

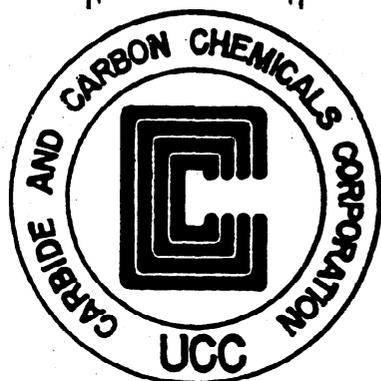
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CRITICAL MASS STUDIES, PART IV

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K-25 PLANT

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A B S T R A C T

Some exploratory experiments are reported describing the conditions under which enriched uranium, contained in aqueous solutions of uranyl fluoride, becomes critical in two right cylindrical reactors having parallel axes. Values of the height of solution and the critical mass have been obtained for reactors with diameters ranging from 5 to 20 inches. Data were obtained at four chemical concentrations corresponding to H:U-235 atomic ratios varying between 30 and 330 and with reactor separations up to 50 cm. In some of the experiments, the reactors were submerged in a water bath while others were done with no reflector.

The critical mass of an unenclosed two reactor system was found to depend upon the distance between the components when they were separated even as much as 50 cm, however, the mass in each was then more than 90% of that required to make it singly critical.

Interposition of water between two reactors reduced their interaction due to the attenuation of the neutron flux by hydrogen. Two water enclosed reactors which could be made singly critical when approximately equilateral were found to be effectively isolated when separated 15 cm or more. Two which could not be made individually critical were isolated by a separation of a few centimeters. Those which were singly critical at heights large compared to their diameters showed apparent interaction at more than 20 cm spacing. The effect is attributed to the equivalence of a few interaction neutrons in the two component system and the relatively large quantity of uranium which must be placed at the end of long reactors to produce small increases in reactivity.

The smallest mass accumulated at criticality in these experiments was 680 gm U-235 contained in each of two water enclosed reactors ten inches in diameter with sides in contact, each filled to a height of 16.9 cm. The chemical concentration of the fuel corresponded to an H:U-235 atomic ratio of 329.

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I. INTRODUCTION

This report presents additional data on the critical mass of U-235 in aqueous solutions of uranyl fluoride. Experiments in which the fluoride solution was made critical in single right cylindrical reactors have been reported previously.¹ The present series of experiments was undertaken to determine the conditions necessary for criticality in a system of two right cylindrical reactors with parallel axes. The mass contained in either reactor at criticality may be significantly less than that required for a single isolated one under otherwise identical conditions. This phenomenon, known as interaction, is a result of an exchange of leakage neutrons between the two reactors. It is important to know the degree of interaction in order to specify the conditions for safe storage and handling of fissionable materials. The experimental variables which were investigated in the present program included the degree of moderation of the U-235, the diameter of the reactors and the separation of their axes. Some experiments were done with the reactors enclosed in a water reflector while others had no reflector.

The data reported here are incomplete and are derived from a group of experiments designed to show major trends rather than details.

II. EXPERIMENTAL MATERIALS

A. Uranium Solution

About 13 kg of uranium as uranyl fluoride, containing 93.4% U-235 and 1.1% U-234 were available for these experiments. Aqueous solutions of this compound were prepared having concentrations corresponding to hydrogen to U-235 atomic ratios of 30, 53, 169 and 329. Reference is made in Part III for a discussion of the nuclear properties of the solutions and of the contaminants present.¹

B. Materials of Construction

Solution storage and transport facilities were fabricated principally from stainless steel, type 347, because of its low corrosivity. Most of the reactors were made from aluminum, type 3S, in view of its favorably low thermal neutron absorption cross section. It was necessary, however, to coat the aluminum with Bakelite varnish to reduce the corrosion. The reactors twenty inches in diameter were made of stainless steel for structural strength. This size was used without a water reflector and previous work had shown little difference between aluminum and stainless steel under these conditions.

1. Beck, Clifford K., A. D. Callihan, J. W. Morfitt, R. L. Macklin, "Critical Mass Studies, Part III", Carbide and Carbon Chemicals Corporation, K-25 Plant, Report K-343, April 19, 1949. In subsequent references to this report, it will be designated as Part III.

Flexible connections were made with heavy wall gum rubber tubing which proved satisfactory during the short time of use. Fluorothene and tygon tubing were used for transparent sections of liquid flow lines.

III. APPARATUS

The general arrangement of the apparatus was that described in Part III. However, certain modifications were made in the reactor assembly in order to accommodate the additional equipment required for these experiments.

A. Reactors and Reflector

The set of interchangeable reactors used in the earlier experiments together with the small water reflector tanks for the respective top surfaces were retained without change. A second set of reactors having diameters of 5, 5-1/2, 6, 8, 10 and 15 inches was fabricated from type 3S aluminum. An additional reactor 20 inches in diameter was made of stainless steel.

The main water reflector tank was replaced by one 4'-5" x 2'-3" x 3'-6" deep. Near one end of the tank one of the first set of reactors could be rigidly mounted. To the top edges of this tank and parallel to its long dimension were attached steel rails on which a dolly could travel. Each of the second set of interchangeable reactors was flanged at the top for rigid attachment to the dolly. It was possible to separate the adjacent edges of reactor pairs, having diameters up to fifteen inches, from contact to 33 cm, maintaining at least 10 cm of water on all sides. In the absence of the water reflector, it was possible to achieve separations exceeding 60 cm with the 15 inch reactor since the rails extend beyond one end of the tank. A schematic drawing of the reactor assembly is shown in Fig. 1 and a photograph of the apparatus in Fig. 2.

