Reactivity Contributions of Various Materials in Topsy, Godiva, and Jezebel

L. B. Engle, G. E. Hansen, and H. C. Paxton

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

Received June 6, 1960

This report brings together an extensive accumulation of reactivity contribution data for the various critical assemblies at Pajarito. Corresponding values of effective absorption and transport cross sections are derived, and relationships between critical mass and volume fraction of diluents are obtained in terms of these cross sections. In some favorable cases, inelastic scattering contributions to the effective absorption cross sections are estimated.

I. EXPERIMENTAL

Within the past ten years, information on the effects of local additions of materials has been accumulated for a number of Pajarito critical assemblies. This report presents results of material replacement measurements on five critical assemblies, four in which the active material is Oy (~94) metal, and one of δ-phase Pu metal. One of the assemblies (Topsy Oy-U) is enriched uranium in a thick natural uranium reflector, a variant had a core at one-half normal density, and in another case (Topsy Oy-Ni), thick nickel was substituted for the uranium reflector. Another (Godiva) is enriched uranium without reflector, and the final assembly is bare δ-phase plutonium. Reported by Linenberger et al. (1), are series of similar results for assemblies with cores effectively of Oy (93.15%), H2HfC17(025), and reflectors of thick U and of thick Ni.

METHOD

A reactivity contribution measurement is the determination of the reactivity change when a small cavity within the system is filled by a sample of the material of interest. Actually, the difference in control rod settings for delayed critical operation with the cavity filled and empty is observed. As changes are made manually, disassembly of the system is required between such observations. By means of a control rod calibration curve, the result is converted to units of cents per mole of perturbing material, where 100 cents is the reactivity interval between delayed and prompt critical (21). Consistency of standards (cavity empty) which bracket each determination with the sample in place, is required for an acceptable measurement. With the assembly reproducing satisfactorily, drifts in series of standards correlate with changes in ambient temperature (for Jezebel, constant temperature was maintained). At the operating level for these tests (~1 w), self-heating of the oralloy assemblies is negligible.

TOSPY MEASUREMENTS

Three of the critical assemblies to be considered were set up on the Topsy machine (3). The system in which the more complete series of replacement measurements was made consisted of a roughly spherical core built up of ¼-in. units of Oy, surrounded intimately by a full density U reflector of 0 in. average thickness (effectively almost infinite). The critical mass was 17.4 kg Oy (94.1%) at an average core density of 18.7 g cm⁻³. The replacements generally were made in ¼-in.³ cavities in a layer centered about ½ in. below the median plane of the assembly, although Oy, Pu, and U checks were made in ½ in. diam × ¼ in. long cylindrical cavities in the radial "glory hole" of the assembly. Most samples were ¼-in. cubes or ½ in. diam × ¼ in. cylinders of full density metal or of pressed powder.
and wherever possible were of high-purity material. Results of measurements on this Oy-U assembly are presented in Table I. The few existing data for a similar assembly with core at 50% normal density are given in Table II.

The other Topsy assembly had a similar "pseudospherical" Oy core in a full density Ni reflector of about 8\(\frac{1}{2}\) in. average thickness. In this case, the critical mass was 21.2 kg Oy (94%) at an average density of 18.7 g/cm\(^3\). Samples were \(\frac{1}{2}\)-in. cubes of

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topsy Oy - Thick U Assembly (Oy Radius = 2.39 in.)</td>
</tr>
</tbody>
</table>

| Radius (in.) | \(0.44^{*}\) | \(0.67^{*}\) | \(1.09^{*}\) | \(1.56^{*}\) | \(2.05^{*}\) | \(2.50^{*}\) | \(2.54^{*}\) | \(2.93^{*}\) | \(3.34^{*}\) | \(4.30^{*}\) | \(4.71^{*}\) | \(D.\psi\) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \(L_iSO_4\) | -0.5 |
| \(Li_2SO_4\) | -9.9 |
| \(Be\) | 8.8 |
| \(BeO\) | 10.7 |
| \(B\) | -0.8 |
| \(C\) | 2.9 |
| \(CH_2\) | 125 |
| \(CD_2\) | 46.2 |
| \(UF_2\) | 11.0 |
| \(Mg\) | 2.1 |
| \(Mg_2\) | 13.8 |
| \(Al\) | 1.4 |
| \(Al_2\) | 2.0 |
| \(S\) | -7.2 |
| \(Ca\) | -6.6 |
| \(Ti\) | -0.7 |
| \(V\) | 1.2 |
| \(Cr\) | -0.5 |
| \(Fe\) | -0.45 |
| \(Mn\) | 0.2 |
| \(Co\) | -1.2 |
| \(Ni\) | -4.38 |
| \(Cu\) | -1.67 |
| \(Zn\) | -2.86 |
| \(Ga\) | -2.3 |
| \(Ge\) | -2.7 |
| \(As\) | 0.0 |
| \(Zr\) | -0.6 |
| \(Nb\) | -5.1 |
| \(Mo\) | -1.5 |
| \(Re\) | -10.0 |
| \(Pb\) | -7.8 |
| \(Ag\) | -14.0 |
| \(Cd\) | -6.9 |
| \(In\) | -21.0 |
| \(Sn\) | -2.5 |
| \(Sb\) | -11.9 |
| \(La\) | -10.3 |
| \(Ce\) | -2.5 |
| \(Nb\) | -6.0 |
### TABLE I (Continued)

**Topsy Oy - Thick U Assembly (Oy Radius = 2.39 in.)**

Reactivity Contributions in Cents/mole

| Radius (in.) | Ta | W | Fe | Pt | Au | Pb | Bi | Th | U | Oy | U233 | Np237 | Pu239 |
|--------------|----|---|----|----|----|----|----|----|---|----|      |       |  |
| 0.44* (0.01) | -15.6 | -7.7 | -0.9 | -7.0 | -12.4 | 0.4 | -1.7 | 24.5 | 184.4 | 328.1 | 16258 |  |
| 0.67* (0.113) | -4.8 | 3.0 | -0.6 | 1.0 | -2.6 | 2.4 | 22.6 | 25.6 | 179.5 | 314 | 16258 |  |
| 1.09* (1.12) | 12.9 | 15.5 | 27.3 | 12.0 | 13.5 | 20.0 | 14.0 | 30.2 | 127.0 | 284.3 | 424.6 |  |
| 1.56* (1.38) | 11.6 | 14.4 | 30.1 | 12.0 | 12.2 | 22.1 | 14.0 | 20.2 | 127.0 | 232.1 | 172.1 | 98.3 |
| 2.05* (2.06) | 3.2 | 14.4 | 30.2 | 6.9 | 3.1 | 5.5 | 1.5 | 20.2 | 72.0 | 172.1 | 77.5 |  |
| 2.50 | 0.0 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |
| 2.54* (2.55) | 3.2 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |
| 2.93 | 0.0 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |
| 3.54 | 0.0 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |
| 4.71 | 0.0 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |
| 5.29 | 0.0 | 14.4 | 30.2 | 1.5 | 0.7 | 2.2 | 1.0 | 4.3 | 46.0 | 105.6 | 46.3 |  |

Note on errors: The estimated probable error of a determination is about ±0.02 cent/sample. As indicated by the following list of moles per sample, probable errors in cent/mole range from ±0.1 to ±0.3 except for extreme cases:

- Li2SO4 -- 0.02; Li2SO4 -- 0.02; Be -- 0.41; BeO -- 0.22; B -- 0.24; C (graphite) -- 0.28; CI2 (polythene) -- 0.13;
- CD2 (polythene) -- 0.10; CF2 (Teflon) -- 0.09; Mg -- 0.15; MgO -- 0.06; Al -- 0.20; Al2O3 -- 0.06; S -- 0.12; Ca -- 0.04;
- Ta -- 0.15; Cr -- 0.20; Fe -- 0.28; Mo -- 0.19; Mn -- 0.25; U -- 0.31; Cu -- 0.26; Zn -- 0.20; Ga -- 0.17;

In addition to the normal error, "wild" values occasionally may appear as a result of masked irreproducibility. Although it is believed that most of these cases have been spotted and eliminated by repetition, it is possible that a few values still may be in error by 1 - 2 cents/mole.

### TABLE II

**Topsy Oy-Thick U Assembly with Core at One-Half Normal Density**

| Radius (in.) | Ta | W | Fe | Pt | Au | Pb | Bi | Th | U | Oy | U233 | Np237 | Pu239 |
|--------------|----|---|----|----|----|----|----|----|---|----|      |       |  |
| 0.374* | -69.5 | 64.8 | 7.4 | 1.77 | 60.7 | 2.26 | 55.7 | 2.76 | 49.8 | 3.26 | 42.7 | 3.76 | 37.4 |
| 0.66* (0.38) | 35.5 | 35.5 | 7.2 | 3.89 (interface) | 41.1 | 3.55 | 3.5 | 4.65 | 3.6 | 0.96 | 1.2 | 0.7 |  |
| 0.67* (0.39) | 7.5 | 7.6 | 8.1 | 2.1 | 3.26 | 42.7 | 8.1 | 2.1 | 3.26 | 42.7 | 8.1 | 2.1 |  |
| 0.71 | 2.2 | 2.2 | 6.0 | 6.0 | 3.5 | 3.5 | 1.9 | 1.9 | 3.5 | 3.5 | 1.9 | 1.9 |  |
| 0.72 | 1.2 | 1.2 | 0.7 | 0.7 | 1.2 | 1.2 | 0.7 | 0.7 | 1.2 | 1.2 | 0.7 | 0.7 |  |

* Replacements in the core were made in lattice positions normally occupied by Oy.
A few checks near the interface indicated that dependence on type of lattice position is within experimental error.
full density or compacted material, and replacements were made in \( \frac{1}{2} \)-in.\(^3\) cavities. The limited set of results for this assembly is summarized in Table III.

**Godiva and Jezebel Measurements**

Other measurements were made in Godiva, the nearly spherical bare Oy critical assembly (4), and in Jezebel, the bare Pu assembly that is described in the companion article by Jarvis et al. (5). The critical mass of Godiva is 52.65 kg Oy (93.7\%) at an effective density of 18.7 g/cm\(^3\). For Jezebel measurements and the principal series of Godiva measurements, replacements were made in a \( \frac{1}{2} \times \frac{1}{2} \)-in. cylindrical cavity within the diametral "glory hole." The radial position of the cavity was adjusted by proper arrangement of fillers of various length which fit into the glory hole. Table IV gives the results of this series of tests as functions of distance from the center of Godiva, and Table V gives the corresponding Jezebel data. Samples, in the form of \( \frac{1}{2} \)-in. diam \( \times \frac{1}{2} \)-in cylinders of compact material, generally are of c.p. grade material or the equivalent.

A second group of measurements on Godiva, designed to give more complete information about an apparent periodicity of central reactivity coefficients for odd \( Z \) - odd \( A \) elements as a function of \( Z \), is reported in Section IV. Finally, transport effects of fissionable materials, deduced from reactivity coefficients at the Godiva surface, are given in Appendix I.

**II Reduction of Data**

Generally, the replacement samples \( (x) \) used for the collection of data, \( \Delta k(r, x) \), given in Tables I, II, III, IV, and V, consisted of normal density \( \frac{1}{2} \)-in. cubes or cylinders. This sample size is sufficiently large to permit accurate reactivity change measurements, yet not so large as to introduce serious perturbations in the neutron distribution within the critical assembly. The small corrections to the observed reactivity contributions which remove the effects of these perturbations (hereinafter called "sample size" corrections) have been computed by the methods given in the preceding paper (6) and the resulting reactivity coefficients,

\[
\Delta k_{0}(r, x)
\]

are presented in Tables VI, VII, VIII, and IX. Radial variations for selected materials in Topsy-Y and Godiva are shown in Figs. 1 and 2, and similar curves for Jezebel are given by Jarvis et al. (5).
TABLE IV

Godiva Bare Oy Assembly (Oy Radius = 3.44 in.)

Reactivity Contributions \( \Delta k \) of Various Materials in Cents/Hole

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>0.030</th>
<th>1.242</th>
<th>1.930</th>
<th>2.512</th>
<th>3.142</th>
<th>3.206</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>6.7</td>
<td>9.4</td>
<td>11.6</td>
<td>10.8</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>( ^{9}B )</td>
<td>-6.3</td>
<td>-0.7</td>
<td>4.9</td>
<td>8.0</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>( ^{20}Ca )</td>
<td>-42.1</td>
<td>-27.3</td>
<td>-12.0</td>
<td>-0.1</td>
<td>6.6</td>
<td>6.2</td>
</tr>
<tr>
<td>C</td>
<td>2.2</td>
<td>0.0</td>
<td>9.0</td>
<td>0.4</td>
<td>9.7</td>
<td></td>
</tr>
<tr>
<td>CH(_2)</td>
<td>88.6</td>
<td>76.6</td>
<td>62.0</td>
<td>43.6</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>0.4</td>
<td>4.6</td>
<td>9.3</td>
<td>9.0</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>-0.1</td>
<td>3.9</td>
<td>8.2</td>
<td>10.6</td>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>-0.5</td>
<td>9.7</td>
<td>12.3</td>
<td>11.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>-6.0</td>
<td>2.0</td>
<td>8.0</td>
<td>10.7</td>
<td>9.6</td>
<td></td>
</tr>
<tr>
<td>( ^{28}Co )</td>
<td>-1.0</td>
<td>4.1</td>
<td>9.3</td>
<td>11.7</td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>Zn</td>
<td>-2.3</td>
<td>4.5</td>
<td>9.0</td>
<td>11.7</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>-3.2</td>
<td>8.4</td>
<td>14.1</td>
<td>13.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>2.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>6.7</td>
<td>2.2</td>
<td>11.6</td>
<td>17.8</td>
<td>17.8</td>
<td></td>
</tr>
<tr>
<td>Bi</td>
<td>-1.5</td>
<td></td>
<td>16.6</td>
<td>21.8</td>
<td>19.8</td>
<td></td>
</tr>
<tr>
<td>Th</td>
<td>-1.2</td>
<td>8.7</td>
<td>18.8</td>
<td>21.6</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>31.9</td>
<td>81.4</td>
<td>27.0</td>
<td>25.6</td>
<td>21.2</td>
<td></td>
</tr>
<tr>
<td>( ^{239}Pu )</td>
<td>6229</td>
<td>229.4</td>
<td>155.4</td>
<td>100.2</td>
<td>42.6</td>
<td></td>
</tr>
<tr>
<td>( ^{240}Pu )</td>
<td>16217</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note on errors: The estimated probable error of a determination is about \( \pm 0.025 \) cents/sample.

