A. GOODWIN, JR., AND C. L. SCHUSKE, "PLUTONIUM GRAPHITE ASSEMBLIES,"
DOW CHEMICAL CO., ROCKY FLATS PLANT REPORT RFP-158, (AUGUST 1959).
Plutonium Graphite Assemblies

Part II

by

A. Goodwin, Jr.

C. L. Schuske

THE DOW CHEMICAL COMPANY

ROCKY FLATS PLANT     DENVER, COLORADO

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PLUTONIUM GRAPHITE ASSEMBLIES - PART II

by

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ABSTRACT

Neutron multiplication measurements were made on a number of cylindrical assemblies of plutonium and graphite discs. $S_n$ calculations were made on homogeneous mixtures of plutonium and graphite with varying C/Pu ratios and varying reflector thickness.

ACKNOWLEDGMENTS

These tests were made possible by the cooperation of Mr. I. B. Venable and staff. The $S_n$ calculations were performed at the Los Alamos Scientific Laboratory with the cooperation of Dr. Hugh Paxton, Dr. Gordon Hansen, and Mr. Bengt Carlson.

We would also like to thank Mr. H. V. Duba for his assistance in illustrating the report.
1. INTRODUCTION

Neutron multiplication measurements were made on cylindrical disc assemblies constructed of alternate layers of graphite and plutonium metal sheet. The thicknesses of the graphite and plutonium layers were changed in order to study the effect of inhomogeneity.

$S_4$ calculations were made on homogeneous mixtures of plutonium and graphite with varying graphite reflector thicknesses. The C/Pu ratios used in the calculations were 0, 5, 10, 15, and 20. These calculations are correlated with the experiments.

This report is a continuation of RFP-123. (1)

2. EXPERIMENTAL MATERIALS

The measuring equipment used in these experiments included scalers, Atomic Model 1050-A, coupled to G.E. $^{10}\text{B}$-lined counters encased in 8-in. diameter polystyrene moderators and a LiI (Eu) scintillator.

2.1 Materials

2.1.1 Moderator and Reflector (Graphite National Carbon Grade CS-312)

A) Moderator and reflector dimensions:

Discs 14 in. in diameter and 1/2-in. thick.

B) Moderator and reflector density: 1.76 g/cm$^3$.

2.1.2 Fuel (Plutonium)

A) Fuel dimensions: Discs ~13.25 x 0.056-in.

B) Fuel density: ~15.8 g/cm$^3$

C) Average weight of fuel pieces: 2075 g.

3. EXPERIMENTAL RESULTS AND PROCEDURES

Figure 1 is a schematic of the assemblies. Regions A are graphite of equal thickness on each end, regions B are plutonium discs, and regions C are graphite discs. Table I summarizes the results of the experiments.

<table>
<thead>
<tr>
<th>Thickness of Region A in.</th>
<th>Number of Sheets of Pu in Region B</th>
<th>Thickness of Region C in.</th>
<th>Number of Sheets in (Extrapolated)</th>
<th>Critical Number of Sheets</th>
<th>Mass Exp. Core (Extrapolated)</th>
<th>C/Pu Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1</td>
<td>1/2</td>
<td>36</td>
<td>~ $\infty$</td>
<td>~ $\infty$</td>
<td>20</td>
</tr>
<tr>
<td>1-1/2</td>
<td>5</td>
<td>2-1/2</td>
<td>35</td>
<td>~ $\infty$</td>
<td>~ $\infty$</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
<td>30</td>
<td>34</td>
<td>70.55</td>
<td>6.67</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>1-1/2</td>
<td>27</td>
<td>31.5</td>
<td>69.5</td>
<td>6.67</td>
</tr>
<tr>
<td>1-1/2</td>
<td>8</td>
<td>2</td>
<td>32</td>
<td>37</td>
<td>76.8</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>26</td>
<td>31.2</td>
<td>64.7</td>
<td>5</td>
</tr>
</tbody>
</table>

The C/Pu ratios in Table I were calculated assuming the core to be as shown in Figure 1a. Under this assumption the C/Pu ratio remains constant as the core is assembled.
Figures 2 through 7 show the extrapolations to critical of $\frac{1}{M}$ versus the number of sheets for the above experiments. An indication of the experimental uncertainty can be seen from these curves.