Uranyl fluoride solution was transferred by air pressure from the four inch storage cylinders to the reactors through the main feed line which terminated in the three inch lead to the stationary reactor. The solution was admitted to the movable reactor through a line attached to the three inch lead. The section of this connecting line external to the reflector water tank was stainless steel pipe provided with a valve and a short length of transparent tygon tubing as shown in Fig. 3. The portion of the connection within the water tank was double wall rubber tubing permanently attached to the reactor as illustrated in Fig. 4. When the reactors were changed, this connection was broken at a flange outside the water tank, reducing the possibility of fuel leaking into the reflector water. Difficulty was experienced in establishing

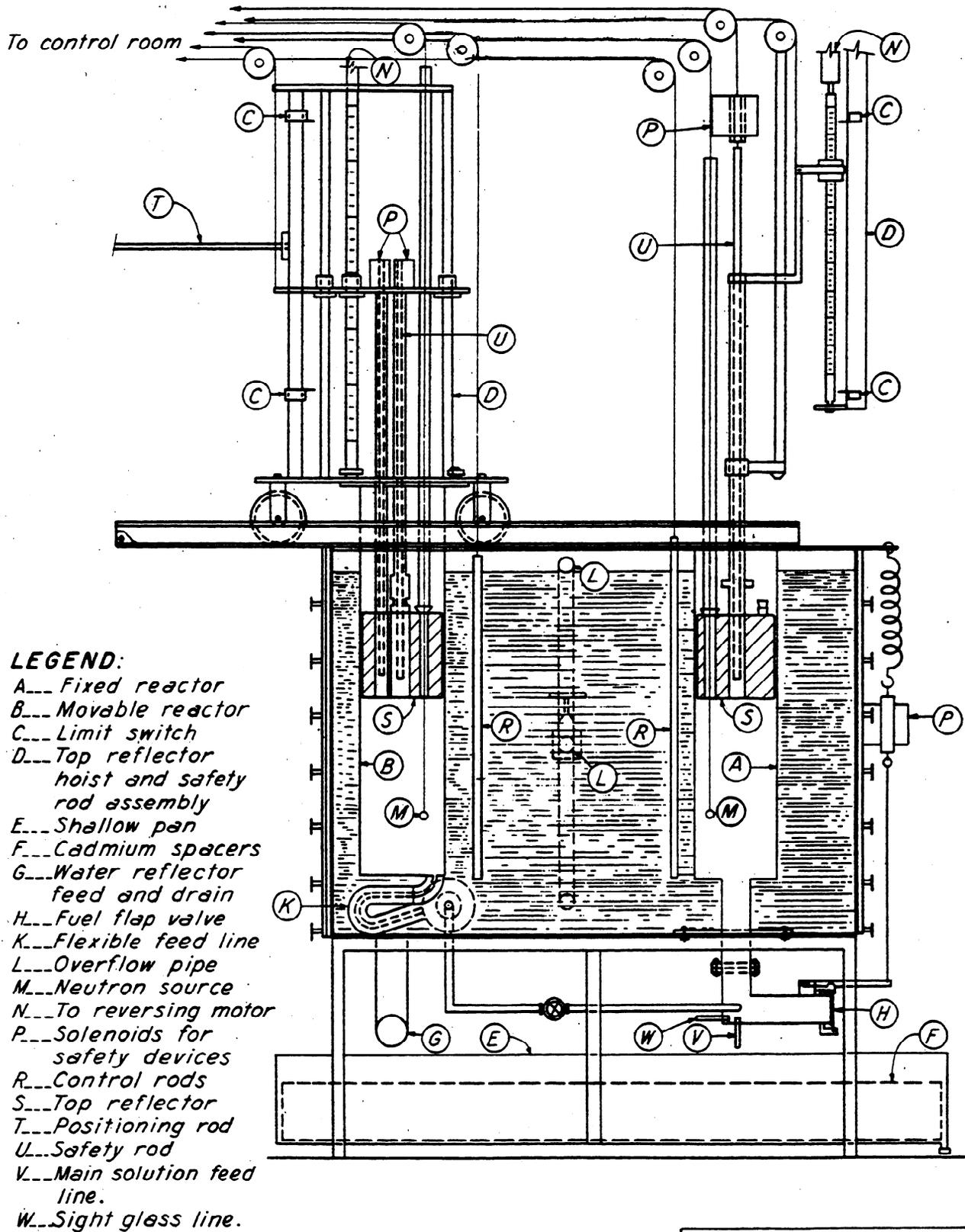
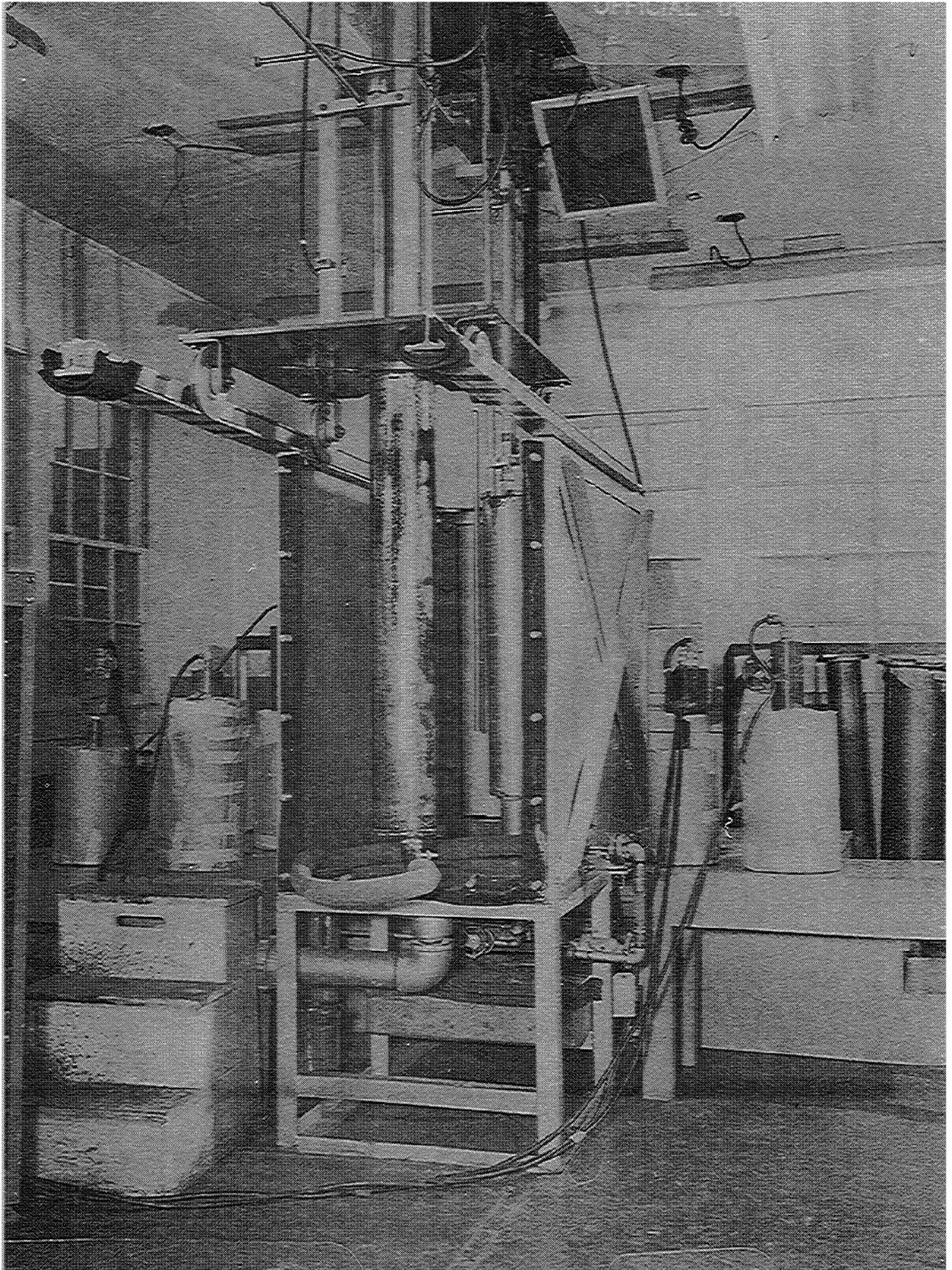
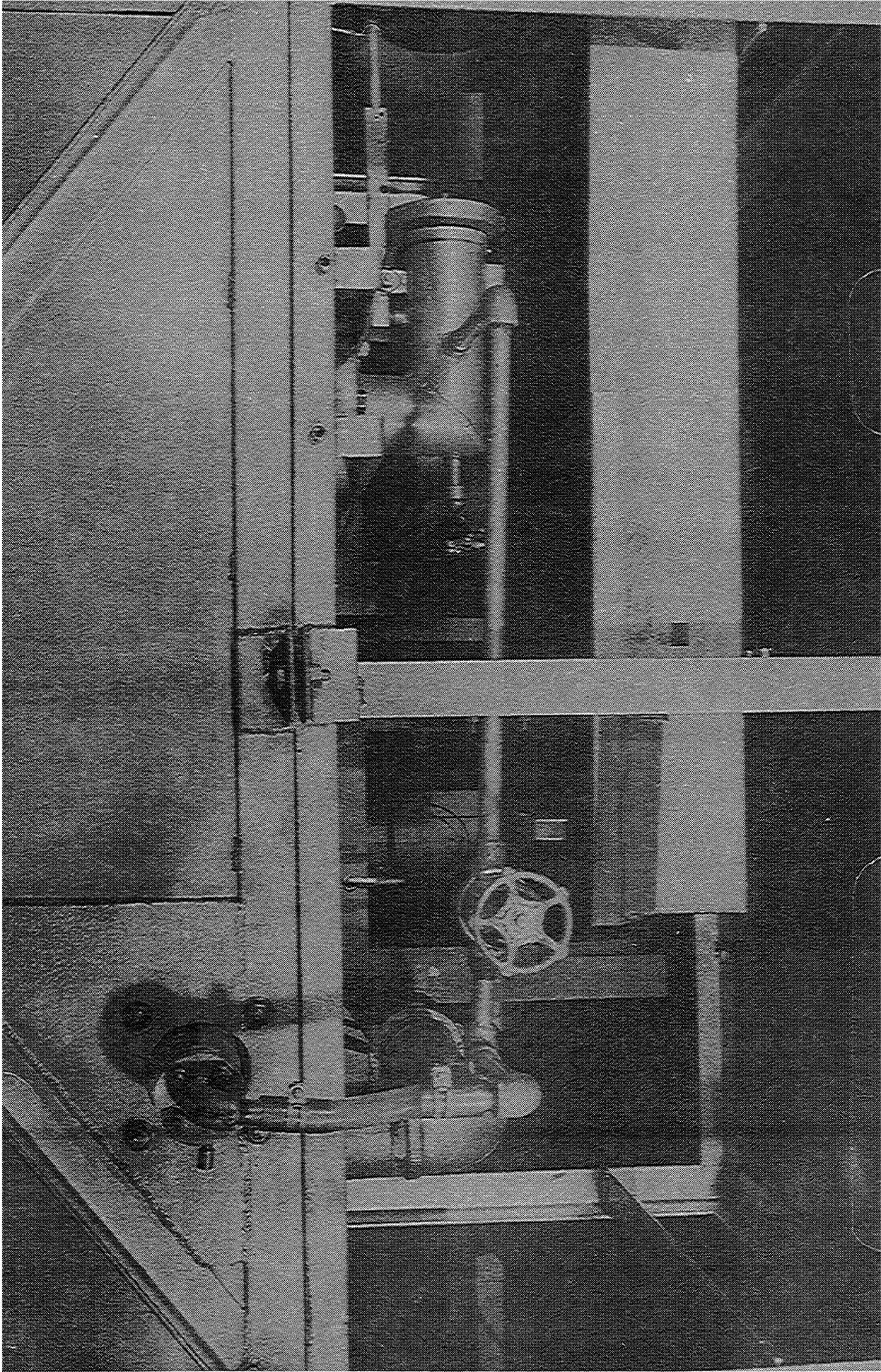