From the following list of moles/sample, it is seen that for most materials the probable errors in
cents/mole range from \( \pm 0.1 \) to \( \pm 0.3 \): Be \( = 0.32 \); \( ^{9}B \) \( = 0.24 \); \( ^{20}Ca \) \( = 0.27 \); C (graphite) \( = 0.21 \);
CH\(_2\) (polythene) \( = 0.10 \); Al \( = 0.16 \); Fe \( = 0.22 \); Co \( = 0.23 \); Ni \( = 0.24 \); Cu \( = 0.22 \); Zn \( = 0.16 \);
Ag \( = 0.15 \); Au \( = 0.16 \); Bi \( = 0.07 \); Th \( = 0.08 \); U \( = 0.12 \); Oy \( = 0.12 \); Pu \( = 0.09 \) mole/sample.

As for Topsy, the possibility of an occasional "wild" value in error by 1 - 2 cents/mole cannot
be excluded.

---

#### ONE-GROUP, OR INTEGRAL ABSORPTION AND TRANSPORT CROSS SECTIONS

With the major exceptions of hydrogenous materials, the functions \( \Delta k_0(r, x) \), for Godiva, Topsy, and Jezebel cores, are given very closely by equations of the form

\[
\Delta k_0(r, x) = -\sigma_n(x) f_0(r) + \sigma_t(x) f_1(r)
\]

where \( f_0(r) \) and \( f_1(r) \) are functions of radius alone as would be expected from one-group diffusion theory. However, the one-group relations

\[
f_0(r) = c (\sin k r, k r)^2
\]

and

\[
f_1(r) = c 3 \frac{[(r - 1) \Sigma_t - \Sigma_i]}{d \sin k r}^2
\]

hold only in the region \( r > 0 \). In these expressions, \( k \simeq 0.29 \) cm\(^{-1}\) for Oy (93) and \(~0.38\) for Pu\(^{239}\), and \( (r - 1) \Sigma_t - \Sigma_i \simeq 0.885 \) cm\(^{-1}\) for Oy or \( \sim 0.149 \) cm\(^{-1}\) for Pu. As a rule, then, measurements at many different radial positions disclose only two numbers characteristic of \( (k) \), viz., \( \sigma_n(x) \) and \( \sigma_t(x) \). With a one-energy-group model, if the constant \( c \) is chosen as the central reactivity change associated with a one-barn capture cross section, then \( \sigma_n(x) \) and \( \sigma_t(x) \) would represent, respectively, capture and transport cross sections of \( (x) \) in barns. In an actual case, processes which transfer neutrons from one energy to another also contribute to reactivity changes, and unit capture in different energy regions gives different reactivity contributions. Nevertheless, it is convenient to assign a value to \( c \) and obtain numerical values of \( \sigma_n(x) \) and \( \sigma_t(x) \). Such values then represent a condensed version of the corrected replacement data. We have chosen the constants \( c \) (Godiva) and \( c \) (Topsy) such that \(-\sigma_n\) Oy \( \simeq \)}
<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>0.00</th>
<th>0.72</th>
<th>1.22</th>
<th>1.78</th>
<th>2.26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>13.1</td>
<td>34.8</td>
<td>61.4</td>
<td>77.3</td>
<td>67.1</td>
</tr>
<tr>
<td>BeO</td>
<td>5.25</td>
<td>48.4</td>
<td>104.8</td>
<td>143.3</td>
<td>134.0</td>
</tr>
<tr>
<td>B</td>
<td>-199.2</td>
<td>-3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-5.8</td>
<td>18.2</td>
<td>46.6</td>
<td>71.35</td>
<td>68.0</td>
</tr>
<tr>
<td>CH₂</td>
<td>100.7</td>
<td>133.4</td>
<td>174.2</td>
<td>189.8</td>
<td>151.1</td>
</tr>
<tr>
<td>Cl₂</td>
<td>-14.7</td>
<td>46.1</td>
<td>125.2</td>
<td>190.0</td>
<td>183.7</td>
</tr>
<tr>
<td>CH₄</td>
<td>-36.8</td>
<td>155.3</td>
<td>224.8</td>
<td>-2.6a</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>-11.9</td>
<td>15.0</td>
<td>48.6</td>
<td>77.0</td>
<td>71.3</td>
</tr>
<tr>
<td>S</td>
<td>-13.7</td>
<td>11.1</td>
<td>49.6</td>
<td>77.3</td>
<td>72.7</td>
</tr>
<tr>
<td>SiO₂</td>
<td>-31.3</td>
<td>47.9</td>
<td>146.9</td>
<td>221.0</td>
<td>211.8</td>
</tr>
<tr>
<td>S</td>
<td>-59.0</td>
<td>36.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tl</td>
<td>-21.8</td>
<td>12.0</td>
<td>49.2</td>
<td>78.0</td>
<td>73.5</td>
</tr>
<tr>
<td>Ta₂O₅</td>
<td>-40.8</td>
<td>26.9</td>
<td>71.2</td>
<td>147.6</td>
<td>130.7</td>
</tr>
<tr>
<td>V</td>
<td>-13.0</td>
<td>20.2</td>
<td>40.1</td>
<td>89.9</td>
<td>87.8</td>
</tr>
<tr>
<td>Fe</td>
<td>-18.1</td>
<td>10.1</td>
<td>45.9</td>
<td>76.0</td>
<td>73.5</td>
</tr>
<tr>
<td>Co</td>
<td>-24.0</td>
<td>19.2</td>
<td>39.0</td>
<td>84.0</td>
<td>79.4</td>
</tr>
<tr>
<td>Ni</td>
<td>-40.2</td>
<td>-6.3</td>
<td>39.0</td>
<td>78.8</td>
<td>78.0</td>
</tr>
<tr>
<td>Cu</td>
<td>-27.0</td>
<td>6.3</td>
<td>49.2</td>
<td>84.0</td>
<td>82.9</td>
</tr>
<tr>
<td>Zn</td>
<td>-33.9</td>
<td>1.2</td>
<td>46.4</td>
<td>90.4</td>
<td>84.8</td>
</tr>
<tr>
<td>Zr₂O₅</td>
<td>-90.0</td>
<td>27.5</td>
<td>70.2</td>
<td>128.6</td>
<td>122.7</td>
</tr>
<tr>
<td>Mo</td>
<td>-37.0</td>
<td>6.4</td>
<td>88.0</td>
<td>138.8</td>
<td>115.3</td>
</tr>
<tr>
<td>Ag</td>
<td>-76.2</td>
<td>-27.0</td>
<td>97.1</td>
<td>91.8</td>
<td>88.4</td>
</tr>
<tr>
<td>Cd</td>
<td>-55.0</td>
<td>-6.6</td>
<td>58.6</td>
<td>103.6</td>
<td>108.3</td>
</tr>
<tr>
<td>Sn</td>
<td>-56.8</td>
<td>4.5</td>
<td>64.4</td>
<td>104.1</td>
<td>108.0</td>
</tr>
<tr>
<td>La</td>
<td>-30.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ce</td>
<td>-58.3 (9)</td>
<td></td>
<td></td>
<td></td>
<td>119.7a</td>
</tr>
<tr>
<td>Dy₂O₃</td>
<td>-175.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho₂O₃</td>
<td>-589.7</td>
<td></td>
<td></td>
<td></td>
<td>440.4</td>
</tr>
<tr>
<td>Yb₂O₃</td>
<td>-156.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>-44.1</td>
<td>-27.4</td>
<td>50.5</td>
<td>114.4</td>
<td>121.1</td>
</tr>
<tr>
<td>W</td>
<td>-60.9</td>
<td>-6.2</td>
<td>67.4</td>
<td>125.3</td>
<td>129.1</td>
</tr>
<tr>
<td>Au</td>
<td>-73.0</td>
<td>-16.1</td>
<td>53.6</td>
<td>121.0</td>
<td>129.1</td>
</tr>
<tr>
<td>Bi</td>
<td>-19.7</td>
<td>34.0</td>
<td>99.8</td>
<td>124.3</td>
<td>147.1</td>
</tr>
<tr>
<td>Th</td>
<td>-54.4</td>
<td>13.9</td>
<td>88.6</td>
<td>154.2</td>
<td>145.1</td>
</tr>
<tr>
<td>U (normal)</td>
<td>94.4</td>
<td>134.4</td>
<td>189.0</td>
<td>233.6</td>
<td>200.3</td>
</tr>
<tr>
<td>Pu (239)</td>
<td>1227</td>
<td></td>
<td></td>
<td>642.7</td>
<td></td>
</tr>
<tr>
<td>Pu (239)</td>
<td>676.9</td>
<td>632.5</td>
<td>547.4</td>
<td>497.2</td>
<td>251.6</td>
</tr>
<tr>
<td>Pu (239)</td>
<td>669</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Measured at r = 1.72 in. instead of 1.78 in.
* Small sample.
* Sample 1/2 in. thick x 1/2 in. diam.

Note: Samples c.p. grade or better; probable error ±0.3 cents/g-atom for standard-size samples, except 0.4% for Pu and Oy; few "wild" values noted.
**Table VI**

Topsy GY - Thick U Assembly (By Radius = 2 39 in )

| Element | Reactivity Coefficients $\phi$ of Various Elements in U/cm²-g
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radius (in.) 0.00</td>
</tr>
<tr>
<td>F</td>
<td>67.6</td>
</tr>
<tr>
<td>Cs</td>
<td>19.0</td>
</tr>
<tr>
<td>La</td>
<td>-13.4</td>
</tr>
<tr>
<td>Ce</td>
<td>9.2</td>
</tr>
<tr>
<td>B</td>
<td>-12.4</td>
</tr>
<tr>
<td>C</td>
<td>2.4</td>
</tr>
<tr>
<td>O</td>
<td>1.4</td>
</tr>
<tr>
<td>Y</td>
<td>4.0</td>
</tr>
<tr>
<td>W</td>
<td>1.4</td>
</tr>
<tr>
<td>Al</td>
<td>1.0</td>
</tr>
<tr>
<td>Si</td>
<td>-8.9</td>
</tr>
<tr>
<td>Ca</td>
<td>-6.6</td>
</tr>
<tr>
<td>Ti</td>
<td>-0.9</td>
</tr>
<tr>
<td>V</td>
<td>0.0</td>
</tr>
<tr>
<td>Cr</td>
<td>-1.7</td>
</tr>
<tr>
<td>Fe</td>
<td>-4.2</td>
</tr>
<tr>
<td>Hm</td>
<td>-1.0</td>
</tr>
<tr>
<td>Co</td>
<td>-2.8</td>
</tr>
<tr>
<td>Ni</td>
<td>-7.5</td>
</tr>
<tr>
<td>Cu</td>
<td>-3.9</td>
</tr>
<tr>
<td>Zn</td>
<td>-5.2</td>
</tr>
<tr>
<td>Ga</td>
<td>-4.0</td>
</tr>
<tr>
<td>Ge</td>
<td>-4.5</td>
</tr>
<tr>
<td>As</td>
<td>-3.9</td>
</tr>
<tr>
<td>Se</td>
<td>-4.5</td>
</tr>
<tr>
<td>Kr</td>
<td>-5.5</td>
</tr>
<tr>
<td>Rb</td>
<td>-7.5</td>
</tr>
<tr>
<td>Sr</td>
<td>-5.6</td>
</tr>
<tr>
<td>Yb</td>
<td>-14.1</td>
</tr>
<tr>
<td>Y</td>
<td>-20.3</td>
</tr>
<tr>
<td>Nd</td>
<td>-9.5</td>
</tr>
<tr>
<td>Sm</td>
<td>-25.8</td>
</tr>
<tr>
<td>Eu</td>
<td>-4.4</td>
</tr>
<tr>
<td>Tb</td>
<td>-14.5</td>
</tr>
<tr>
<td>Lr</td>
<td>-13.5</td>
</tr>
<tr>
<td>Ce</td>
<td>-1.8</td>
</tr>
<tr>
<td>Pr</td>
<td>-4.5</td>
</tr>
<tr>
<td>Nd</td>
<td>-17.2</td>
</tr>
<tr>
<td>Sm</td>
<td>-10.8</td>
</tr>
<tr>
<td>La</td>
<td>-21.3</td>
</tr>
<tr>
<td>Pm</td>
<td>-10.7</td>
</tr>
<tr>
<td>Eu</td>
<td>-15.8</td>
</tr>
<tr>
<td>Gd</td>
<td>-4.1</td>
</tr>
<tr>
<td>Tb</td>
<td>-7.6</td>
</tr>
<tr>
<td>Dy</td>
<td>386.7</td>
</tr>
<tr>
<td>Er</td>
<td>298.4</td>
</tr>
<tr>
<td>Tm</td>
<td>256.7</td>
</tr>
<tr>
<td>Yb</td>
<td>175.2</td>
</tr>
<tr>
<td>Lu</td>
<td>482.6</td>
</tr>
</tbody>
</table>

* From CH²-C
+ From CH²-C
/ Average of values from NH²-Mg and Aľ₂O₃-Mg.
+ Average of values from CH²-C and Wp₂-Wc.
### TABLE VII

