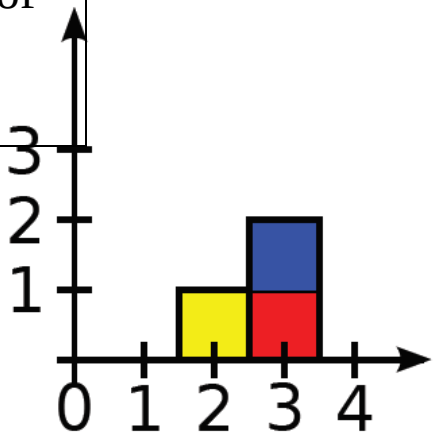


Sub-critical Data Analysis: Construct Feynman Histogram

Data are separated into gates



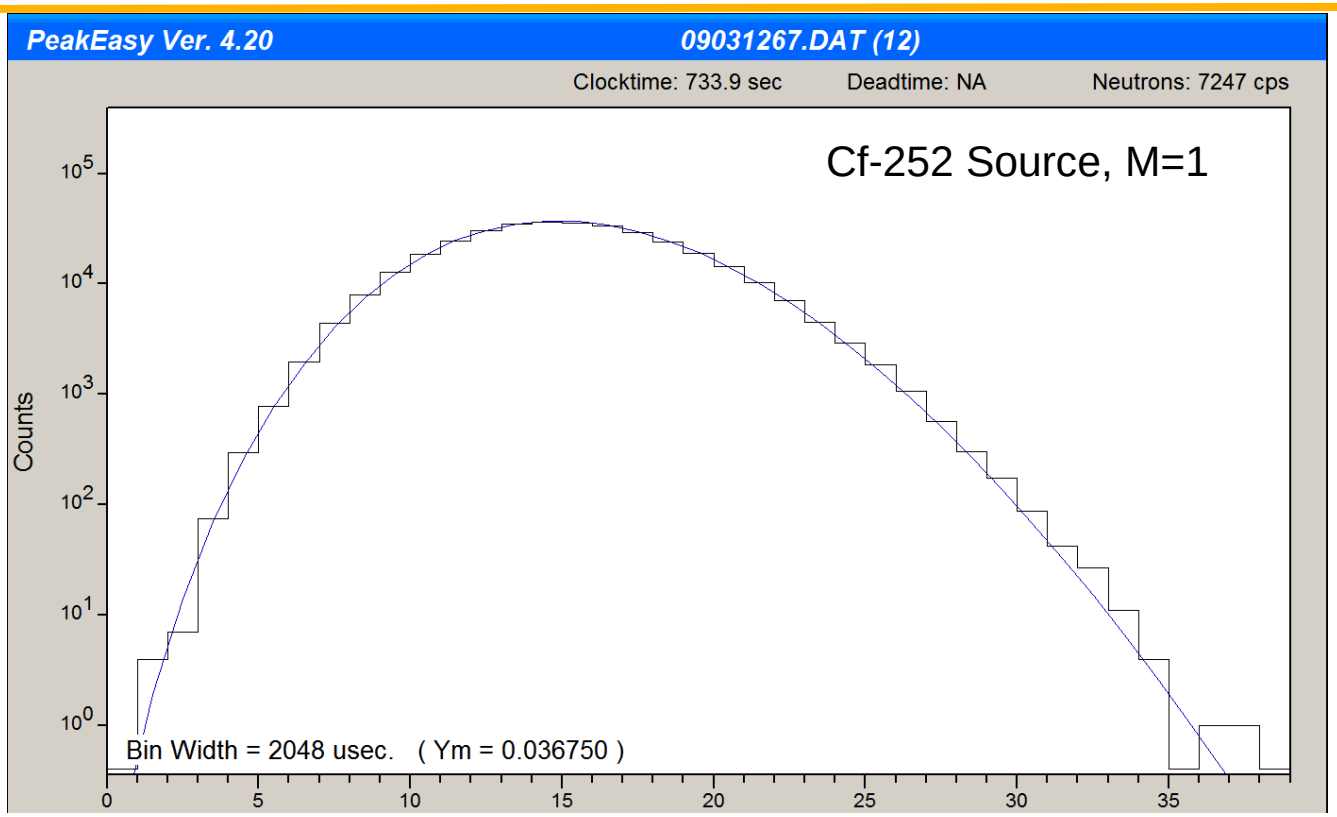
y-axis is the number of gates that contained exactly n events (C_n)



x-axis is the number of neutrons recorded in the gate (n)