FIGURE 1

| | |
|---------------------------------|-----------------|
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| INTERACTION APPARATUS | |
| K-25 PLANT | |
| CARBIDE & CARBON CHEMICALS CORP | |
| DRWN. <i>B. Hammer</i> | APP'VD. _____ |
| DATE: MAY 25, 1949 | DWG NO: LD-8861 |





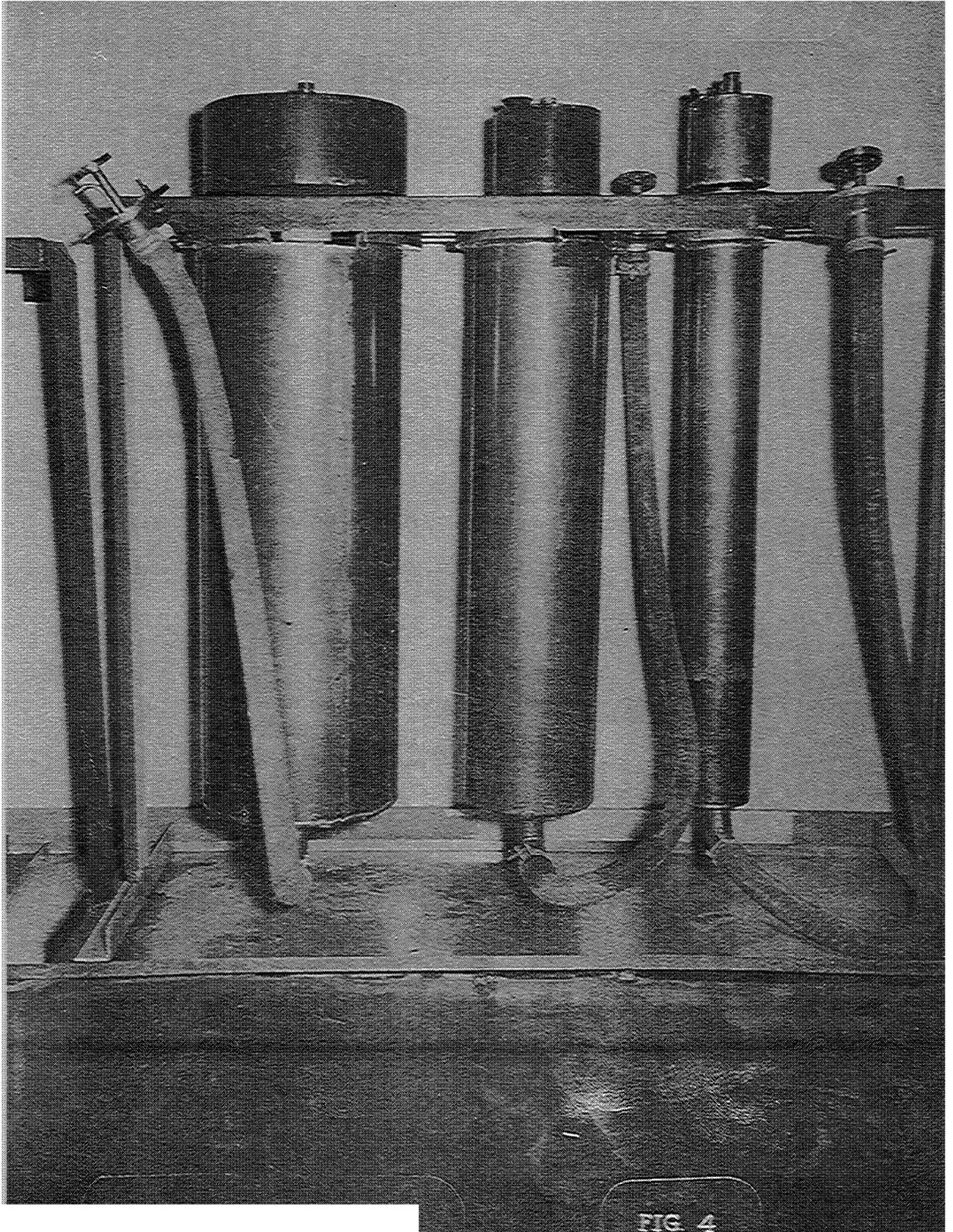


FIG. 4

a flow of solution through the rubber tubing because of air locks, requiring manual manipulation of the tubing until a continuous column of liquid was present. Solution was then readily transported between the reactors.