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>0</th>
<th>0.030</th>
<th>1.242</th>
<th>1.290</th>
<th>2.512</th>
<th>3.142</th>
<th>3.206</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B</strong></td>
<td>-12.2</td>
<td>-3.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>B</strong>&lt;sup&gt;10&lt;/sup&gt;</td>
<td>-20.8</td>
<td>-19.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>2.7</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Al</strong></td>
<td>0.9</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ti</strong></td>
<td>0.1</td>
<td>2.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Zr</strong></td>
<td>-1.5</td>
<td>1.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cu</strong></td>
<td>-3.0</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ga</strong></td>
<td>-1.1</td>
<td>9.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Au</strong></td>
<td>-11.0</td>
<td>-4.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rb</strong></td>
<td>-1.8</td>
<td>9.6</td>
<td></td>
<td></td>
<td></td>
<td>91.1</td>
<td></td>
</tr>
<tr>
<td><strong>Th</strong></td>
<td>-2.0</td>
<td>3.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>W</strong></td>
<td>-4.4</td>
<td>-3.0</td>
<td>-1.1</td>
<td>3.7</td>
<td>0.2</td>
<td>0.0</td>
<td>7.6</td>
</tr>
<tr>
<td><strong>U</strong>&lt;sup&gt;233&lt;/sup&gt;</td>
<td>372.8</td>
<td>333.0</td>
<td>309.7</td>
<td>261.1</td>
<td>200.7</td>
<td>120.5</td>
<td>66.5</td>
</tr>
<tr>
<td><strong>U</strong>&lt;sup&gt;235&lt;/sup&gt;</td>
<td>223.5</td>
<td>209.4</td>
<td>186.3</td>
<td>163.1</td>
<td>194.6</td>
<td>86.8</td>
<td>61.3</td>
</tr>
<tr>
<td><strong>Pu</strong>&lt;sup&gt;238&lt;/sup&gt;</td>
<td>31.8</td>
<td>33.3</td>
<td>35.0</td>
<td>35.5</td>
<td>36.6</td>
<td>31.5</td>
<td>22.0</td>
</tr>
<tr>
<td><strong>Pu</strong>&lt;sup&gt;239&lt;/sup&gt;</td>
<td>426.0</td>
<td>361.5</td>
<td>351.8</td>
<td>295.1</td>
<td>234.2</td>
<td>141.1</td>
<td>73.5</td>
</tr>
</tbody>
</table>

### TABLE VIII

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>0.030</th>
<th>1.242</th>
<th>1.300</th>
<th>2.512</th>
<th>3.142</th>
<th>3.206</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>He</strong></td>
<td>47.9</td>
<td>29.5</td>
<td>37.7</td>
<td>16.0</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>17.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Be</strong></td>
<td>7.3</td>
<td>9.4</td>
<td>11.0</td>
<td>9.0</td>
<td>8.4</td>
<td></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>6.9</td>
<td>2.4</td>
<td>3.4</td>
<td>6.4</td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td><strong>He</strong>&lt;sup&gt;10&lt;/sup&gt;</td>
<td>-55.3</td>
<td>-57.6</td>
<td>-144</td>
<td>-4.7</td>
<td>4.5</td>
<td>4.1</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>2.4</td>
<td>4.5</td>
<td>7.8</td>
<td>8.9</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td><strong>Al</strong></td>
<td>0.5</td>
<td>3.9</td>
<td>7.9</td>
<td>8.2</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td><strong>Fe</strong></td>
<td>-0.2</td>
<td>3.1</td>
<td>6.9</td>
<td>9.0</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td><strong>Co</strong></td>
<td>-0.8</td>
<td>8.4</td>
<td>10.7</td>
<td>9.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ni</strong></td>
<td>-4.4</td>
<td>1.0</td>
<td>6.5</td>
<td>9.1</td>
<td>8.2</td>
<td></td>
</tr>
<tr>
<td><strong>Cu</strong></td>
<td>-1.8</td>
<td>3.3</td>
<td>7.9</td>
<td>10.0</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td><strong>Zn</strong></td>
<td>-2.5</td>
<td>3.5</td>
<td>7.3</td>
<td>9.7</td>
<td>9.9</td>
<td></td>
</tr>
<tr>
<td><strong>Ag</strong></td>
<td>-10.2</td>
<td>6.1</td>
<td>11.6</td>
<td>11.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>W</strong></td>
<td>-4.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Au</strong></td>
<td>-7.4</td>
<td>0.5</td>
<td>9.3</td>
<td>15.2</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td><strong>Rb</strong></td>
<td>-2.7</td>
<td>13.3</td>
<td>17.9</td>
<td>16.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Th</strong></td>
<td>-1.4</td>
<td>6.9</td>
<td>15.5</td>
<td>17.7</td>
<td>16.7</td>
<td></td>
</tr>
<tr>
<td><strong>U</strong>&lt;sup&gt;235&lt;/sup&gt;</td>
<td>149.3</td>
<td>118.3</td>
<td>88.4</td>
<td>59.3</td>
<td>31.9</td>
<td>20.3</td>
</tr>
<tr>
<td><strong>U</strong>&lt;sup&gt;238&lt;/sup&gt;</td>
<td>24.3</td>
<td>27.6</td>
<td>25.7</td>
<td>19.9</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td><strong>Pu</strong>&lt;sup&gt;239&lt;/sup&gt;</td>
<td>285.2</td>
<td>223.2</td>
<td>154.8</td>
<td>97.9</td>
<td>39.8</td>
<td></td>
</tr>
<tr>
<td><strong>Pu</strong>&lt;sup&gt;240&lt;/sup&gt;</td>
<td>172.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a From CH<sub>3</sub>-C.

b From CU<sub>2</sub>-C.
1.78, this being close to the value in barns of

\[(x - 1)\sigma - \sigma s\]

Similarly c (Jezebel) is chosen such that

\[\sigma_s(Pu^{239}) \approx 3.60 \text{ barns,}\]

in this case allowing \(-0.15 \text{ barn for inelastic scattering contribution. Table X lists the } \sigma_n(x) \text{ and }\]

\[\sigma_{el}(x) \text{ values corresponding to the corrected data }\]

\[\Delta \rho_d(x, x) \text{ of Tables VI, VII, VIII, and IX; the empirical functions } f_0(r) \text{ and } f_1(r) \text{ are graphed in}\]

### Table IX

<table>
<thead>
<tr>
<th>Element</th>
<th>Reactivity Coefficients $\Delta \rho_d$ of Various Elements in Cents/g-atom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radius (in.) 0.00</td>
</tr>
<tr>
<td>H</td>
<td>62.8</td>
</tr>
<tr>
<td>O</td>
<td>-5.3</td>
</tr>
<tr>
<td>Be</td>
<td>35.5</td>
</tr>
<tr>
<td>N</td>
<td>-251</td>
</tr>
<tr>
<td>C</td>
<td>-6.9</td>
</tr>
<tr>
<td>N</td>
<td>-22.6</td>
</tr>
<tr>
<td>O</td>
<td>-9.9</td>
</tr>
<tr>
<td>P</td>
<td>-18.4</td>
</tr>
<tr>
<td>Al</td>
<td>-14.1</td>
</tr>
<tr>
<td>Si</td>
<td>-16.3</td>
</tr>
<tr>
<td>S</td>
<td>-70.2</td>
</tr>
<tr>
<td>Ti</td>
<td>-25.9</td>
</tr>
<tr>
<td>V</td>
<td>-15.4</td>
</tr>
<tr>
<td>Fe</td>
<td>-21.5</td>
</tr>
<tr>
<td>Co</td>
<td>-29.0</td>
</tr>
<tr>
<td>Ni</td>
<td>-48.0</td>
</tr>
<tr>
<td>Cu</td>
<td>-34.7</td>
</tr>
<tr>
<td>Sn</td>
<td>-60.3</td>
</tr>
<tr>
<td>Zn</td>
<td>-22.6</td>
</tr>
<tr>
<td>Zr</td>
<td>-44.0</td>
</tr>
<tr>
<td>Ag</td>
<td>-93.4</td>
</tr>
<tr>
<td>Cd</td>
<td>-65.5</td>
</tr>
<tr>
<td>Sn</td>
<td>-43.5</td>
</tr>
<tr>
<td>La</td>
<td>-36.2</td>
</tr>
<tr>
<td>Ce</td>
<td>-33.5</td>
</tr>
<tr>
<td>Dy</td>
<td>-89</td>
</tr>
<tr>
<td>Ho</td>
<td>-153</td>
</tr>
<tr>
<td>Yb</td>
<td>-76</td>
</tr>
<tr>
<td>Sm</td>
<td>-100.5</td>
</tr>
<tr>
<td>Eu</td>
<td>-82.3</td>
</tr>
<tr>
<td>Gd</td>
<td>-87.2</td>
</tr>
<tr>
<td>Tb</td>
<td>-23.4</td>
</tr>
<tr>
<td>Dy</td>
<td>-64.7</td>
</tr>
<tr>
<td>Er</td>
<td>1399</td>
</tr>
<tr>
<td>Tm</td>
<td>233</td>
</tr>
<tr>
<td>Yb</td>
<td>238</td>
</tr>
<tr>
<td>Lu</td>
<td>237</td>
</tr>
<tr>
<td>Pu</td>
<td>239</td>
</tr>
<tr>
<td>Pu</td>
<td>240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Note</th>
</tr>
</thead>
</table>
a From CH2-C. |
b From CD2-C. |
c From TN-Te. |
d Average of values from BeO-Be and SiO2-Si. |
e From CF2-C. |
Figs. 3 and 4. The shapes of these functions are determined by the radial dependences of $\Delta k(r, x)$ with a pure absorber and pure scatterer, respectively. The former is represented by combinations such as $\Delta k_a(r, Pu) = \Delta k_a(r, Oy)$ or $\Delta k_a(r, Oy) = \Delta k_a(r, U)$ since $\sigma_n(Pu) \approx \sigma_n(Oy) \approx \sigma_n(U)$, while the latter may be represented by elements such as $\Delta k_s$ where $\sigma_a \approx 0$. The relative values of $f_a(r)$ and $f_s(r)$ are fixed by requiring agreement with Eq. (11-2) at small values of $r$, and the absolute values then determined by the above-mentioned choice of the constant $c$.

For a majority of elements, $\Delta k_a(r, x)$ has a maximum value in the vicinity of $r \sim 2$ in. (for Oy cores), and the value of $\sigma_n(x)$ is determined mainly by the $\Delta k$ measurements in the range 1.5-2.5 in. Typically, in this range, $\Delta k \sim 10$ cents/g-atom with an experimental precision of $\sim 0.2$-0.3 cents/g-atom, so that the apparent transport cross section $\sigma_n(x)$ can be specified with a relative precision of 2-3%.

The exceptions are (i) Pu in Topsy (through influence on $\sigma_n$ of large spectral changes near the core surface); (ii) the highly fissionable materials Pu, U$^{233}$, and Oy [because a universal function $f_a(r)$ cannot permit sufficiently accurate subtraction of the very large absorptive component, $-\sigma_n(x)f_a(r)$, from $\Delta k_a(r, x)$]; and (iii) hydrogen and deuterium (because of a change with radius of the energy dependence of the adjoint or neutron effective).

Included in the tables of reactivity coefficients, $\Delta k_0(r, x)$ (Tables VI and VII) for the Topsy and Topsy Ni assemblies, is the additional column $\Delta k_0(r = 0, x)$. The normal stacking of the Topsy assemblies prevented access to the core centers, and the values in this additional column are obtained from $\Delta k_0(r = 0, x) \equiv -\sigma_n(x)f_a(r = 0)$. It may be noted that for the weakly absorbing materials, there is a large percentage difference between $\Delta k_0(r = 0, x)$ and the nearest observed value $\Delta k_0(r \sim 0.5$ in., $x)$. The difference is due almost entirely to the scattering contribution to reactivity change at the $r \sim 0.5$ in. sample position. Since $\sigma_n(x)$ values are accurately determined in the same experiment, the extrapolation to $\Delta k_0(r = 0, x)$ is unambiguous.

The detailed significance of $\sigma_n$, and the breakdown of $\sigma_n$ into components arising from capture (and fission) and from energy degradation by scattering are discussed by Hansel and Axel (6). The scattering contribution to $\sigma_n$ is evaluated by Byers (7) in cases where appropriate capture cross sections have been measured and his results are used in Section IV.