4. THEORETICAL CORRELATION

Some $S_4$ calculations were done on homogeneous mixtures of carbon and plutonium reflected by carbon. Table II gives a description and the results of these $S_4$ calculations. The densities were calculated assuming the plutonium metal density to be 15.8 g/cm$^3$ and the graphite density to be 1.76 g/cm$^3$.

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Fuel Density</th>
<th>Moderator and/or Reflector Density</th>
<th>Geometry</th>
<th>C/Pu Ratio</th>
<th>Core Radius or Half Thickness cm</th>
<th>Reflector Thickness cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.8</td>
<td>1.76</td>
<td>slab</td>
<td>0</td>
<td>1.83</td>
<td>0.91</td>
</tr>
<tr>
<td>2*</td>
<td>15.8</td>
<td>0</td>
<td>slab</td>
<td>0</td>
<td>2.18</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4.86</td>
<td>1.22</td>
<td>cylinder</td>
<td>5</td>
<td>10.14</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>4.86</td>
<td>1.22</td>
<td>slab</td>
<td>5</td>
<td>5.50</td>
<td>0.92</td>
</tr>
<tr>
<td>5*</td>
<td>2.81</td>
<td>1.43</td>
<td>slab</td>
<td>10</td>
<td>8.05**</td>
<td>1.07</td>
</tr>
<tr>
<td>6*</td>
<td>2.81</td>
<td>1.43</td>
<td>cylinder</td>
<td>10</td>
<td>14.29</td>
<td>1.06</td>
</tr>
<tr>
<td>7</td>
<td>2.04</td>
<td>1.53</td>
<td>cylinder</td>
<td>15</td>
<td>17.65</td>
<td>1.10</td>
</tr>
<tr>
<td>8</td>
<td>2.04</td>
<td>1.53</td>
<td>slab</td>
<td>15</td>
<td>9.98</td>
<td>1.42</td>
</tr>
<tr>
<td>9</td>
<td>1.58</td>
<td>1.58</td>
<td>cylinder</td>
<td>20</td>
<td>20.36</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>1.58</td>
<td>1.58</td>
<td>slab</td>
<td>20</td>
<td>11.84</td>
<td>1.32</td>
</tr>
</tbody>
</table>

* Also reported in RFP-123.
** This core thickness is given incorrectly in Table III of RFP-123. It should be 16.10 cm instead of 17.10 cm.
These calculations were interpolated to finite geometries by use of a constant buckling and extrapolation length formula. The buckling and extrapolation length were calculated from the overall dimensions (core + reflector) of the cylinder and slab for each C/Pu ratio. Table III lists these parameters.

<table>
<thead>
<tr>
<th>C/Pu</th>
<th>Buckling ( (B^2) ) cm(^{-2} )</th>
<th>Extrapolation Length cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.150*</td>
<td>1.87*</td>
</tr>
<tr>
<td>5</td>
<td>0.0323</td>
<td>2.32</td>
</tr>
<tr>
<td>10</td>
<td>0.0182</td>
<td>2.46</td>
</tr>
<tr>
<td>15</td>
<td>0.0127</td>
<td>2.59</td>
</tr>
<tr>
<td>20</td>
<td>0.00986</td>
<td>2.66</td>
</tr>
</tbody>
</table>

* Values reported in RFP-123 obtained from the sizes of a bare slab and a bare cylinder of plutonium metal (density 15.6 g/cm\(^3\)) calculated by S\(_4\) using six group cross sections.