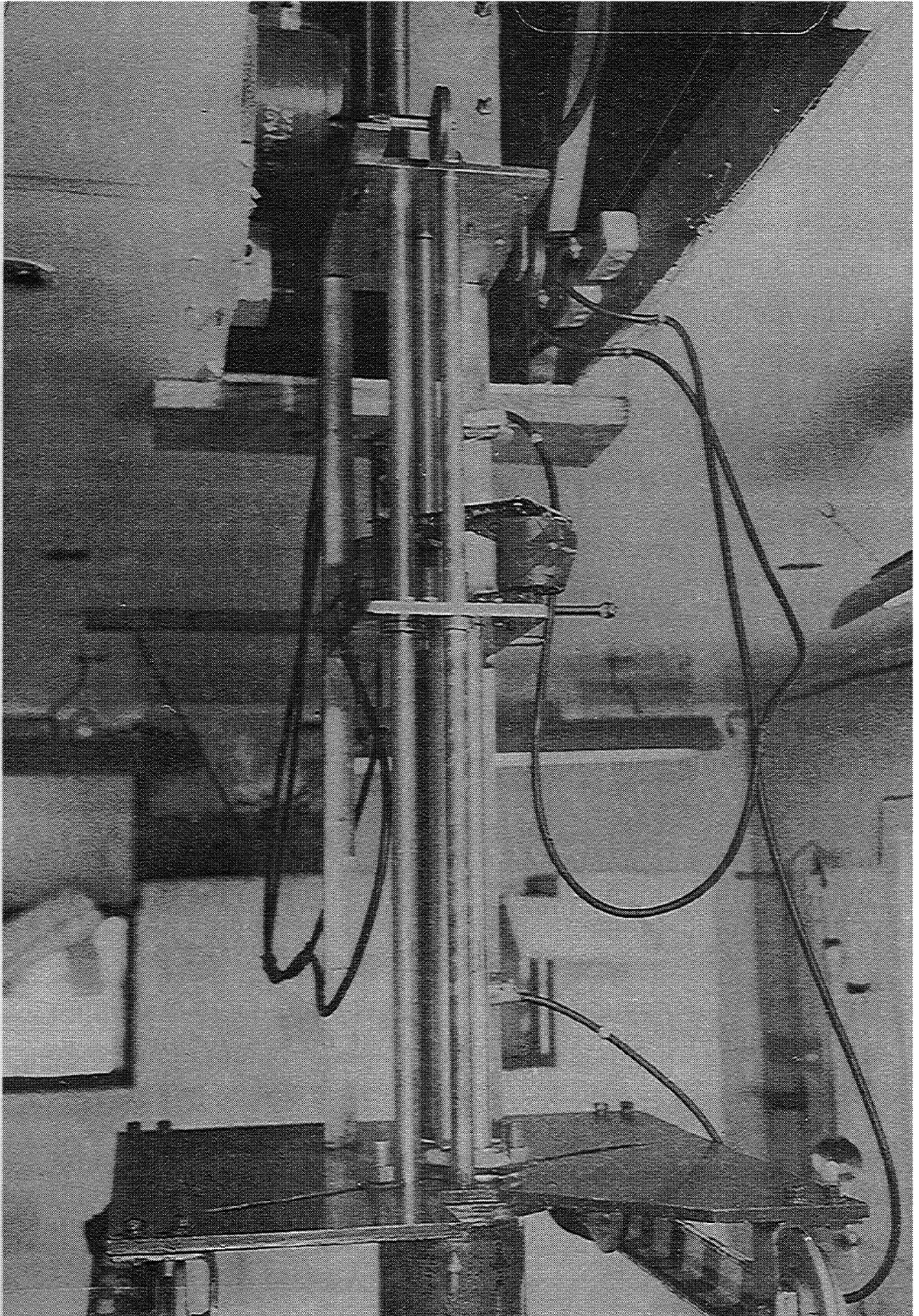
Small aluminum water tanks, six inches deep and fitting snugly into the respective reactors, provided reflectors for the top surface of the fuel. The top reflector was positioned by a motor driven screw, mounted on the dolly, which was operated from the Control Room. Some of the movable reactors and their top reflector tanks are shown in Fig. 4.

The position of the surface of the fuel was indicated in a transparent plastic sight glass mounted on a scale in the Control Room and connected to the reactor assembly. Indicators showing the location of the lower surfaces of both top reflector tanks utilized the same scale in a manner such that, when either tank was in contact with the fuel surface, its indicator and the liquid level in the sight glass were in juxtaposition. Bringing either top reflector tank into contact with the solution in the reactor disturbed the solution level in the sight glass, thereby providing an independent determination of the position of the fuel surface.

B. Safety and Control Devices

The magnetically supported safety rod, the control rod, the neutron source for the stationary reactor, and the arrangement for rapid dispersal of the fuel into a shallow pan are described in Part III. Fig. 3 is a photograph of the section below the reflector tank showing on the right the flap valve for emptying the fuel into the shallow pan.

Similar safety and control devices were provided for the movable reactor. A number of these devices are shown in Fig. 5. The plate in the center of the photograph could be moved vertically by a remotely controlled reversible electric motor through a threaded rod and nut. The motion was constrained by limit switches. The plate supported the top reflector tank referred to above and a stainless steel tube used to guide the neutron source. Two electromagnets, mounted on the plate, held the safety rods. The latter consisted of cadmium sheet sealed in stainless steel tubing. The magnets were de-energized if the neutron intensity at either of two detectors exceeded a predetermined value permitting the rods to fall into the fuel. Another stainless steel encased cadmium rod - the control rod- and a neutron source supported by wires, could be manually adjusted from the Control Room. The source was placed in the reactor and the control rod near the periphery of the reactor. In the absence of a water reflector the control rod was located in the fuel. Appropriate adjustable pulleys were provided to compensate for changes in the positions



of the source and control rod in the reactor due to horizontal motion of the dolly.