Then frequency binning is performed

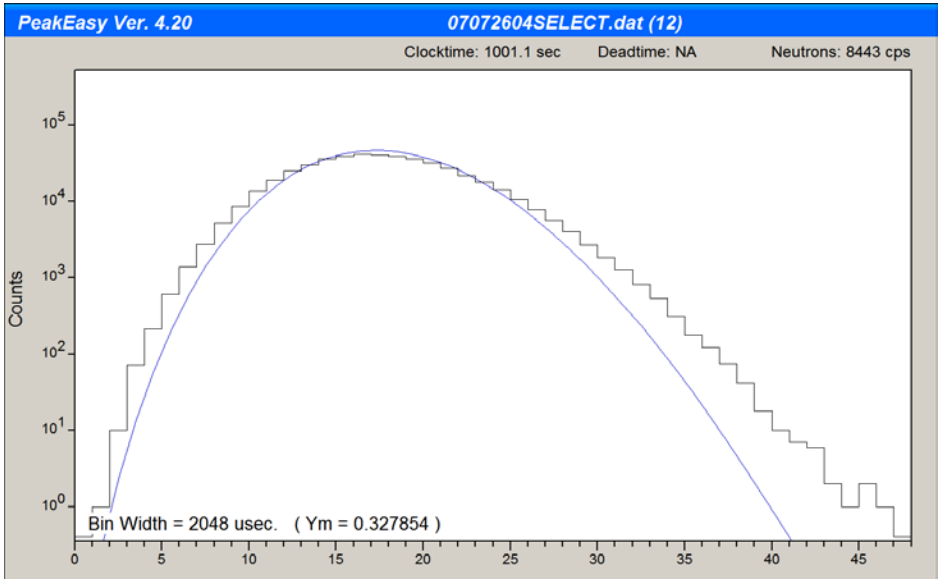
Sub-critical Data Analysis: Feynman Histogram Randomly fissioning source follows Poisson Distribution



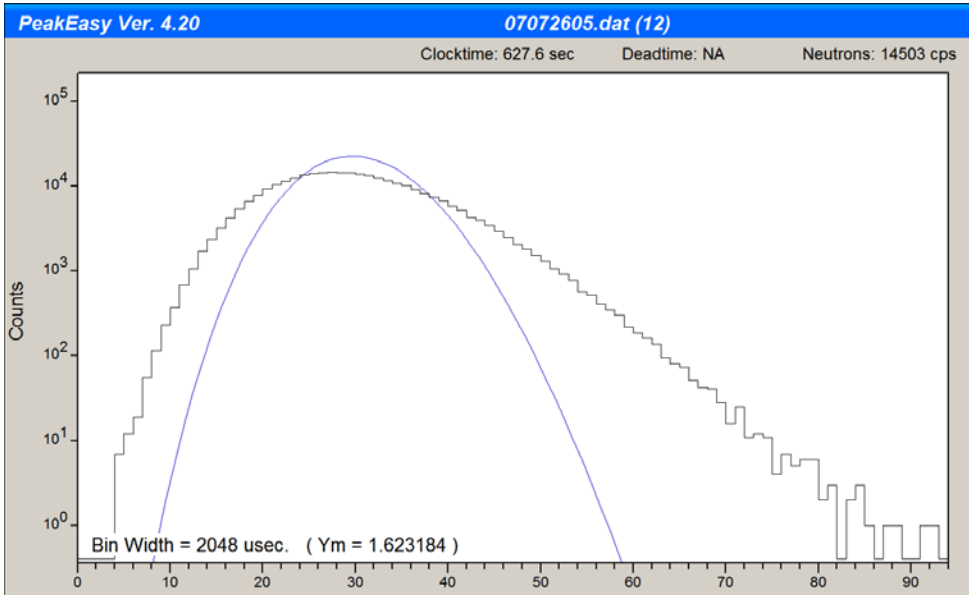
Feynman distributions are constructed and the deviation of the distribution from a Poisson gives us information about the multiplication of the system.