**III. CRITICAL MASSES OF MILDLY DILUTED $^{235}$U AND $^{239}$Pu SYSTEMS DERIVED FROM REACTIVITY COEFFICIENTS**

The relationship for critical mass versus concentration of a diluent is obtained as follows for Topsy: Godiva, and Jezebel. Let the reactivity coefficient (per gram atom) of enriched uranium or plutonium at a given radius ($r$) be $\Delta k(r, Oy$ or Pu). The reactivity contribution per unit volume is then

$$\Delta k(r, Oy$ or Pu)$\rho/A,$$

where $\rho$ is the density and $A$ is the average atomic mass of the uranium or plutonium. Similarly, the reactivity contribution per unit volume for an ele-
### Reactivity Contributions

**Table X**

Apparent Absorption and Transport Cross Sections

<table>
<thead>
<tr>
<th>Element</th>
<th>$\sigma_A$ (barns)</th>
<th>$\sigma_{tr}$ (barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Topsay-U</td>
<td>Topsay-Ni</td>
</tr>
<tr>
<td>H</td>
<td>-0.010</td>
<td>-0.216</td>
</tr>
<tr>
<td>D</td>
<td>-0.216</td>
<td>-0.223a</td>
</tr>
<tr>
<td>Li$^6$</td>
<td>0.097</td>
<td></td>
</tr>
<tr>
<td>Li$^7$</td>
<td>-0.035</td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>-0.005</td>
<td>-0.002</td>
</tr>
<tr>
<td>B$^{10}$</td>
<td>0.790</td>
<td>0.721</td>
</tr>
<tr>
<td>B</td>
<td>0.110</td>
<td>0.100</td>
</tr>
<tr>
<td>C</td>
<td>-0.022</td>
<td>-0.023</td>
</tr>
<tr>
<td>N</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>-0.013</td>
<td>-0.012b</td>
</tr>
<tr>
<td>F</td>
<td>-0.004</td>
<td>-0.004b</td>
</tr>
<tr>
<td>Mg</td>
<td>-0.003</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>-0.006</td>
<td>-0.006a</td>
</tr>
<tr>
<td>Si</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Ca</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>0.017</td>
<td>-0.001l</td>
</tr>
<tr>
<td>Cr</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Mn</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>0.020</td>
<td>0.013</td>
</tr>
<tr>
<td>Co</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>0.066</td>
<td>0.056</td>
</tr>
<tr>
<td>Cu</td>
<td>0.035</td>
<td>0.025</td>
</tr>
<tr>
<td>Zn</td>
<td>0.004</td>
<td>0.030</td>
</tr>
<tr>
<td>Ga</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>As</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>Zr</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>Nb</td>
<td>0.068</td>
<td></td>
</tr>
<tr>
<td>Mo</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>Ru</td>
<td>0.127</td>
<td></td>
</tr>
<tr>
<td>Pd</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td>Ag</td>
<td>0.162</td>
<td>0.127a</td>
</tr>
<tr>
<td>Cd</td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>0.233</td>
<td></td>
</tr>
<tr>
<td>Sn</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td>Sb</td>
<td>0.151</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>La</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>Ce</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>Nd</td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>Dy</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Ho</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Yb</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>0.155</td>
<td>0.112</td>
</tr>
</tbody>
</table>
TABLE X (Continued)

<table>
<thead>
<tr>
<th>Element</th>
<th>Topsy-U</th>
<th>Topsy-Ni</th>
<th>Godiva</th>
<th>Jezebel</th>
<th>Topsy-U</th>
<th>Topsy-Ni</th>
<th>Godiva</th>
<th>Jezebel</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.097</td>
<td></td>
<td>0.050</td>
<td>0.179</td>
<td>4.40</td>
<td>4.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>0.097</td>
<td></td>
<td></td>
<td></td>
<td>4.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>0.143</td>
<td>0.093</td>
<td>0.093</td>
<td>0.196</td>
<td>4.17</td>
<td>4.11</td>
<td>4.43</td>
<td>4.48</td>
</tr>
<tr>
<td>Pb</td>
<td>0.016</td>
<td></td>
<td>0.005</td>
<td></td>
<td>4.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi</td>
<td>0.037</td>
<td>0.015</td>
<td>0.021</td>
<td>0.053</td>
<td>4.64</td>
<td>4.32</td>
<td>4.69</td>
<td>4.62</td>
</tr>
<tr>
<td>Th</td>
<td>0.069</td>
<td>0.017</td>
<td>0.017</td>
<td>0.139</td>
<td>4.48</td>
<td>4.64</td>
<td>4.92</td>
<td>5.00</td>
</tr>
<tr>
<td>U233</td>
<td>-3.250</td>
<td>-3.147</td>
<td></td>
<td>-3.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U235</td>
<td>-1.897d</td>
<td>-1.887d</td>
<td>-1.858</td>
<td>-1.813</td>
<td></td>
<td></td>
<td>5.3e</td>
<td></td>
</tr>
<tr>
<td>U238</td>
<td>-2.240</td>
<td>-2.071</td>
<td>-0.202</td>
<td>-0.258</td>
<td></td>
<td></td>
<td>5.0d</td>
<td>5.1de</td>
</tr>
<tr>
<td>Np237</td>
<td>-0.558</td>
<td>-0.502</td>
<td>-0.522</td>
<td>-0.553</td>
<td></td>
<td></td>
<td>5.2e</td>
<td></td>
</tr>
<tr>
<td>Pu239</td>
<td>-2.660c</td>
<td>-2.103</td>
<td>-2.121</td>
<td>-2.103</td>
<td></td>
<td></td>
<td>5.1c</td>
<td></td>
</tr>
<tr>
<td>Pu240</td>
<td>-2.660c</td>
<td>-2.103</td>
<td>-2.121</td>
<td>-2.103</td>
<td></td>
<td></td>
<td>5.1c</td>
<td></td>
</tr>
</tbody>
</table>

For the system to remain at delayed critical,

\[ \Delta k_1 + \Delta k_2 = 0 \]  

Combining Eqs. (III-1), (III-2), and (III-3), and noting that the fractional change in critical mass of enriched uranium is

\[ \frac{\Delta m}{m_e} = \frac{\Delta F}{F} + 3 \frac{\Delta R_0}{R_0} \]  

For the Topsy assembly (enriched uranium core with normal uranium reflector), the reactivity change per mole at the core-reflector interface is defined as \( \Delta k_2 \). With \( U \) (normal) of density \( \rho \) and atomic mass \( A \), the reactivity change per mole at the core-reflector interface is

\[ \Delta k_2 = \Delta V \left[ \frac{\Delta k(R_0, Y) \rho}{A} - \frac{\Delta k(R_0, U) \rho}{A} \right] \]  

For the Godiva or Jezebel assemblies (bare enriched uranium or plutonium) \( \Delta k_2 \) is simply

\[ \Delta k_2 = \Delta V \left[ \frac{\Delta k(R_0, Y \text{ or } Pu) \rho}{A} \right] \]  

and one obtains the expression
where \( f_0(r) \) and \( f_1(r) \) are functions of the radius alone for any given assembly, and \( \sigma_{v}(x) \) and \( \sigma_{a}(x) \) are the effective absorption and transport cross sections of an element \( (x) \). When Eq. (III-8) is substituted into Eqs. (III-6) and (III-7), and results of graphical integrations of

\[
\Delta k(r, s) = -\sigma_v(r) f_0(r) + \sigma_a(r) f_1(r)
\]

and \( f_1(r)^2 dr \) are inserted, the following expressions are obtained:

For Topsy,

\[
n_T = 1.20 - \frac{\rho_T}{\rho_s} \left[ 0.735 \sigma_v(x, T) - 12.82 \sigma_a(x, T) \right]
\]

for Godiva,

\[
n_G = 2.00 - \frac{\rho_G}{\rho_s} \left[ 12.25 \sigma_v(x, G) - 14.27 \sigma_a(x, G) \right]
\]

and for Jezebel,

\[
n_J = 2.00 - \frac{\rho_J}{\rho_s} \left[ 1.846 \sigma_v(x, J) - 9.96 \sigma_a(x, J) \right]
\]

where the exponent \((n)\) is defined by

\[
\frac{\Delta m}{m_e} = -n \frac{\Delta E}{F}, \text{ or } m_e = \text{const } F^{-n}
\]

Again, \( F \) is the volume fraction of enriched uranium or plutonium within the core.

Values for the exponent \((n)\) for the Topsy and Godiva assemblies are given in Table XI and for Jezebel in Table X11. These values were obtained by using the above expressions with the listed effective cross section data for the different elements for which replacement data exist. Data for the different elements are maximum values from the Metals Handbook. Effective cross sections for Zwent and Zephyr are included in the tables to indicate sensitivity to dilution \( (8) \).

The tabulated exponent \( n = 1.20 \) for density change of the Topsy core was confirmed by gross

Dr. R. D. Smith of Harwell has kindly provided these cross sections. Zwent and Zephyr consisted of rods packed to form hexagonal-cylindrical cores surrounded by thick reflectors primarily of natural uranium. The Zeus (No. I) core averaged about 49 v/o (volume per cent) U, 25.5 v/o Nb, 2.5 v/o Nb, 12 v/o Al, and 30.5 v/o void. The Zephyr (No. I) core is estimated to be 50 v/o 3-phase Pu, 33 v/o U, 10 v/o Ni (with a small quantity of Cu), and 7 v/o void.
density changes that extended to 50% of normal density. Similarly, gross dilution by natural uranium (to one-half volume fraction) gave 0.73 for the $U^{238}$ dilution exponent, which compares with 0.74 from Table XI.

### Table XI

Dilution Exponents for Topsy and Godiva with Comparative Cross Sections for Zeus

<table>
<thead>
<tr>
<th>Element</th>
<th>Density g-atom/cm$^3$</th>
<th>$c_n(x)$ barn</th>
<th>$c_{\text{c}}(x)$ barn</th>
<th>Dilution exponent n(x)</th>
<th>$c_n(x)$ barn</th>
<th>$c_{\text{c}}(x)$ barn</th>
<th>Dilution exponent n(x)</th>
<th>$c_n(x)$ barn</th>
<th>$c_{\text{c}}(x)$ barn</th>
<th>Dilution exponent n(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.077</td>
<td>0.015</td>
<td>0.016</td>
<td>0.74</td>
<td>-0.139</td>
<td>1.05</td>
<td>0.010</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li</td>
<td>0.077</td>
<td>0.015</td>
<td>0.016</td>
<td>0.74</td>
<td>-0.139</td>
<td>1.05</td>
<td>0.010</td>
<td>1.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>0.202</td>
<td>-0.063</td>
<td>1.82</td>
<td>0.72</td>
<td>-0.093</td>
<td>2.17</td>
<td>0.75</td>
<td>-0.035</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.212</td>
<td>0.110</td>
<td>0.88</td>
<td>0.86</td>
<td>-0.028</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B$_{10}$</td>
<td>0.212</td>
<td>0.110</td>
<td>0.88</td>
<td>0.86</td>
<td>-0.028</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.185</td>
<td>-0.022</td>
<td>2.13</td>
<td>0.96</td>
<td>-0.012</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>0.103</td>
<td>0.120</td>
<td>0.96</td>
<td>0.96</td>
<td>-0.012</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.04</td>
<td>0.80</td>
<td>0.96</td>
<td>0.96</td>
<td>-0.012</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na</td>
<td>0.135</td>
<td>0.004</td>
<td>2.76</td>
<td>0.95</td>
<td>-0.017</td>
<td>2.17</td>
<td>1.02</td>
<td>-0.017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg</td>
<td>0.100</td>
<td>-0.006</td>
<td>2.12</td>
<td>1.04</td>
<td>-0.006</td>
<td>2.14</td>
<td>1.01</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>0.083</td>
<td>-0.080</td>
<td>2.04</td>
<td>1.15</td>
<td>-0.080</td>
<td>2.04</td>
<td>1.15</td>
<td>-0.080</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si</td>
<td>0.029</td>
<td>0.090</td>
<td>2.18</td>
<td>1.16</td>
<td>-0.090</td>
<td>2.18</td>
<td>1.16</td>
<td>-0.090</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>0.064</td>
<td>-0.080</td>
<td>2.04</td>
<td>1.15</td>
<td>-0.080</td>
<td>2.04</td>
<td>1.15</td>
<td>-0.080</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.039</td>
<td>0.060</td>
<td>2.13</td>
<td>1.13</td>
<td>-0.060</td>
<td>2.13</td>
<td>1.13</td>
<td>-0.060</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cl</td>
<td>0.017</td>
<td>0.090</td>
<td>2.19</td>
<td>1.04</td>
<td>-0.090</td>
<td>2.19</td>
<td>1.04</td>
<td>-0.090</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>0.138</td>
<td>0.015</td>
<td>2.41</td>
<td>0.98</td>
<td>-0.020</td>
<td>2.41</td>
<td>0.98</td>
<td>-0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kr</td>
<td>0.135</td>
<td>0.004</td>
<td>2.70</td>
<td>0.95</td>
<td>-0.017</td>
<td>2.70</td>
<td>0.95</td>
<td>-0.017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xe</td>
<td>0.137</td>
<td>0.020</td>
<td>2.29</td>
<td>1.01</td>
<td>-0.006</td>
<td>2.29</td>
<td>1.01</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Br</td>
<td>0.151</td>
<td>0.093</td>
<td>2.73</td>
<td>0.94</td>
<td>-0.007</td>
<td>2.73</td>
<td>0.94</td>
<td>-0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kr</td>
<td>0.152</td>
<td>0.086</td>
<td>2.77</td>
<td>1.02</td>
<td>-0.006</td>
<td>2.77</td>
<td>1.02</td>
<td>-0.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rn</td>
<td>0.141</td>
<td>0.055</td>
<td>2.49</td>
<td>0.90</td>
<td>-0.002</td>
<td>2.49</td>
<td>0.90</td>
<td>-0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Po</td>
<td>0.109</td>
<td>0.047</td>
<td>2.86</td>
<td>1.04</td>
<td>-0.030</td>
<td>2.86</td>
<td>1.04</td>
<td>-0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>At</td>
<td>0.084</td>
<td>0.026</td>
<td>2.67</td>
<td>1.06</td>
<td>-0.016</td>
<td>2.67</td>
<td>1.06</td>
<td>-0.016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tl</td>
<td>0.076</td>
<td>0.018</td>
<td>3.01</td>
<td>1.00</td>
<td>-0.003</td>
<td>3.01</td>
<td>1.00</td>
<td>-0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>0.071</td>
<td>0.000</td>
<td>3.07</td>
<td>1.00</td>
<td>-0.011</td>
<td>3.07</td>
<td>1.00</td>
<td>-0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hf</td>
<td>0.092</td>
<td>0.068</td>
<td>3.99</td>
<td>1.01</td>
<td>-0.021</td>
<td>3.99</td>
<td>1.01</td>
<td>-0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>0.106</td>
<td>0.092</td>
<td>4.88(7)</td>
<td>0.80</td>
<td>-0.021</td>
<td>4.88(7)</td>
<td>0.80</td>
<td>-0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nb</td>
<td>0.121</td>
<td>0.127</td>
<td>3.38</td>
<td>1.10</td>
<td>-0.095</td>
<td>3.38</td>
<td>1.10</td>
<td>-0.095</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mo</td>
<td>0.112</td>
<td>0.132</td>
<td>3.61</td>
<td>1.04</td>
<td>-0.127</td>
<td>3.61</td>
<td>1.04</td>
<td>-0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>0.097</td>
<td>0.162</td>
<td>3.26</td>
<td>1.17</td>
<td>0.127</td>
<td>3.26</td>
<td>1.17</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Os</td>
<td>0.077</td>
<td>0.000</td>
<td>3.01</td>
<td>1.09</td>
<td>0.127</td>
<td>3.01</td>
<td>1.09</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td>0.113</td>
<td>0.093</td>
<td>2.51</td>
<td>1.04</td>
<td>0.127</td>
<td>2.51</td>
<td>1.04</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nd</td>
<td>0.113</td>
<td>0.093</td>
<td>2.51</td>
<td>1.04</td>
<td>0.127</td>
<td>2.51</td>
<td>1.04</td>
<td>0.127</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sm</td>
<td>0.127</td>
<td>0.155</td>
<td>3.55</td>
<td>1.40</td>
<td>0.268</td>
<td>3.55</td>
<td>1.40</td>
<td>0.268</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eu</td>
<td>0.102</td>
<td>0.162</td>
<td>4.5</td>
<td>1.40</td>
<td>0.122</td>
<td>4.5</td>
<td>1.40</td>
<td>0.122</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