It was shown in RFP-123 that for a C/Pu ratio of 10 the reflector savings is approximately equal to the reflector thickness for thin reflectors (<1 cm). Therefore for the C/Pu ratios \( \geq 10 \) the above bucklings and extrapolation lengths can be used for bare systems. They were also used for the bare C/Pu = 5 systems although without verification. We are primarily interested here in systems which are partially reflected since all of the experiments had some graphite on the ends and also had graphite extending radially beyond the plutonium discs. Therefore, the above bare buckling and extrapolation length
were not verified by making a completely bare calculation for a system with $C/Pu = 5$.

The slab for $C/Pu = 15$ in Table II has a rather large reflector thickness and is probably too thick to use the overall size of the system to calculate the buckling and extrapolation length. Therefore, the above buckling and extrapolation length for $C/Pu = 15$ were obtained by subtracting $0.05 \text{ cm}$ (refer to Figure 8) from the overall size of the slab with $C/Pu = 15$.

In order to determine the effect of thicker graphite reflectors some $S_n$ calculations were made at a $C/Pu$ of 10. The results of these calculations are given in Table IV.

**TABLE IV**

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Geometry</th>
<th>Core Radius or Half Thickness cm</th>
<th>Reflector Thickness cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Slab</td>
<td>7.28</td>
<td>1.94</td>
</tr>
<tr>
<td>12</td>
<td>Cylinder</td>
<td>13.81</td>
<td>1.50</td>
</tr>
<tr>
<td>13</td>
<td>Slab</td>
<td>5.55</td>
<td>5.05</td>
</tr>
<tr>
<td>14</td>
<td>Cylinder</td>
<td>12.35</td>
<td>3.86</td>
</tr>
</tbody>
</table>

Knowing the sizes of the bare geometries (slab and cylinder obtained using buckling and extrapolation length in Table II) we can obtain reflector savings for the above reflector thicknesses. In order to extend these calculations to larger reflector thicknesses a one-group formula was fitted
to the $S_n$ slab calculations. The critical equation for a one-group reflected slab reactor is:

$$\tan \left( \frac{B}{2} \right) = \frac{D_K}{D_C} \coth \left[ K_r (T + \delta_r) \right]$$

where $B^2 =$ core buckling

$D =$ diffusion coefficient (subscripts denote core or reflector)

$K_r =$ reciprocal diffusion length for reflector

$\delta_r =$ extrapolation length for reflector.

The following parameters were obtained to fit the $S_n$ calculations to this formula:

$$B^2 = 0.0182 \text{ cm}^{-2} \text{ (see Table III)} \quad K_r = 0.075 \text{ cm}^{-1}$$

$$\frac{D_r}{D_c} = 0.893 \quad \delta_r = 2.3 \text{ cm}$$

Using this method the reflector savings for an infinite reflector is 5.89 cm.

The curve of reflector savings versus reflector thickness is plotted in Figure 8. The $S_n$ cylinder calculations also fall on this curve showing that the reflector savings is not greatly dependent on geometry for the sizes calculated.

The above formula can be used to determine the reflector savings for finite cylinders reflected on the ends if the buckling in equation 1 is reduced by subtracting

$$B_r^2 = \left( \frac{2.4048}{R + \delta_r} \right)^2 .$$

Using this method the reflector savings for a 14-in. diameter cylinder reflected on the ends by infinite graphite was calculated to be 5.34 cm whereas for the infinitely reflected
infinite slab the reflector savings was 5.89 cm. The difference in reflector savings between infinite slabs and finite end reflected cylinders would be less for finite reflectors.