Sub-critical Data Analysis: Feynman Variance-to-Mean Multiplying source Deviates from Poisson Distribution

- **As the multiplication of a system increases, the deviation from a Poisson distribution will increase.**



BeRP Ball (4.5 kg Pu sphere), M=4.4



BeRP Ball reflected by 3” polyethylene, M=13.5

Hage-Cifarelli formulism

- **Moments are a measure of the shape of data points.**
  - 1st moment is the mean, 2nd is the variance or width.
- **Reduced factorial moments:**

$$\overline{m_{1!}} = \frac{\sum nC_n}{1!\sum C_n}$$

$$\overline{m_{2!}} = \frac{\sum n(n-1)C_n}{2!\sum C_n}$$

$$\overline{m_{3!}} = \frac{\sum n(n-1)(n-2)C_n}{3!\sum C_n}$$

Numerator is the total number of neutrons detected.

Hage-Cifarelli formulism

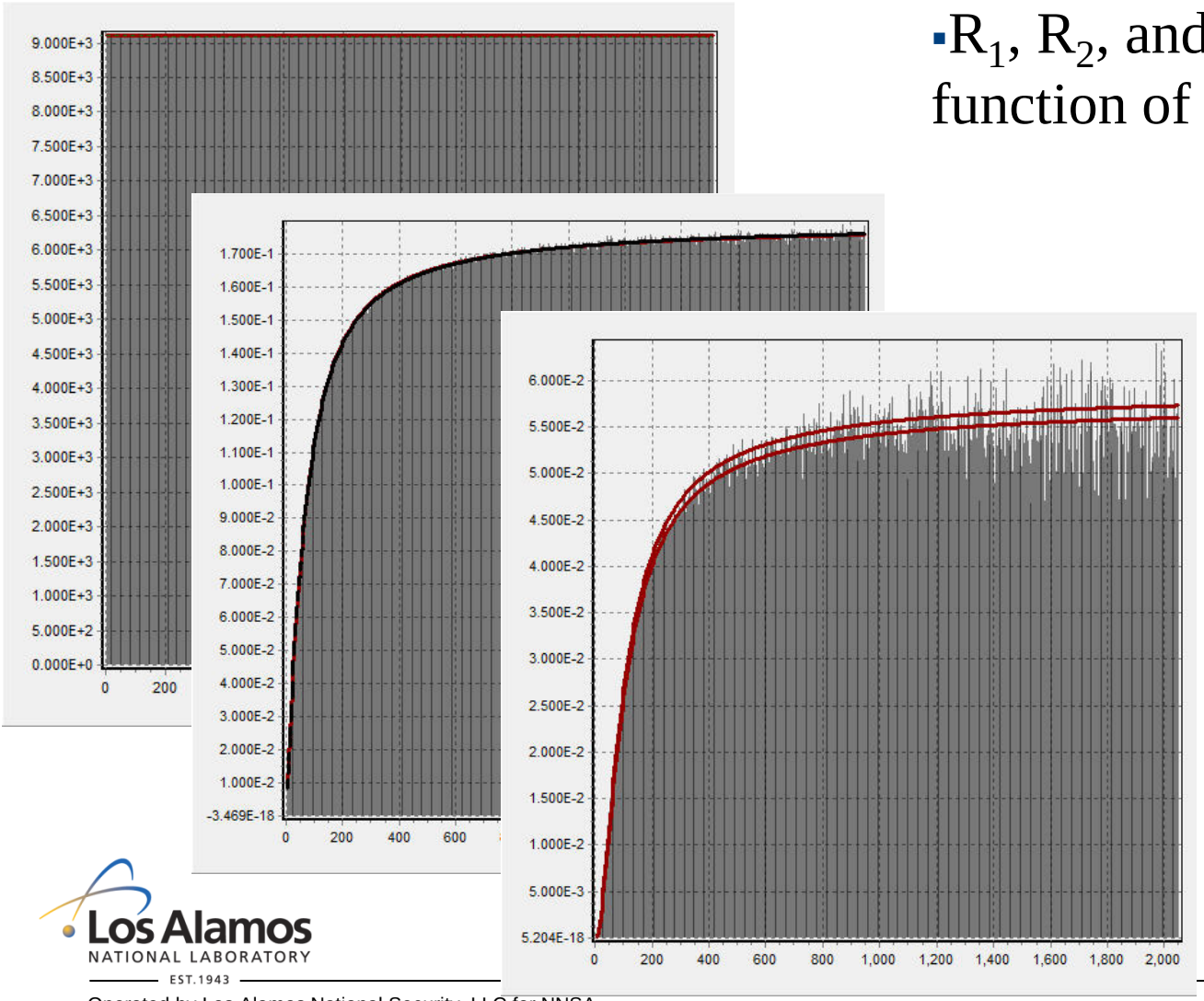
$$R_1 = \frac{\overline{m_{1!}}}{\tau\,\omega_1} \qquad \omega_1 = 1$$