IV LARGE-CAVITY MEASUREMENTS AT GODIVA CENTER

### Central Godiva Values as Function of Z

A second group of measurements on Godiva was designed to give more detail about effective absorp-
REACTIVITY CONTRIBUTIONS

### Table XI (Continued)

<table>
<thead>
<tr>
<th>Element (x)</th>
<th>Density g-atom/cm²</th>
<th>$o_0(x)$ barn</th>
<th>$o_1(x)$ barn</th>
<th>Dilution exponent n(x)</th>
<th>$o_0(x)$ barn</th>
<th>$o_1(x)$ barn</th>
<th>Dilution exponent n(x)</th>
<th>$o_0(x)$ barn</th>
<th>$o_1(x)$ barn</th>
<th>Dilution exponent n(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>0.064</td>
<td>0.233</td>
<td>3.33</td>
<td>1.24</td>
<td>0.183</td>
<td>0.279</td>
<td>5.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sn</td>
<td>0.061</td>
<td>0.040</td>
<td>3.31</td>
<td>1.08</td>
<td>0.063</td>
<td>0.184</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sb</td>
<td>0.054</td>
<td>0.131</td>
<td>3.31</td>
<td>1.16</td>
<td>0.080</td>
<td>0.180</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>0.039</td>
<td>0.122</td>
<td>3.55</td>
<td>1.16</td>
<td>0.076</td>
<td>0.200</td>
<td>4.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.083</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ba</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>La</td>
<td>0.044</td>
<td>0.016</td>
<td>3.47</td>
<td>1.10</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ce</td>
<td>0.049</td>
<td>0.041</td>
<td>3.54</td>
<td>1.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pr</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.017</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nd</td>
<td>0.049</td>
<td>0.078</td>
<td>3.41</td>
<td>1.13</td>
<td>0.112</td>
<td>0.250</td>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>0.092</td>
<td>0.155</td>
<td>3.91</td>
<td>1.12</td>
<td>0.050</td>
<td>0.117</td>
<td>4.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>0.105</td>
<td>0.097</td>
<td>4.40</td>
<td>0.99</td>
<td>0.162</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ir</td>
<td>0.110</td>
<td>0.097</td>
<td>4.34</td>
<td>0.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>0.098</td>
<td>0.143</td>
<td>4.17</td>
<td>1.06</td>
<td>0.093</td>
<td>0.200</td>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.079</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>0.055</td>
<td>0.014</td>
<td>4.26</td>
<td>1.04</td>
<td>0.009</td>
<td>0.025</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi</td>
<td>0.047</td>
<td>0.037</td>
<td>4.64</td>
<td>1.06</td>
<td>0.021</td>
<td>0.026</td>
<td>6.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Th</td>
<td>0.049</td>
<td>0.069</td>
<td>4.48</td>
<td>1.08</td>
<td>0.017</td>
<td>0.133</td>
<td>6.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.080</td>
<td>-0.242</td>
<td>0.73⁴</td>
<td>-0.310</td>
<td>0.018</td>
<td>5.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>233U</td>
<td>0.080</td>
<td>-3.220</td>
<td></td>
<td>-3.380</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>235U</td>
<td>0.080</td>
<td>-1.893³</td>
<td></td>
<td>-1.890</td>
<td>-1.890</td>
<td>10.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>238U</td>
<td>0.080</td>
<td>-0.228</td>
<td>2.10⁴</td>
<td>-0.229</td>
<td>0.032</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>232Th</td>
<td></td>
<td>-1.40</td>
<td></td>
<td>2.38⁷</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>230Th</td>
<td></td>
<td>-2.06b</td>
<td></td>
<td>-2.05⁶</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>228Th</td>
<td></td>
<td>-2.08b</td>
<td></td>
<td>-2.06⁶</td>
<td>-2.05⁶</td>
<td>2.08b</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>224Ra</td>
<td></td>
<td>-2.56(?)</td>
<td></td>
<td>-2.56(?)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Void</td>
<td></td>
<td></td>
<td></td>
<td>1.20⁵</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

a Information kindly provided by Dr. R. D. Smith of Harwell.
b From large-sample measurements of Section IV.
c Integrated separately.
d Used for normalization.
e From appendix.
f Compares with 0.73 observed directly from Oy(47) to Oy(94).
g Compares with 1.20 observed directly from 50% to 100% normal density.

...
The pattern corresponds in general to similar data for fission neutrons, reported by Hughes et al. (10) for the Los Alamos fast reactor (9), Clementine, and for Harwell's Zeus and Zephyr (8). In view of the behavior of capture cross sections for fission neutrons, reported by Hughes et al. (10) it is not surprising that there is strong suggestion of a magic number influence on reactivity contributions by this isotopically simple class of elements.

\[
\begin{array}{cccccc}
\text{Element} & \text{Density} & \sigma_e(n) & \sigma_{tr}(n) & n(x) & \sigma_e(n) & \sigma_{tr}(n) \\
\text{(x)} & \text{g-atom/cm}^3 & \text{barn} & \text{barn} & \text{exponent} & \text{barn} & \text{barn} \\
\hline
\text{H} & -0.141 & 1.19 & 0.999 & \\
\text{D} & 0.113 & 1.08 & \\
\text{Li} & 0.796 & \\
\text{Be} & 0.222 & 2.25 & 1.09 & 0.031 & 2.0 \\
\text{B} & 0.185 & 0.012 & 1.88 & 0.031 & 2.0 \\
\text{C} & 0.016 & 2.15 & 1.30 & \\
\text{N} & 0.05 & 1.72 & 0.80 & 1.8 \\
\text{O} & 0.023 & 2.22 & 0 & 2.5 \\
\text{F} & 0.041 & 2.72 & 0.002 & 2.8 \\
\text{Na} & 0.021 & 2.9 & \\
\text{Mg} & 0.014 & 2.6 & \\
\text{Al} & 0.033 & 2.30 & 1.61 & 0.026 & 2.7 \\
\text{Si} & 0.014 & 2.34 & 1.68 & 0.032 & 2.5 \\
\text{P} & 0.041 & 2.72 & 0.002 & 2.8 \\
\text{S} & 0.064 & 1.60 & 1.91 & 0.085 & 2.5 \\
\text{Cl} & 0.067 & 2.2 & 0.061 & 2.5 \\
\text{K} & 0.076 & 2.5 & 0.061 & 2.5 \\
\text{Ca} & 0.040 & 2.2 & 0.076 & 2.5 \\
\text{Sc} & 0.035 & 2.6 & \\
\text{Ti} & 0.055 & 2.48 & 1.62 & 0.031 & 2.6 \\
\text{V} & 0.034 & 2.72 & 1.45 & 0.034 & 3.2 \\
\text{Cr} & 0.035 & 2.6 & 0.034 & 3.2 \\
\text{Mn} & 0.023 & 2.8 & 0.023 & 2.8 \\
\text{Fe} & 0.030 & 2.44 & 1.45 & 0.055 & 2.9 \\
\text{Co} & 0.058 & 2.76 & 1.32 & 0.051 & 3.0 \\
\text{Ni} & 0.111 & 2.77 & 1.39 & 0.093 & 3.2 \\
\text{Cu} & 0.074 & 2.83 & 1.37 & 0.074 & 3.2 \\
\text{Zn} & 0.093 & 2.85 & 1.53 & 0.083 & 3.3 \\
\text{Ga} & 0.093 & 2.33 & 0.081 & 3.3 \\
\text{Ge} & 0.077 & 3.1 & 0.077 & 3.4 \\
\text{As} & 0.114 & 4.1 & 0.114 & 4.1 \\
\text{Se} & 0.068 & 4.1 & 0.068 & 4.1 \\
\text{Br} & 0.148 & 3.4 & 0.148 & 3.4 \\
\text{Rb} & 0.073 & 3.3 & 0.073 & 3.3 \\
\text{Sr} & 0.061 & 4.9 & 0.061 & 4.9 \\
\text{Y} & 0.045 & 5.9 & 0.045 & 5.9 \\
\text{Zr} & 0.071 & 6.10 & 1.51 & 0.040 & 4.6 \\
\text{Nb} & 0.105 & 4.9 & 0.105 & 4.9 \\
\text{Mo} & 0.105 & 3.33 & 3.33 & 0.105 & 4.4 \\
\text{Ru} & 0.126 & 3.6 & 0.126 & 3.6 \\
\text{Rh} & 0.217 & \\
\end{array}
\]
### TABLE XII (Continued)

Dilution Exponents for Jezebel with Comparative Cross Sections for Zephyr

<table>
<thead>
<tr>
<th>Element</th>
<th>Density g-atom/cm³</th>
<th>Jezbel (bare Pu)</th>
<th>Zephyr (No. 1)²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>oₐ(x)</td>
<td>oₐ₂(x)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>barn</td>
<td>barn</td>
</tr>
<tr>
<td></td>
<td></td>
<td>barn</td>
<td>barn</td>
</tr>
<tr>
<td>Pd</td>
<td>0.097</td>
<td>0.212</td>
<td>3.54</td>
</tr>
<tr>
<td>Ag</td>
<td>0.077</td>
<td>0.148</td>
<td>3.65</td>
</tr>
<tr>
<td>Cd</td>
<td>0.063</td>
<td>0.102</td>
<td>3.49</td>
</tr>
<tr>
<td>In</td>
<td>0.064</td>
<td>0.093</td>
<td>3.16</td>
</tr>
<tr>
<td>Sn</td>
<td>0.049</td>
<td>0.077</td>
<td>3.67</td>
</tr>
<tr>
<td>Sb</td>
<td>0.061</td>
<td>0.102</td>
<td>3.49</td>
</tr>
<tr>
<td>Ta</td>
<td>0.052</td>
<td>0.232</td>
<td>4.24</td>
</tr>
<tr>
<td>W</td>
<td>0.103</td>
<td>0.182</td>
<td>4.00</td>
</tr>
<tr>
<td>Re</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Os</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ir</td>
<td>0.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pt</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>0.098</td>
<td>0.199</td>
<td>4.48</td>
</tr>
<tr>
<td>Hg</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tl</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi</td>
<td>0.047</td>
<td>0.054</td>
<td>4.62</td>
</tr>
<tr>
<td>Th</td>
<td>0.049</td>
<td>0.141</td>
<td>5.00</td>
</tr>
<tr>
<td>U</td>
<td>0.060</td>
<td>-0.249</td>
<td>5.1</td>
</tr>
<tr>
<td>N²³³</td>
<td>0.080</td>
<td>-2.68</td>
<td></td>
</tr>
<tr>
<td>N²³⁵</td>
<td>0.080</td>
<td>-1.828</td>
<td>5.3</td>
</tr>
<tr>
<td>N²³⁸</td>
<td>0.080</td>
<td>-2.238</td>
<td>5.1⁶</td>
</tr>
<tr>
<td>Np²³⁷</td>
<td>-1.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pu²³⁹</td>
<td>-3.60⁶</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>Pu²⁴⁰</td>
<td>-2.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Void</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Maxima (or plateaus in generally rising regions) appear in the neighborhood of Z = 20, 50 and 82 (or 80) and N = 20, 50, 82, and 126.

**Capture and Scattering Components**

A central reactivity contribution may, in principle, be separated into a capture effect and scattering effect. Capture, of course, gives a negative contribution, but in an assembly such as Godiva, energy degradation by scattering is expected to reduce the probability that neutrons escape fission thus increasing the reactivity (11).