C. Reactor Separation

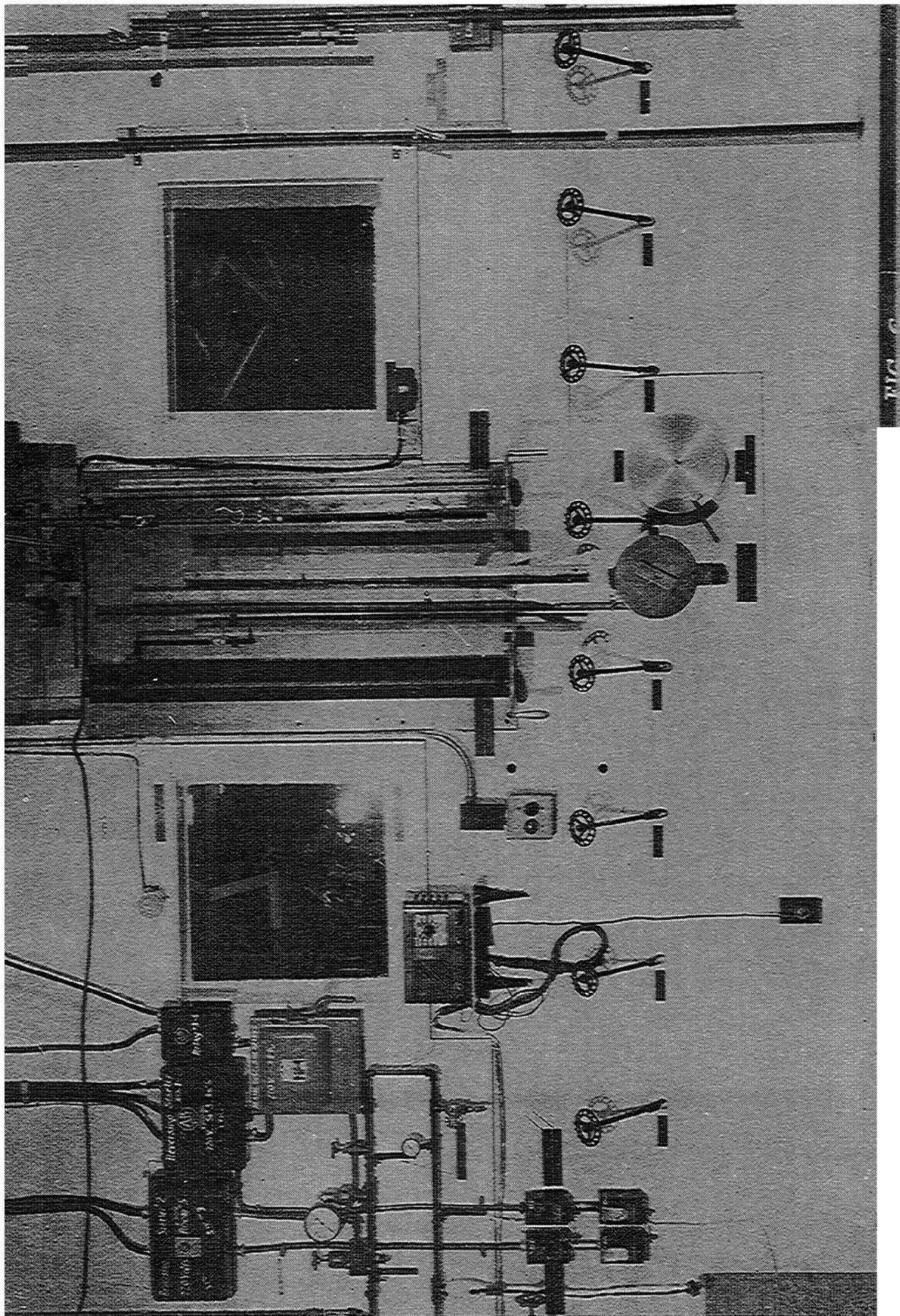
Motion of the dolly was effected through a steel rod, one half inch in diameter, extending into the Control Room. The threaded end of the rod was fitted with a sprocket wheel which was chain driven from a conveniently located hand wheel. It was not possible for the dolly to move except when the controls were operated, thereby minimizing the danger of the two containers of fuel inadvertently approaching each other. A wire connecting the dolly to a weighted pointer in the Control Room permitted the position of the movable reactor to be observed on a scale.

D. Operational Controls

Fig. 6 shows the assembly controls located on the wall separating the Experimental and Control Rooms. The regulated air pressure supply and the motor control switches for positioning the top reflector tanks are at the left. The solution flow control valve handles extend through the lower section of the wall. In the center are the positioners and indicators for the safety and control rods and for the dolly. The adjustments for the two neutron sources are at the right. Across the bottom of the picture is the emergency foot bar which opens the flap valve emptying the fuel into the shallow pan below the reactor. The sight glass, some of the detection instruments, and the reflector water control valves are shown in Fig. 7.

IV. EXPERIMENTAL PROCEDURE

The procedure followed in approaching criticality in these experiments was the same as that described in Part III. Fuel was added stepwise, noting continuously the value of the neutron flux as indicated by the trace on the recorders, and measuring the reactivity with counters between fuel additions. Graphs, showing the dependence of the reciprocal multiplication on the height of solution in the reactor, were extrapolated as a guide in approaching the critical height. Near criticality the source of neutrons was removed after which the quantity of fuel and the control rod position were adjusted to maintain constant neutron intensity, the criterion for criticality. These procedures were not altered by the presence of the second reactor except that two control rods and two sources of neutrons were used. It is imperative to have these dual controls especially if the conditions of an experiment are such that the two reactors can be effectively isolated and made critical singly. It should be pointed out that the two neutron sources must be of comparable strength in order that the removal of one from a sub-critical system will effect a change in the total neutron flux.