$$R_2 = \frac{1}{\tau\,\omega_2}\left[\overline{m_{2!}} - \frac{1}{2}\overline{m_{1!}}^{2}\right] \qquad \omega_2 = 1 - \frac{1-e^{-\lambda\tau}}{\lambda\tau}$$

$$R_3 = \frac{1}{\tau\,\omega_3}\left[\overline{m_{3!}} - \overline{m_{2!}m_{1!}} + \frac{1}{3}\overline{m_{1!}}^{3}\right] \qquad \omega_3 = 1 - \frac{3-4e^{-\lambda\tau}+e^{-2\lambda\tau}}{2\lambda\tau}$$

- R_1, R_2, and R_3 are the singles, doubles, and triples counting rates. R_1 is the count rate in the detector in n/sec.
- τ is the gate width.
- λ is the decay constant of the of the system (including neutron interactions inside the detector system).

Hage-Cifarelli formulism: What gate width should we use?



- R_1, R_2, and R_3 as a function of gate width

- R_1 = 9,104
- R_2 = 1,632
- R_3 = 525

Hage-Cifarelli formulism: Identifying the fissionable materials

- **Average number of neutrons (denoted as $\overline{\nu}$) released in a fission depends upon:**
  - Energy of the incident neutron
  - Nuclide that fissions
- **From 0-8 neutrons released per fission. Number of neutrons released is statistical**
- **Probability distribution of the number of neutrons released per fission has been measured for many isotopes of interest (for both s.f. and induced fission).**
- **P_ν is the probability that ν neutrons are released during fission. U-235 is below.**

ν	P_ν	νP_ν
0	0.032	0.000
1	0.172	0.172
2	0.336	0.672
3	0.304	0.912
4	0.127	0.508
5	0.027	0.133
6	0.003	0.016
7	0.000	0.001
		2.414

- **$\overline{\nu}$ is the first factorial moment of the P_ν distribution.**
- **Other moments of the P_ν distribution can be determined from:**

$$\overline{\nu_{(\mu)}} = \sum_{\nu=\mu}^{\infty} \binom{\nu}{\mu} P_\nu$$

- **So the first two factorial moments are:**

$$\overline{\nu_{(1)}} = \sum_{\nu=1}^{\infty} \binom{\nu}{1} P_\nu = \sum_{\nu=1}^{\infty} \frac{\nu}{1!} P_\nu$$

$$\overline{\nu_{(2)}} = \sum_{\nu=2}^{\infty} \binom{\nu}{2} P_\nu = \sum_{\nu=2}^{\infty} \frac{\nu(\nu-1)}{2!} P_\nu$$