For some elements of the class being considered, capture cross sections (σₐ) for the Godiva central
<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>Contribution, center/gram</th>
<th>Sample wt - g</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>+4.12 ± 0.05</td>
<td>see CH&lt;sub&gt;2&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>+1.89 ± 0.02</td>
<td>see D&lt;sub&gt;2&lt;/sub&gt;O</td>
<td></td>
</tr>
<tr>
<td>Li</td>
<td>3</td>
<td>-0.04 ± 0.01</td>
<td>0.66</td>
<td>c.p. (Fisher), massive</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>+0.80 ± 0.00</td>
<td>see Al&lt;sub&gt;2&lt;/sub&gt;O, PBO</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>8</td>
<td>+0.80 ± 0.02</td>
<td>3.33, 1.625</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>+2.6 ± 0.01</td>
<td>see KF, PCl</td>
<td></td>
</tr>
<tr>
<td>Na</td>
<td>11</td>
<td>+0.9 ± 0.1</td>
<td>0.02</td>
<td>An. Reag. (Baker), massive</td>
</tr>
<tr>
<td>Al</td>
<td>13</td>
<td>+0.35 ± 0.06</td>
<td>9.75</td>
<td>(Baker), granular</td>
</tr>
<tr>
<td>P</td>
<td>15</td>
<td>+1.2 ± 0.1</td>
<td>7.05</td>
<td>yellow purified (Baker), massive</td>
</tr>
<tr>
<td>Cl</td>
<td>17</td>
<td>-0.6 ± 0.1</td>
<td>see CO&lt;sub&gt;2&lt;/sub&gt;, NaCl, PBO</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>19</td>
<td>+2.6 ± 0.15</td>
<td>5.02</td>
<td>(Baker), massive</td>
</tr>
<tr>
<td>Sc</td>
<td>21</td>
<td>+0.0 ± 0.04</td>
<td>see Sc&lt;sub&gt;2&lt;/sub&gt;O</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>23</td>
<td>+0.80 ± 0.08</td>
<td>12.48</td>
<td>99.7% (Wackay), massive</td>
</tr>
<tr>
<td>Mn</td>
<td>25</td>
<td>+1.10 ± 0.06</td>
<td>19.47</td>
<td>96%, C-free (Baker), massive</td>
</tr>
<tr>
<td>Co</td>
<td>27</td>
<td>+0.58 ± 0.06</td>
<td>18.70</td>
<td>98% - 100% (Baker), massive</td>
</tr>
<tr>
<td>Ni</td>
<td>28</td>
<td>+2.0 ± 0.12</td>
<td>see NiCl, Cr</td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>29</td>
<td>+1.13 ± 0.04</td>
<td>4.33</td>
<td>see CrCl</td>
</tr>
<tr>
<td>As</td>
<td>31</td>
<td>-0.80 ± 0.08</td>
<td>16.12</td>
<td>from Rondell, massive</td>
</tr>
<tr>
<td>La</td>
<td>33</td>
<td>+0.2 ± 0.1</td>
<td>14.07</td>
<td>pure (Walling), granular</td>
</tr>
<tr>
<td>Br</td>
<td>35</td>
<td>-1.25 ± 0.25</td>
<td>see NaBr</td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td>76</td>
<td>+3.35 ± 0.5</td>
<td>see RbCl</td>
<td></td>
</tr>
<tr>
<td>Nb</td>
<td>41</td>
<td>+0.7 ± 0.4</td>
<td>see TaCl&lt;sub&gt;5&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Ta</td>
<td>91</td>
<td>-1.4 ± 0.2</td>
<td>26.15</td>
<td>91% grade (Fansteel), vac. dried</td>
</tr>
<tr>
<td>Zr</td>
<td>40</td>
<td>-2.4 ± 0.7</td>
<td>see ZrCl&lt;sub&gt;2&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Hf</td>
<td>72</td>
<td>-1.9 ± 0.1</td>
<td>see HfCl&lt;sub&gt;4&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Ir</td>
<td>77</td>
<td>-3.9 ± 0.1</td>
<td>see Ir&lt;sub&gt;2&lt;/sub&gt;O</td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>79</td>
<td>-5.6 ± 0.2</td>
<td>22.20</td>
<td>c.p. (Fisher), massive</td>
</tr>
<tr>
<td>Ti</td>
<td>81</td>
<td>-0.66 ± 0.15</td>
<td>37.75</td>
<td>c.p. (Faust and Bauer), massive</td>
</tr>
<tr>
<td>Pb</td>
<td>82</td>
<td>-0.51 ± 0.10</td>
<td>41.80</td>
<td>c.p. (Brazil), granular</td>
</tr>
<tr>
<td>Bi</td>
<td>83</td>
<td>-0.9 ± 0.10</td>
<td>20.22</td>
<td>c.p. (Baker and Co.), granular</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compound</th>
<th>Contribution, center/gram</th>
<th>Sample wt - g</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH&lt;sub&gt;2&lt;/sub&gt;</td>
<td>+84.1 ± 0.8</td>
<td>2.698</td>
<td>polythene, corr. from special geom.</td>
</tr>
<tr>
<td>D&lt;sub&gt;2&lt;/sub&gt;O</td>
<td>+30.48 ± 0.10</td>
<td>6.603</td>
<td>99.8%</td>
</tr>
<tr>
<td>Al&lt;sub&gt;2&lt;/sub&gt;O</td>
<td>+2.0 ± 0.2</td>
<td>5.99</td>
<td>An. Reag. (Baker), heat dried</td>
</tr>
<tr>
<td>PBO</td>
<td>+0.3 ± 0.2</td>
<td>22.99</td>
<td>An. Reag. (Walling.), heat dried</td>
</tr>
<tr>
<td>NaF</td>
<td>+4.6 ± 0.1</td>
<td>7.38</td>
<td>c.p. (Baker), heat dried</td>
</tr>
<tr>
<td>KF, PCl</td>
<td>+10.5 ± 0.2</td>
<td>11.75</td>
<td>fluoroethylene G</td>
</tr>
<tr>
<td>NaCl</td>
<td>-13.9 ± 0.40</td>
<td>0.04</td>
<td>An. Reag. (Walling.)</td>
</tr>
<tr>
<td>NaI</td>
<td>+2.9 ± 0.15</td>
<td>6.73</td>
<td>An. Reag. (Walling.), heat dried</td>
</tr>
<tr>
<td>PBO</td>
<td>-6.8 ± 0.4</td>
<td>16.57</td>
<td>Reag. (Horst), undried powder</td>
</tr>
<tr>
<td>SeO&lt;sub&gt;2&lt;/sub&gt;</td>
<td>+3.4 ± 0.6</td>
<td>4.29</td>
<td>Spec. An. (Pfaetz and Bauer), heat dried</td>
</tr>
<tr>
<td>NaBr</td>
<td>-0.45 ± 0.2</td>
<td>8.14</td>
<td>An. Reag. (Walling.), heat dried</td>
</tr>
<tr>
<td>NH&lt;sub&gt;3&lt;/sub&gt;</td>
<td>-2.6 ± 0.5</td>
<td>5.19</td>
<td>c.p. (Fisher), heat dried</td>
</tr>
<tr>
<td>KF</td>
<td>+3.8 ± 0.6</td>
<td>6.05</td>
<td>99% (Rea. Chem., Inc.), heat dried</td>
</tr>
<tr>
<td>KI</td>
<td>+9.3 ± 0.3</td>
<td>9.63</td>
<td>c.p. (Jenser), next ores</td>
</tr>
<tr>
<td>Pr&lt;sub&gt;2&lt;/sub&gt;O</td>
<td>+4.2 ± 0.1</td>
<td>20.21</td>
<td>99% (Rea. Chem., Inc.), heat dried</td>
</tr>
<tr>
<td>IrO&lt;sub&gt;2&lt;/sub&gt;</td>
<td>-6.3 ± 0.8</td>
<td>10.92</td>
<td>(Baker and Co.), heat dried</td>
</tr>
</tbody>
</table>
spectrum have been measured by Byers (7) and others may be estimated from published data.

For comparison with \( \sigma_e \), effective absorption cross sections may be obtained from reactivity contribution values by means of Eq. (II-1). For the central position, this relation may be rewritten

\[ \sigma_e(x) = \sigma_e(Oy) \left[ \frac{\Delta k_e(x)}{\Delta k_e(Oy)} \right] \]

where \( \sigma_e(x) \) is the effective absorption cross section of nuclide \( x \). As before, \( \sigma_e(Oy) = 1.77 \) barns, and by taking \( \Delta k_e(Oy) = 117 \) cents/mole, the value for a small Oy sample in a \( \frac{1}{8} \times \frac{1}{8} \) in. cylindrical cavity, \( \Delta k_e(x) \) for a nonfissileable (low absorption) material is simply the observed reactivity contribution Table XIV lists \( \sigma_e \), corresponding estimates of \( \sigma_e \) for Godiva, and the scattering residues, \( \sigma_e - \sigma_a \).

**Scattering Residues as Function of Z**

The resulting estimates of inelastic scattering component for odd \( Z \) – odd \( A \) elements are shown vs \( Z \) in Fig. 6. Again a periodicity with \( Z \) is suggested, such that maximum inelastic scattering (at least in the two most pronounced cases) coincides with maximum absorption. Two cases of negative scattering residues fall at magic numbers.

**APPENDIX I: RELATIVE TRANSPORT CROSS SECTIONS OF Pu, Oy (93.5%), AND U IN GODIVA**

The reactivity contributions of samples of Pu, Oy (93.5%), and U placed just beyond the Godiva surface were measured in an attempt to improve the information on the relative transport cross sections of these materials. (These heavy elements were known to have nearly equal transport cross sections). For such a sample position, the reactivity contribution is proportional to the number of neutrons sent back to the core (per core fission or other flux level unit) and is thus a measure of the cross section \( \nu \sigma + \delta \). Although the highly anisotropic flux distribution beyond the Godiva surface gives rise to a complicated weighting of the angular distribution of elastically scattered neutrons, one expects \( \delta \) to be comparable to the transport scattering cross section, and hence that \( \sigma_e + \sigma + \sigma_{\text{scat}} = \delta \) be comparable to the transport cross section.

Table XV lists the measured reactivity contributions for both the externally positioned samples and samples placed within but near the Godiva surface. It is seen that the ratio \( \Delta k(\text{Pu})/\Delta k(\text{Oy}) \) or the ratio \( \Delta k(\text{Oy})/\Delta k(\text{U}) \) varies with radial sample position, \( r \), until the Godiva surface is reached, then remains constant at 1.37 ± 0.02. Because the self-multiplication of the Pu sample is the largest and that of the U sample the smallest, the limiting values of these experimental ratios exceed those of the reactivity coefficient or \( \nu \sigma + \delta \) ratios. With the customary “size correction” on the experimental values, these latter ratios become

\[
\begin{align*}
\langle \nu - 1 \rangle \sigma_f - (\nu \sigma_\gamma) + \delta |_{\text{Pu}} &= 1.28 \pm 0.02 \\
\langle \nu - 1 \rangle \sigma_f - (\nu \sigma_\gamma) + \delta |_{\text{Oy}} &= 1.30 \pm 0.02 \\
\langle \nu - 1 \rangle \sigma_f - (\nu \sigma_\gamma) + \delta |_{\text{U}} &= 1.32 \pm 0.02
\end{align*}
\]
<table>
<thead>
<tr>
<th>Element</th>
<th>Apparent absorption cross section ($\sigma_a$, barns)</th>
<th>$\sigma_p$ (barns)</th>
<th>Scattering effect ($\sigma_a - \sigma_p$, barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>-0.623 ± 0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-0.325 ± 0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li</td>
<td>0.0157 ± 0.0009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-0.0272 ± 0.0011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>-0.012 ± 0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-0.034 ± 0.0013</td>
<td>0.0009</td>
<td>0.0043</td>
</tr>
<tr>
<td>Na</td>
<td>-0.014 ± 0.0015</td>
<td>(0.0014)$^b$</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Al</td>
<td>-0.0033 ± 0.0009</td>
<td>(0.0008)$^b$</td>
<td>(0.009)</td>
</tr>
<tr>
<td>P</td>
<td>0.018 ± 0.0015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.007 ± 0.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.039 ± 0.0002</td>
<td>&lt; 0.0004</td>
<td>~ 0.035</td>
</tr>
<tr>
<td>Sc</td>
<td>-0.008 ± 0.0006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>-0.0121 ± 0.0012</td>
<td>0.0029</td>
<td>0.015</td>
</tr>
<tr>
<td>Mn</td>
<td>-0.0104 ± 0.0009</td>
<td>0.0033</td>
<td>0.020</td>
</tr>
<tr>
<td>Co</td>
<td>0.0088 ± 0.0009</td>
<td>~ 0.081</td>
<td>~ 0.077</td>
</tr>
<tr>
<td>Ni</td>
<td>0.0334 ± 0.0008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu</td>
<td>0.0172 ± 0.0011</td>
<td>0.0127</td>
<td>-0.009</td>
</tr>
<tr>
<td>Cs</td>
<td>0.0181 ± 0.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>As</td>
<td>-0.0030 ± 0.0015</td>
<td>0.0562</td>
<td>0.059</td>
</tr>
<tr>
<td>Br</td>
<td>0.0006 ± 0.0004</td>
<td>0.064</td>
<td>0.047</td>
</tr>
<tr>
<td>Rb</td>
<td>0.019 ± 0.0008</td>
<td>- 0.045</td>
<td>- 0.025</td>
</tr>
<tr>
<td>Y</td>
<td>-0.011 ± 0.0004</td>
<td>0.0004</td>
<td>0.050</td>
</tr>
<tr>
<td>Nb</td>
<td>0.021 ± 0.0003</td>
<td>0.037</td>
<td>0.016</td>
</tr>
<tr>
<td>Sn</td>
<td>0.045 ± 0.0005</td>
<td>0.164$^b$</td>
<td>0.084</td>
</tr>
<tr>
<td>Ag</td>
<td>0.123 ± 0.0003</td>
<td>0.194$^b$</td>
<td>0.071</td>
</tr>
<tr>
<td>Ta</td>
<td>0.183 ± 0.0005</td>
<td>0.243$^d$</td>
<td>0.060</td>
</tr>
<tr>
<td>Sb</td>
<td>0.089 ± 0.0005</td>
<td>0.120$^d$</td>
<td>0.040</td>
</tr>
<tr>
<td>I</td>
<td>0.076 ± 0.0003</td>
<td>0.104</td>
<td>0.028</td>
</tr>
<tr>
<td>Cs</td>
<td>0.083 ± 0.0005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>La</td>
<td>0.021 ± 0.0003</td>
<td>0.009</td>
<td>-0.012</td>
</tr>
<tr>
<td>Pr</td>
<td>0.017 ± 0.0005</td>
<td>0.0185$^b$</td>
<td>0.000</td>
</tr>
<tr>
<td>Tm</td>
<td>0.112 ± 0.0002</td>
<td>0.152</td>
<td>0.040</td>
</tr>
<tr>
<td>Re</td>
<td>0.161 ± 0.0005</td>
<td>- 0.196</td>
<td>- 0.036</td>
</tr>
<tr>
<td>Ir</td>
<td>0.120 ± 0.0014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>0.065 ± 0.0003</td>
<td>0.123</td>
<td>0.038</td>
</tr>
<tr>
<td>Tl</td>
<td>0.019 ± 0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pb</td>
<td>0.009 ± 0.0015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi</td>
<td>0.014 ± 0.0002</td>
<td>0.001</td>
<td>0.013</td>
</tr>
</tbody>
</table>

The ratios $\hat{\sigma}_p/\hat{\sigma}_a$ and $\hat{\sigma}_i/\hat{\sigma}_a$ are now determinable from the sample replacement measured values of $\langle \sigma_a \rangle_p/\langle \sigma_a \rangle_a$ and $\langle \sigma_a \rangle_i/\langle \sigma_a \rangle_a$ given in Table X together with one value of $[(\nu - 1)\bar{\sigma} - \sigma_{a,\bar{\sigma}}]/\bar{\sigma}$. We adopt $[(\nu - 1)\bar{\sigma} - \sigma_{a,\bar{\sigma}}]\hat{\sigma}_{103} = 0.34 \pm 0.01$ as the value of the excess number of neutrons produced per $\bar{\sigma}$ collision, this value being consistent...
with cross-section measurements if \( \sigma \) is identified with the transport cross section. We then obtain:

\[
\sigma_{\text{tu}} / \sigma_{\text{tu}} = 1.02 \pm 0.03
\]

\[
\tilde{\psi}_{\text{c}} / \psi_{\text{c}} = 0.98 \pm 0.02
\]

**APPENDIX II: CORRELATION BETWEEN REFLECTOR SAVINGS AND CROSS SECTIONS FROM REACTIVITY COEFFICIENTS**

From critical masses of 5\( \frac{1}{2} \)-in. diameter cylinders of Oy(93.5) in \( \frac{1}{2} \) and 1-in. thick reflectors of various materials, corresponding reflector savings for spherical cores have been deduced (12). For a thin reflector of material \( r \), the savings \( (\Delta R)_{r} \) in cm Oy is proportional to \( N(\sigma(1 + f))_{x} \), where \( N \) is the nuclear density, \( f \) is the thickness, \( \sigma \) is the collision cross section (assumed to be the transport average), and \( f \) is the number of excess neutrons emitted per collision. In terms of the parameters for natural uranium, for which \( \sigma(1 + f) = 5.4 \) barns, then

\[
(\sigma(1 + f))_{x} = 5.4 \left( \frac{Nt}{(\Delta R)_{u}} \right) \text{ barns}
\]

As \( \sigma(1 + f) \cong \sigma_{\text{tu}}(x) - \sigma_{\text{n}}(x) \), values deduced from the reflector savings measurements may be compared with corresponding values from reactivity coefficient data (Table X). The listing in Table XVI of both types of results show reasonable agreement except in the cases of Mo and the Topsy value for Be.