Figure 9 shows calculated curves and the experimental points of critical mass versus C/Pu ratio. The curve labeled "bare" in Figure 9 was calculated from the bucklings and extrapolation lengths of Table III. The curves labeled 1/2-in., 1-in., and 1-1/2-in. end reflector were obtained from the bare curves and using Figure 8 to obtain reflector savings at C/Pu = 10. These reflected cylinders were also assumed to have 3/8-in. radial reflectors. At C/Pu = 0 cylinders with 1/2-in., 1-in., and 1-1/2-in. end reflector thickness can be estimated from Figure 6 of RFP-123. This was done using a 13.25-in. diameter cylinder. In using Figure 6 of RFP-123 the mass used for the bare 13.25-in. cylinder was obtained using the buckling and extrapolation length for C/Pu = 0 of Table III, then a curve was drawn parallel to the experimental curves. Finally, all curves in Figure 9 ("bare", 1/2 in., 1 in., 1-1/2 in.) must go to infinity for the same C/Pu ratio.

The points of the arrows in Figure 8 represent the experimental C/Pu ratios and masses listed in Table I plus some data taken from RFP-123. From this method of displaying the data one would conclude that there is a rather large decrease in critical mass as one lumps the fuel into large pieces keeping the C/Pu ratio constant. For the fast systems
described here it would be difficult to explain such an effect as being due to inhomogeneity.

Another method of calculating the C/Pu ratios of a homogeneous core was tried in order to determine if the experimental points could be made to fit the theoretical calculations better. In this method the C/Pu ratios were calculated for a core as illustrated in Figure 1b. Both methods of calculating C/Pu ratios assumed that the graphite which extended radially beyond the fuel sheets was reflector. It is evident that the C/Pu ratios do not remain constant as the assembly is constructed when using the method illustrated in Figure 1b. Therefore, the C/Pu ratio has to be calculated for the system after it has been extrapolated to critical. The C/Pu ratios of the inscribed points in Figure 9 were calculated by this latter method.

The scatter of the experimental points is due mostly to experimental uncertainties and whatever effect is actually due to inhomogeneity would be masked by these experimental errors. Disagreement between calculation and experiment can be due to calculational error, experimental uncertainty, or both. The method of converting from the infinite geometries calculated by $S_n$ to the finite geometries of the experiments would also contribute to the error. The conversion from non-homogeneous systems to homogeneous systems greatly affects the comparison between experiment and calculation.
The equivalent homogeneous core would seem to be best represented by Figure 1b, that is, the core surface is the smallest cylinder which would enclose all the fuel.
FIGURE 1
REGION A—1/2" Graphite
" B—1 Sheet Pu (See Fig. 1)
" C—1/2" Graphite

NUMBER OF Critical Sheets \( \sim \infty \)
Critical Mass \( \sim \infty \)

FIGURE 2

REGION A—1-1/2" Graphite
" B—5 Sheets Pu (See Fig. 1)
" C—2-1/2" Graphite

NUMBER OF Critical Sheets \( \sim \infty \)
Critical Mass \( \sim \infty \)

FIGURE 3
Region A—1" Graphite
" 8—6 Sheets Pu (See Fig.1)
" C—1" Graphite

Number Of Critical Sheets 34
Critical Mass 70.55 Kg

Region A—1" Graphite
" B—9 Sheets Pu (See Fig.1)
" C—1-1/2" Graphite

Number Of Critical Sheets 31.5
Critical Mass 69.5 Kg
INVERSE MULTIPLICATION Vs. NUMBER OF SHEETS OF Pu

Region A-1-1/2" Graphite
" B-8 Sheets Pu (See Fig.1)
" C-2" Graphite

Number OF Critical Sheets 37
Critical Mass 76.8 Kg

INVERSE MULTIPLICATION Vs. NUMBER OF SHEETS OF Pu

Region A-1" Graphite
" B-8 Sheets Pu (See Fig.1)
" C-1" Graphite

Number OF Critical Sheets 31.2
Critical Mass 64.7 Kg
REFLECTOR SAVINGS vs. REFLECTOR THICKNESS

Core — Graphite Plutonium Mixture C/Pu = 10
Reflector — Graphite (Density 1.76 gm/cm³).
Calculated for infinite cylinders and slabs by SNL.

FIGURE 8
CRITICAL MASS Vs. C/Pu RATIO

14 Inch Diameter Cylinders

FIGURE 9