Hage-Cifarelli formulism

$$R_1 = \varepsilon\left[b_{11}F_S + b_{12}S_\alpha\right]$$

$$R_2 = \varepsilon^2\left[b_{21}F_S + b_{22}S_\alpha\right]$$

with

$$b_{11} = M_L\,\bar{\nu}_{S(1)}$$

$$b_{21} = M_L^2\left[\bar{\nu}_{S(2)} + \frac{M_L - 1}{\bar{\nu}_{I(1)} - 1}\bar{\nu}_{S(1)}\bar{\nu}_{I(2)}\right]$$

$$b_{12} = M_L$$

$$b_{22} = M_L^2\,\frac{M_L - 1}{\bar{\nu}_{I(1)} - 1}\bar{\nu}_{I(2)}$$

The subscripts S and I refer to s.f and induced fission.

- ε is the detector efficiency.
- F_s is the spontaneous fission rate of the system.
- S_α is the (α,n) neutron emission rate.
- M_L is the leakage multiplication of the system.
- Four unknowns (ε,F_s,S_α,M_L) and only two equations.
  - R_1 and R_2 are measured
  - Estimate: M_L and ε
  - Use additional moments of the counting distribution (R_3 and/or R_4).

Hage-Cifarelli formulism: Some choices…(1)

- **(α,n) neutron emission rate (S_α):**
  - For "pure" metal systems S_α is approximately 0.
  - For systems in which $S_\alpha > 0$ it may be possible to estimate S_α using other methods (gamma spectroscopy).
- **Detector efficiency (ε):**
  - Calibration measurements with neutron sources can be performed.
  - Efficiency determined via gross counting (detected neutron count rate/"known" neutron emission rate) or using the singles and doubles counting rates (in this case the neutron emission rate does not need to be known).
- **The downside to this approach is that it may be difficult to accurately determine the (α ,n) emission rate or the detector efficiency for some measurements.**

Hage-Cifarelli formulism: Some choices…(2)

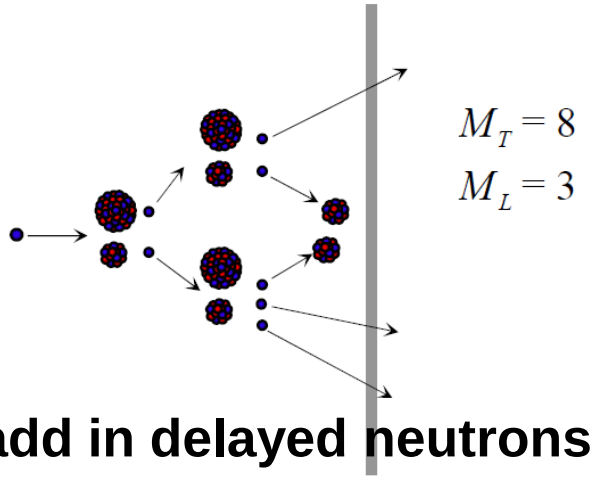
- **Use R_3 and R_4:**
  - The downside to this approach is that it is usually difficult (or may not be possible at all) to obtain "quality" triples counting rates (R_3).
  - Doing so would require many counts to provide decent statistics for each n in the Feynman histogram (C_n would have to be large for all n).
  - It is impossible for most systems to have meaningful quad counting rates (R_4).

- **After the approach is decided we can rearrange the equations and will then have solutions for ε, F_s, S_α, and M_L.**
- **What about M_T, keff, SNM mass…**
- **What else can we learn about the system now that we know these parameters:**
  - Total multiplication (M_T).
  - Mass

Hage-Cifarelli formulism: What about M_T, k_{eff}, SNM mass…?