**APPENDIX III: ESTIMATION OF THE AVERAGE NUMBER, \( n \), OF NEUTRONS PRODUCED PER Fission FOR \( \text{U}^{233}, \text{U}^{235}, \text{Pu}^{239}, \text{Pu}^{240}, \text{Np}^{237}, \text{AND} \text{U}^{238} \)**

The central Topsy, Godiva, and Jezabel reactivity coefficient data provide, for several of the fissionable isotopes, accurate values of the net neutron production cross section,

\[
\sigma_{n} = |v - 1 - \alpha| \sigma_{f}.
\]

Since the major neutron reaction parameters of most of these isotopes have already been measured as a function of neutron energy, the question arises as to what new information resides in the reactivity coefficient data. Especially for the threshold fissioners, the values of \( \sigma_{n} \) are sensitive to the neutron flux energy spectra of the critical assemblies, and in lieu of other flux sampling measurements, may best be utilized to characterize these spectra thereby indicating gross inelastic scattering properties of \( \text{U}^{233} \) and \( \text{Pu}^{239} \). Combined with auxiliary measurements of \( \sigma_{f} \), however, one obtains values for the much less spectral-sensitive quantity \( \sigma_{n} = |v - 1 - \alpha| \), and retains the spectral characterization of the critical assemblies. For the majority of \( |v - 1 - \alpha| \) values so obtained, the probable uncertainties in available \( v \) and \( \alpha \) data imply \( \Delta v > \Delta \alpha \), thus our present emphasis is on estimating \( v \).

Of major concern are the sources and magnitudes of the uncertainties accompanying each stage of this data reduction to \( v \) values, not only to indicate the precision with which these values are determined but also to help establish what weak links may be clarified by reactivity coefficients when accurate results from measurements of \( v(E) \) become available. Values of \( v \) for various isotopes, deduced from the reactivity coefficient measurements in Topsy, Godiva, and Jezabel, have been reported elsewhere (13-15) and this provides another incentive for detailing the data reduction procedure.

**A The Ratios \( \sigma_{n}(x), \sigma_{f}(\text{U}^{235}) \)**

Because of the energy dependence of the adjoint or neutron effectiveness functions, \( \phi^* \), characteristic of the critical assemblies, the measured reactivity coefficient ratios, \( \Delta \alpha(x) / \Delta \alpha \)(\text{U}^{235}), differ from the corresponding net neutron production
TABLE XV
Reactivity Contributions of Pu, Oy (93.3%), and U Samples
Positioned near the Godiva Surface ($r_e = 3.44$ in.)

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th>$\Delta k$ (Pu) (d/g-atm)</th>
<th>$\Delta k$ (Oy) (d/g-atm)</th>
<th>$\Delta k$ (U) (d/g-atm)</th>
<th>$\Delta k$ (Pu) (\Delta k) (U)</th>
<th>$\Delta k$ (Oy) (\Delta k) (U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.302</td>
<td>26.8</td>
<td>42.8</td>
<td>66.1</td>
<td>1.63</td>
<td>1.51</td>
</tr>
<tr>
<td>3.142</td>
<td>22.6</td>
<td>33.2</td>
<td>48.2</td>
<td>1.47</td>
<td>1.45</td>
</tr>
<tr>
<td>3.206</td>
<td>21.2</td>
<td>30.6</td>
<td>42.7</td>
<td>1.44</td>
<td>1.40</td>
</tr>
<tr>
<td>3.258a</td>
<td>19.3</td>
<td>27.4</td>
<td>--</td>
<td>1.42</td>
<td>--</td>
</tr>
<tr>
<td>3.285b</td>
<td>20.0</td>
<td>27.8</td>
<td>--</td>
<td>1.39</td>
<td>--</td>
</tr>
<tr>
<td>3.41k</td>
<td>17.3</td>
<td>24.4</td>
<td>--</td>
<td>1.41</td>
<td>--</td>
</tr>
<tr>
<td>3.456</td>
<td>12.8</td>
<td>18.4</td>
<td>25.9</td>
<td>1.44</td>
<td>1.41</td>
</tr>
<tr>
<td>2.541k</td>
<td>10.6 ± 0.1</td>
<td>14.8 ± 0.1</td>
<td>--</td>
<td>1.38 ± 0.02</td>
<td>--</td>
</tr>
<tr>
<td>3.706</td>
<td>7.8 ± 0.1</td>
<td>10.7 ± 0.1</td>
<td>14.7 ± 0.2</td>
<td>1.37 ± 0.02</td>
<td>1.37 ± 0.02</td>
</tr>
</tbody>
</table>

a At these radii, the U sample is a 50.3-g cylinder of length 0.295 in., outer diameter 0.875 in., and inner diameter 0.24 in.; the Oy sample is a 49.7-g cylinder of the same dimensions. At all other radii, the U sample is a 29.10-g solid cylinder of length 0.495 in. and diameter 0.488 in.; the Oy sample is a 30.08-g solid cylinder of length 0.500 in. and diameter 0.488 in.; the Pu sample is a 21.56-g solid cylinder of length 0.454 in. and diameter 0.488 in.

cross-section ratios, and one must compute, for each fissionable isotope, $x$, the quantity

$$C_x = \frac{[\sigma_p(x)/\sigma_p(U^{235})]}/(\Delta \sigma(x)/\Delta \sigma(U^{235}))$$

according to the perturbation theory equation (IIIA-1) given below. In this equation $\phi(E)$ denotes the neutron flux, $\chi(E, E')$ the normalized fission neutron spectrum for incident neutrons of energy $E$, $\sigma^{in}(E, E')$ denotes the differential inelastic scattering cross section, and for compactness the $n - 2n$ reaction is not included explicitly. Because $C_x \approx 1$ is ensured by the weak energy dependence of $\phi^+(E)$; say $|\phi^+(E) - 1| \ll 1$, Eq. (IIIA-1) could be rephrased in terms of the small quantities $(C_x - 1)$ and $(\phi^+(E) - 1)$, thereby better illuminating the extent to which the poorly known neutron reaction cross sections of isotopes such as $\text{Np}^{237}$ affect the precision of the computation of $C_x$. Rather than going into such detail, we shall assume a 50% uncertainty in our computed values of $(C_x - 1)$ for all $x$ except $\text{U}^{238}$, and indicate only qualitatively the source of this uncertainty. For $x = \text{Np}^{237}$ or $\text{Pu}^{240}$, little or no data exist for $\sigma_p(x)$ and $\sigma^{in}(E, E')$ and one is obliged to estimate these quantities from the empirical rule that the radiative capture cross sections of the heavy elements are equal, the nonelastic cross sections of the heavy elements are equal, and the spectrum of neutrons inelastically scattered from some primary energy by any heavy element is the same as that for $\text{U}^{235}$. It is felt that use of this empirical rule is the primary source of the 50% uncertainty ascribed to the computed $(C_x - 1)$ values for these isotopes.

For $x = \text{U}^{235}$ or $\text{Pu}^{239}$, the nuclear properties of $x$...
<table>
<thead>
<tr>
<th>Material and density (g/cm³)</th>
<th>Reflector savings (cm on 93.5)</th>
<th>(1 + f)</th>
<th>Topsy</th>
<th>Godiva</th>
</tr>
</thead>
<tbody>
<tr>
<td>He (MQW)</td>
<td>2.34</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.64</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C (Ca-312)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.67</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg (PS-1)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.67</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al (25S)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti (98.5 w/o)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fe (999 MNQ)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.78 g/cm³</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co (reagent)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ni (electrolytic)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cu (99.9-99.5 w/o)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mo (99.5 w/o)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W (99.1 w/o)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U (natural)</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.70</td>
<td>1.27</td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| a Effect of multiple scattering unknown.  
| b For topsy with Ni reflector.  
| c For the "molecule" W(Cu, Ni)O.  
| d Normalization value: \( \sigma_v > \sigma_l \) renormalized.  

are similar to those of \( ^{125}S_2 \) and the small value of \( C_0 = 1 \) depends significantly on such quantities as \( (dv/dE)_{\epsilon} \) or \( (dv/dE)_{\epsilon 235} \), not only through explicit appearance in Eq. (III-A-1) but for \( ^{238}U \) and \( ^{238}U \), implicitly in the functions \( \phi^*(E) \). It is estimated that the sources of the 50% uncertainty in the computed \( C_0 = 1 \) values for these isotopes are about evenly distributed between the nuclear description of \( X \) and the description of the characteristic functions, \( \phi(E) \) and \( \phi^*(E) \). For \( X = \epsilon \), the value of \( C_0 = 1 \) depends primarily on the accurately known (16) differential scattering cross section, and the estimated 20% uncertainty arises primarily in the uncertainties of \( \phi(E) \). Table XVII lists the observed ratios \( \Delta b_n(x)/\Delta b_n(1_{238}) \) and the inferred ratios \( \sigma_f(x)/\sigma_f(1_{238}) \) from which our computed values of \( C_0 \) may be inferred. Table XVIII lists the multigroup values of \( \phi(E), \phi^*(E), \) and several other quantities which play a significant role in the computation of \( C_0 \) and subsequent computations. It may be noted in passing that the magnitude of uncertainty in \( \phi^*(E) \) given above arises from the claimed precision in the measurements of the nuclear parameters of \( ^{125}S_2 \) and \( ^{238}U \). What few experimental checks on \( \phi^*(E) \) that exist, such as the observed reactivity coefficients of hydrogen and deuterium, indicate smaller uncertainties in \( \phi^*(E) \). (Unfortunately a good experimental check of the value \( \phi^* - 1 \) does not exist for the highest energy group of Table XVIII, as this quantity is especially sensitive to \( (dv/dE) \) of \( ^{238}U \) in the case of Godiva or of \( ^{238}U \) in the case of Jezebel.)

B. The Ratios \( |v - 1 - \alpha_{12} | |v - 1 - \alpha_{13} | \)

The fission ratios

\[
\sigma_f(1_{238})/\sigma_f(1_{235}) \quad \text{and} \quad \sigma_f(Np^{237}) \cdot \sigma_i(1_{235})
\]

have been measured in the three critical assemblies and the results are listed in Table XVIII with corresponding computed values. Agreement between these computed and observed values is not fortuitous as the threshold detector data were utilized in the estimation of the inelastic transfer cross sections of \( ^{235}U \) and \( ^{238}U \), which, in turn, participate in the determination of \( \phi(E) \) and \( \phi^*(E) \). The main purpose for including the computed values is to indicate biases in \( \phi(E) \) which affect the precision of the computed values of the spectrally insensitive ratios

\[
\sigma_{e}(^{235}U)/\sigma_{e}(Np^{237}), \quad \sigma_{e}(^{1_{235}}), \quad \sigma_{e}(^{1_{235}}), \quad \sigma_{e}(^{1_{235}}), \quad \sigma_{e}(^{238}U)/\sigma_{e}(^{238}U).
\]

The computed ratio \( \sigma_{e}(^{235}U)/\sigma_{e}(Np^{237}) \) varies
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Topsy</th>
<th>Godiva</th>
<th>Jezebel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{239}$Pu</td>
<td>$1.932 \pm 0.015$</td>
<td>$1.910 \pm 0.015$</td>
<td>$1.979 \pm 0.015$</td>
</tr>
<tr>
<td>$^{233}$U</td>
<td>$1.721 \pm 0.015$</td>
<td></td>
<td>$1.689 \pm 0.020$</td>
</tr>
<tr>
<td>$^{240}$Pu</td>
<td>$1.37 \pm 0.06$</td>
<td>$1.14 \pm 0.11$</td>
<td>$1.29 \pm 0.06$</td>
</tr>
<tr>
<td>$^{237}$Np</td>
<td>$0.84 \pm 0.04$</td>
<td></td>
<td>$0.98 \pm 0.02$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$0.128 \pm 0.005$</td>
<td>$0.163 \pm 0.005$</td>
<td>$0.142 \pm 0.005$</td>
</tr>
</tbody>
</table>