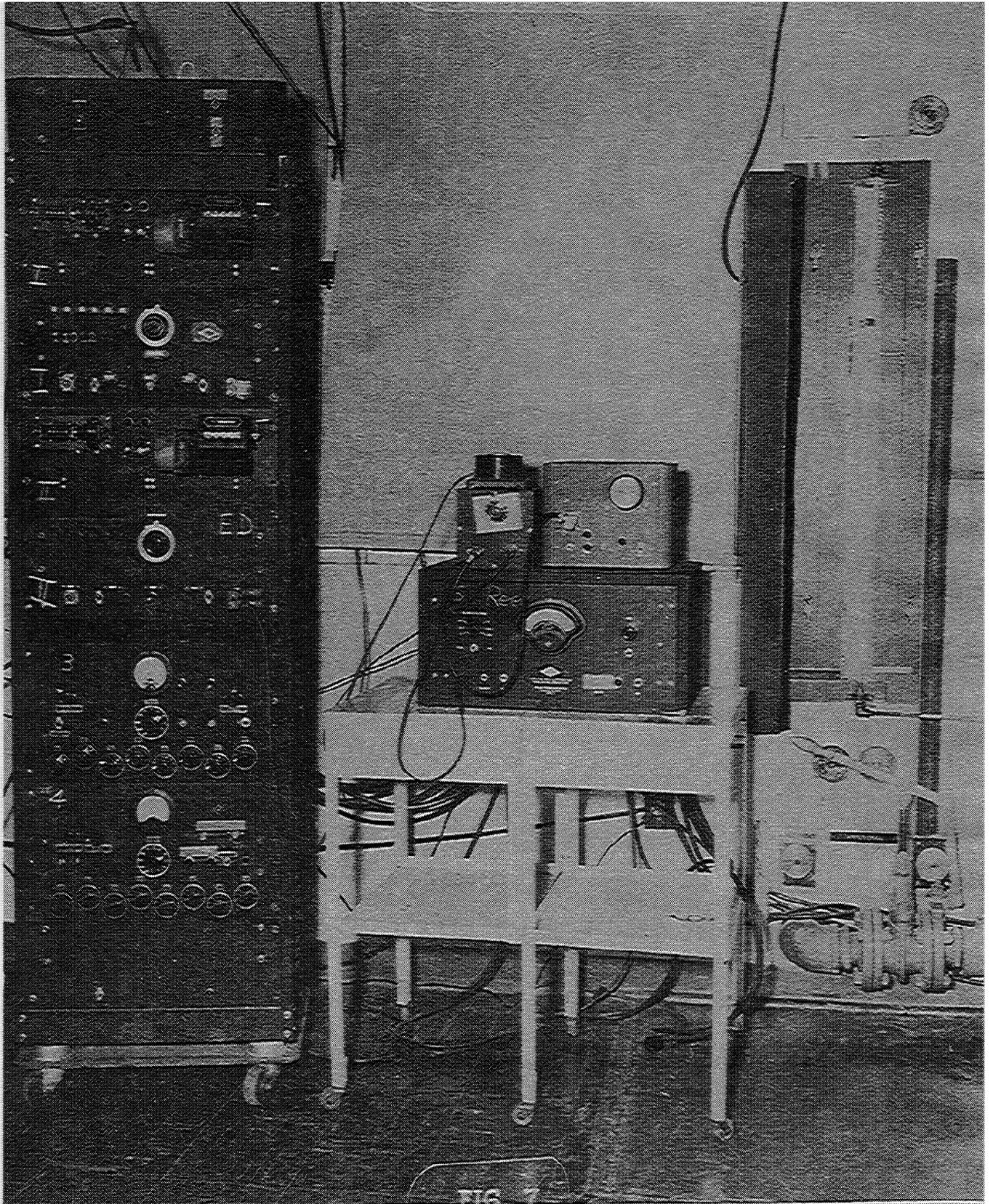


FIG. 7

In general, the critical height was first determined in each of two equal cylinders placed with sides in contact. After inserting control rods the dolly was then moved, separating the reactors a few centimeters with concomitant decrease in reactivity. Fuel was added to make the system critical once more. The procedure was repeated until the critical height was independent of the separation of the reactors. Some experiments were limited by the quantity of U-235 available or by the dimensions of the equipment. Fig. 8 shows a sequence of reciprocal multiplication curves obtained in one experiment.

V. RESULTS AND DISCUSSION

A. Introduction

In these experiments, data were obtained on the critical mass of U-235 in aqueous solutions of UO_2F_2 contained in two cylindrical reactors with axes parallel. The separation of the reactors, the hydrogen content of the fuel, and the nature of the reflector are the experimental variables which were investigated. The data are recorded in Tables 3 and 4 in the Appendix. The values given for the separation of the reactors are the distances between adjacent outer surfaces. Thus the separation reported should be increased by 0.3 cm ($1/8''$) to obtain the actual distance between adjacent edges of columns of fuel. This 0.3 cm represents the thickness of the walls of the reactors. The temperatures of the fuel and surroundings varied between 10°C and 25°C .

The information listed in the two right hand columns of each table is derived from data reported here and in Part III and will be discussed below. Typical curves showing trends in the data have been included.

B. Experimental Results

Figure 9 shows how the critical mass per reactor in an interacting pair of reactors without reflector increases with increasing separation of their edges. As expected, with increasing separation, the critical mass per reactor approaches that of an isolated one, although at a separation of two feet or more the critical mass per reactor may still be as much as 5% less than that in the isolated case. The data are shown differently in Fig. 10, where the ratio of the critical mass per reactor at a particular separation to the critical mass per reactor with the reactors in contact is plotted against the separation. The relative insensitivity of the larger diameter reactors, such as 15 to 20 inches, to interaction, is apparent. This insensitivity results from their slab-like geometry in which leakage of neutrons through the sides approaches zero. Since the critical height of such an isolated reactor is not greatly different from that of one having infinite diameter, the proximity of two of these reactors having coplanar bases does not materially affect the critical mass in one of them.