- **Serber Equation:** $M_T = 1 + Q_f \bar{\nu}_{I(1)}$ $\qquad M_L = 1 + Q_f \left(\bar{\nu}_{I(1)} - 1 - \frac{\sigma_c}{\sigma_f} \right)$
- **Q_f is the number of fissions produced when one neutron is introduced in the system.**

$$M_T = \frac{M_L \bar{\nu}_{I(1)} - 1 - \frac{\sigma_c}{\sigma_f}}{\bar{\nu}_{I(1)} - 1 - \frac{\sigma_c}{\sigma_f}} \qquad M_T = \frac{1}{1 - k_p}$$



- **Measurements are for prompt neutrons. To add in delayed neutrons we look to the case in which a system is at prompt critical:**

$$k_p = 1$$

$$\rho = \beta$$

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \Rightarrow k_{eff} = \frac{1}{1 - \rho} \Rightarrow k_{eff} = \frac{1}{1 - \beta}$$

$$\frac{k_{eff}}{k_p} = \frac{\frac{1}{1-\beta}}{1} \Rightarrow k_{eff} = \frac{k_p}{1 - \beta}$$

Hage-Cifarelli formulism

- F_s is the spontaneous fission rate of the system.
- The number of neutron per second produced by spontaneous fission is equal to $F_S \nu_{S(1)}$
- The spontaneous fission yield is known for many isotopes of interest.
- Dividing the number of neutrons per second from spontaneous fission by the spontaneous fission yield gives the mass of the isotope which is undergoing spontaneous fission.

Isotope	Total Half-Life	Spontaneous Fission Half-Life (yr)	Spontaneous Fission Yield (n/s·g)
^{232}Th	1.41×10^{10} yr	$> 1 \times 10^{21}$	$> 6 \times 10^{-8}$
^{232}U	71.7 yr	8×10^{21}	1.3
^{233}U	1.59×10^{5} yr	1.2×10^{21}	8.6×10^{-4}
^{234}U	2.45×10^{5} yr	2.1×10^{21}	5.02×10^{-3}
^{235}U	7.04×10^{8} yr	3.5×10^{21}	2.99×10^{-4}
^{235}U	6.8×10^{8} yr	1.8×10^{21}	8.0×10^{-4}
^{236}U	2.34×10^{7} yr	1.95×10^{21}	5.49×10^{-3}
^{238}U	4.47×10^{9} yr	8.20×10^{21}	1.36×10^{-2}
^{238}U	4.5×10^{9} yr	8.0×10^{21}	1.6×10^{-2}
^{237}Np	2.14×10^{6} yr	1.0×10^{21}	1.14×10^{-4}
^{238}Pu	87.74 yr	4.77×10^{21}	2.59×10^{3}
^{239}Pu	2.41×10^{4} yr	5.48×10^{21}	2.18×10^{-2}
^{239}Pu [a]	2.4×10^{4} yr	5.5×10^{21}	3.0×10^{-2}
^{240}Pu	6.56×10^{3} yr	1.16×10^{21}	1.02×10^{3}
^{240}Pu [a]	6.6×10^{3} yr	1.2×10^{21}	1.0×10^{3}
^{241}Pu	14.35 yr	(2.5×10^{21}) [b]	(5×10^{-2}) [b]
^{242}Pu	3.76×10^{5} yr	6.84×10^{21}	1.72×10^{3}
^{241}Am	433.6 yr	1.05×10^{21}	1.18
^{242}Cm	163 days	6.56×10^{21}	2.10×10^{7}
^{244}Cm	18.1 yr	1.35×10^{21}	1.08×10^{7}
^{249}Bk	320 days	1.90×10^{21}	1.0×10^{5}
^{252}Cf	2.646 yr	85.5	2.34×10^{12}
^{252}Cf [a]	2.65 yr	66.0	2.3×10^{12}

Data are from the PANDA manual (1991), p. 339, unless otherwise noted.

[a] Data from *DOE Fundamentals Handbook, Nuclear Physics and Reactor Theory,* Module 2, p. 2.

[b] Parameters in parenthesis have estimated accuracies of 2 orders of magnitude.

Summary of Correlated Neutron Analysis

- **Passive neutron analysis can help provide information on many systems of interest. In particular it can give information on:**
  - The location of sources, absorption properties of materials, room return information, the multiplication (total or leakage), neutron or fission rates, and mass.
- **Things that it cannot provide include:**
  - High-quality isotopic information.
  - Elemental information about reflector or moderator materials.
- **Neutron measurements can provide useful information on their own but are even more effective when coupled with other measurement methods (such as gamma spectroscopy).**