**B. Inferred net production cross-section ratios, $\left(\frac{V(1-x)_{1}}{V(1-x)_{235}}\right)$**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Topsy</th>
<th>Godiva</th>
<th>Jezebel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{239}$Pu</td>
<td>$1.923 \pm 0.015$</td>
<td>$1.913 \pm 0.015$</td>
<td>$1.947 \pm 0.019$</td>
</tr>
<tr>
<td>$^{233}$U</td>
<td>$1.726 \pm 0.015$</td>
<td></td>
<td>$1.664 \pm 0.023$</td>
</tr>
<tr>
<td>$^{240}$Pu</td>
<td>$1.33 \pm 0.06$</td>
<td>$1.11 \pm 0.11$</td>
<td>$1.284 \pm 0.06$</td>
</tr>
<tr>
<td>$^{237}$Np</td>
<td>$0.81 \pm 0.04$</td>
<td></td>
<td>$0.98 \pm 0.02$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$0.106 \pm 0.008$</td>
<td>$0.124 \pm 0.012$</td>
<td>$0.175 \pm 0.012$</td>
</tr>
</tbody>
</table>

**C. Inferred ratio, $\left[\frac{V(1-x)_{1}}{V(1-x)_{235}}\right]$, of excess neutrons produced per fission.**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Topsy</th>
<th>Godiva</th>
<th>Jezebel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{239}$Pu</td>
<td>$1.390 \pm 0.033$</td>
<td>$1.358 \pm 0.033$</td>
<td>$1.330 \pm 0.033$</td>
</tr>
<tr>
<td>$^{233}$U</td>
<td>$1.121 \pm 0.033$</td>
<td></td>
<td>$1.094 \pm 0.033$</td>
</tr>
<tr>
<td>$^{240}$Pu</td>
<td>$1.89 \pm 0.11$</td>
<td>$1.46 \pm 0.16$</td>
<td>$1.45 \pm 0.09$</td>
</tr>
<tr>
<td>$^{237}$Np</td>
<td>$1.04 \pm 0.07$</td>
<td></td>
<td>$0.98 \pm 0.05$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$0.76 \pm 0.06$</td>
<td>$0.80 \pm 0.08$</td>
<td>$0.87 \pm 0.06$</td>
</tr>
</tbody>
</table>

**D. Inferred values of $\left[\frac{V(1-x)_{1}}{V(1-x)_{235}}\right]$.**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Topsy</th>
<th>Godiva</th>
<th>Jezebel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{235}$U</td>
<td>$2.47 \pm 0.06$</td>
<td>$2.48 \pm 0.06$</td>
<td>$2.53 \pm 0.06$</td>
</tr>
<tr>
<td>$^{239}$Pu</td>
<td>$3.04 \pm 0.09$</td>
<td>$3.01 \pm 0.09$</td>
<td>$3.03 \pm 0.09$</td>
</tr>
<tr>
<td>$^{233}$U</td>
<td>$2.65 \pm 0.08$</td>
<td></td>
<td>$2.67 \pm 0.08$</td>
</tr>
<tr>
<td>$^{240}$Pu</td>
<td>$3.77 \pm 0.19$</td>
<td>$3.16 \pm 0.24$</td>
<td>$3.21 \pm 0.16$</td>
</tr>
<tr>
<td>$^{237}$Np</td>
<td>$2.53 \pm 0.12$</td>
<td></td>
<td>$2.50 \pm 0.10$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$2.12 \pm 0.10$</td>
<td>$2.18 \pm 0.13$</td>
<td>$2.33 \pm 0.11$</td>
</tr>
</tbody>
</table>

**E. Inferred values of $\left[\frac{V(1-x)_{1}}{V(1-x)_{235}}\right]$.**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Topsy</th>
<th>Godiva</th>
<th>Jezebel</th>
<th>Ave.</th>
<th>$E$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{235}$U</td>
<td>$2.59 \pm 0.06$</td>
<td>$2.59 \pm 0.06$</td>
<td>$2.63 \pm 0.06$</td>
<td>$2.60 \pm 0.06$</td>
<td>$1.45$</td>
</tr>
<tr>
<td>$^{239}$Pu</td>
<td>$3.10 \pm 0.09$</td>
<td>$3.07 \pm 0.10$</td>
<td>$3.06 \pm 0.09$</td>
<td>$3.06 \pm 0.09$</td>
<td>$1.58$</td>
</tr>
<tr>
<td>$^{233}$U</td>
<td>$2.70 \pm 0.08$</td>
<td></td>
<td>$2.72 \pm 0.08$</td>
<td>$2.71 \pm 0.08$</td>
<td>$1.49$</td>
</tr>
<tr>
<td>$^{240}$Pu</td>
<td>$3.68 \pm 0.20$</td>
<td>$3.26 \pm 0.24$</td>
<td>$3.27 \pm 0.17$</td>
<td>$3.16 \pm 0.5$</td>
<td>$2.13$</td>
</tr>
<tr>
<td>$^{237}$Np</td>
<td>$2.62 \pm 0.13$</td>
<td></td>
<td>$2.57 \pm 0.11$</td>
<td>$2.60 \pm 0.11$</td>
<td>$2.09$</td>
</tr>
<tr>
<td>$^{238}$U</td>
<td>$2.61 \pm 0.10$</td>
<td>$2.61 \pm 0.13$</td>
<td>$2.61 \pm 0.11$</td>
<td>$2.61 \pm 0.10$</td>
<td>$3.01$</td>
</tr>
</tbody>
</table>

The computed ratio $\sigma_j^{235}/\sigma_j^{239}$ again varies less than one per cent over the Topsy, Godiva, and Jezebel spectra. Its uncertainty is thus governed almost exclusively by the uncertainties in the basic cross-section measurements of $\sigma_f(E, Np^{237})$ (17) and $\sigma_f(E, Pu^{240})$ (18) which should amount to $\sim 4\%$. We have used

$$\sigma_j(Pu^{240})/\sigma_j(U^{235}) = [\sigma_j(Pu^{240})/\sigma_j(Np^{237})]_{\text{computed}} \times [\sigma_j(Np^{237})/\sigma_j(U^{235})]_{\text{observed}}$$
1.521 for the $^{235}$U thermal fission neutron spectrum, however, is $\sim 5\%$ higher than Richmond's measurement of $1.45 \pm 2.2\%$ (as quoted by Allen and Henkel (18). We adopt the computed ratio; i.e., the implication of the $\sigma_f(E, {\U}^{235})$ and $\sigma_f(E, {\U}^{239})$ data, and assign to it a $3\%$ probable error.

As may be seen in Table XVIII, the computed ratio $\sigma_f({\U}^{235})/\sigma_f({\U}^{239})$, with an estimated $3\%$ uncertainty, agrees well with the measured value (20) which has an estimated $2\%$ uncertainty. The computed ratio for the $^{235}$U thermal fission neutron spectrum agrees well with Richmond's (18) value of $1.423 \pm 2.1\%$. We adopt the mean of the observed and computed ratios listed in Table XVIII, and assign a $2\%$ probable error.

The quantities $|\nu - 1 - \alpha|, |\nu - 1 - \alpha|{\U}^{235}$ obtained by division of $\sigma_f(x)/\sigma_f({\U}^{235})$ by $\sigma_f(x)/\sigma_f({\U}^{239})$ are listed in Table XVIII with their associated probable errors.

### C. Absolute Values of $|\nu - \alpha|$.

According to one-group neutron transport theory, the critical radius of Godiva establishes a relation between the net neutron production cross-section $\sigma_p({\U}^{238})$ and the transport cross-section $\sigma_p({\U}^{235})$. $\sigma_i(E, {\U}^{235})$ has been measured throughout the relevant energy range (18) with a precision of $5\%$ or better. As may be found, for example, from the extrapolated end point method applied to Godiva, a $\sim 5\%$ uncertainty in the scale of $\sigma_i(E, {\U}^{235})$ reflects a $\sim 2.3\%$ uncertainty in $\sigma_i({\U}^{235})$ while the $0.2\%$ uncertainty in the specification of the Godiva critical radius yields a $0.3\%$ uncertainty in $\sigma_i({\U}^{235})$. It thus appears that the Godiva critical radius and available $\sigma_i(E, {\U}^{235})$ data permit the determination of $\sigma_i({\U}^{235})$ with a precision of $\sim 2.3\%$. Present computational techniques, however, cannot fully exploit this possible precision, primarily because of the weak sensitivity of critical radius to weightings of the differential scattering cross sections, $\sigma_i(E, \theta, {\U}^{235})$ other than the transport weighting (it should be mentioned that uncertainties in spectrum-determining parameters such as $\sigma_i(E, {\U}^{235})$ are of little significance in this connection). We have used the $S_4$ method (21) in the $S_4$ and transport approximations with six-group nuclear parameter descriptions of $^{235}$U and $^{238}$U for computation of the Godiva critical radius (and for the central values of $\phi$ and $\phi'$ listed in Table XVIII). This computed radius is
0.3% less than the observed Godiva radius. Here the \( S_1 \) approximation yields critical radii 0.8% less than \( S_6 \) (see, for example, Table 2a of reference 17), while the transport approximation is estimated to produce an additional negative bias of \( \sim 0.4\% \). The nuclear parameter descriptions of \( \text{U}^{238} \) and \( \text{U}^{236} \) of reference 18 thus imply a Godiva critical radius 0.9% larger than that observed. To secure agreement we must raise the correspondingly implied value \( \sigma_f(\text{U}^{238}) = 1.879 \) barns by 1.3%. We conclude that the observed critical radius of Godiva implies \( \sigma_f(\text{U}^{238}, \text{Godiva}) = 1.90 \pm 0.05 \) barns, or combined with \( \sigma_f(\text{U}^{239}, \text{Godiva}) = 1.280 \pm 0.03 \) barns, implies \( \langle \nu - \alpha \rangle_{\text{U}^{238}, \text{Godiva}} = 2.48 \pm 0.06 \). If we normalize to this value, \( \langle \nu - \alpha \rangle_{\text{U}^{239}, \text{Topsy}} = 2.488 \pm 0.06 \) and \( \langle \nu - \alpha \rangle_{\text{U}^{235}, \text{Jezebel}} = 2.526 \pm 0.06 \), and thence from our \( \langle \nu - \alpha \rangle \), \( \langle \nu - \alpha \rangle_{\text{U}^{233}} \) ratios the values of \( \langle \nu - \alpha \rangle \), listed in Table XVII.

D. ABSOLUTE VALUES OF \( \nu(x) \)

Capture to fission ratio data for the critical assemblies exist only for \( \text{U}^{235} \). Here, the observed \( \sigma_{c, y}/\sigma_f \) values listed in Table XVIII together with the observed Godiva value \( \sigma_{c, y}(\text{U}^{235})/\sigma_f(\text{U}^{239}) = 0.05 \) (20) give the \( \nu(\text{U}^{235}) \) values listed in Table XVII. The available differential (23) and/ or integral (24) \( \sigma_{c, y} \) data for \( \text{U}^{232} \) and \( \text{Pu}^{240} \) permit estimation of \( \alpha(\text{U}^{235}) \) and \( \alpha(\text{Pu}^{240}) \) in each critical assembly with a \( \sim 25\% \) accuracy. No fast radiative capture data are available for \( \text{U}^{231} \), \( \text{Pu}^{240} \), and \( \text{NP}^{237} \). Nevertheless, the data for other elements suggest that the fast radiative capture cross sections of the heavy elements are similar and it is felt that the error incurred by our assigning to \( \text{U}^{232} \), \( \text{Pu}^{240} \), and \( \text{NP}^{237} \) the capture cross sections of \( \text{U}^{238} \) is only \( \sim 50\% \). Comparison of the \( \nu(x) \) and \( \nu - \alpha \) listings of Table XVII indicates the relative unimportance of refining \( \alpha \) estimates. Generally, the spread in \( \nu \) values obtained from reactivity measurements in the different fast critical assemblies is small compared to the stated uncertainties indicating that these uncertainties originate mainly in the estimation procedure. The one exception is for \( \nu(\text{Pu}^{240}) \); here the basic reactivity coefficient measurements were made in Godiva and Jezebel with the same \( \text{Pu}^{240} \)-enriched plutonium sample whereas the meas-

\[ \langle \nu - \alpha \rangle_{\text{Pu}^{240}, \text{Godiva}} = 3.72 \pm 0.06 \text{ barns}, \quad \langle \nu - \alpha \rangle_{\text{Pu}^{240}, \text{Jezebel}} = 1.83 \pm 0.06 \text{ barns}, \quad \langle \nu - \alpha \rangle_{\text{Pu}^{240}, \text{Topsy}} = 3.03 \pm 0.07. \]

urements in Topsy involved a separate set of \( \text{Pu}^{240} \)-enriched samples. We have been unable to determine which was incorrectly specified, and have, therefore, increased the estimated uncertainty in the value of the average \( \nu(\text{Pu}^{240}) \) also listed in Table XVII. In this table, the energy \( E(x) \) corresponds to the average energy of the critical assembly neutrons producing fissions in sample \( x \) and is computed as

\[ E(x) = \frac{\int E \sigma_f(x, E) \sum \phi_f(E) \, dE}{\int \sigma_f(x, E) \sum \phi_f(E) \, dE} \]

where the summation extends over the assemblies in which reactivity coefficients of \( x \) were measured.

REFERENCES

3. R H WHITE, Nuclear Sci and Eng 1, 53-61 (1956)
5. G A JAVITS, G A LINENBERGER, J D ORNDOFF, AND H C PAXTON, Two plutonium-metal critical assemblies Nuclear Sci and Eng 8, 525 (1960) This issue
6. G E HANSEN AND C MAIER, Perturbation theory for fast-neutron systems Nuclear Sci. and Eng. 8, 515 (1960) This issue
7. C C BYERS, Cross sections of various materials in the (godiva and Jezebel critical assemblies Nuclear Sci and Eng 8, 608 (1960). This issue
16. L CRANBERG AND J LIVAN, Phys Rev 109, 2063-2070 (1958)