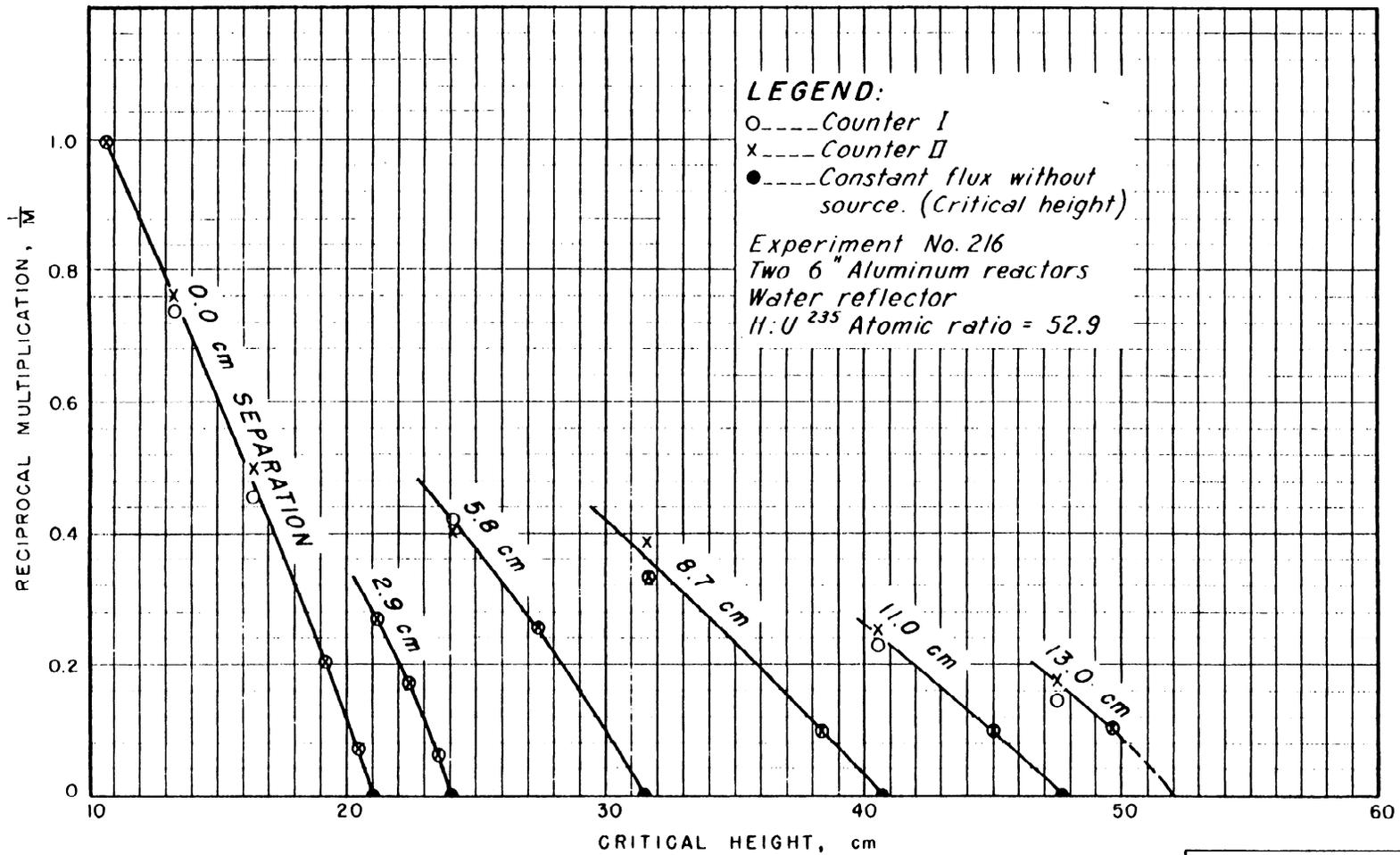
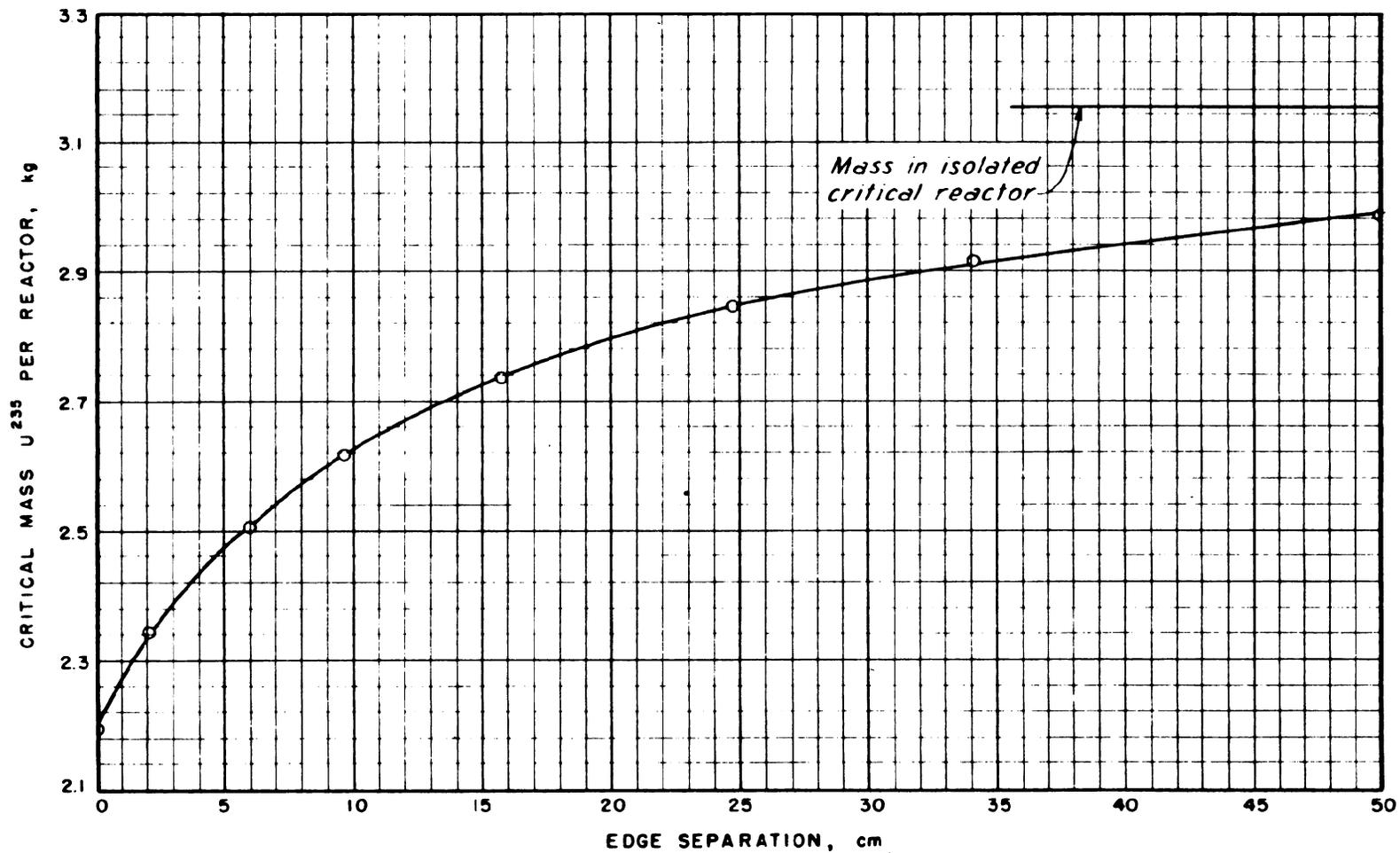


FIGURE 8

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| LABORATORY DIVISION | |
| TYPICAL RECIPROCAL MULTIPLICATION CURVES | |
| K-25 PLANT | |
| CARBIDE & CARBON CHEMICALS CORP | |
| DRWN <i>B. Hammer</i> | APPV'D _____ |
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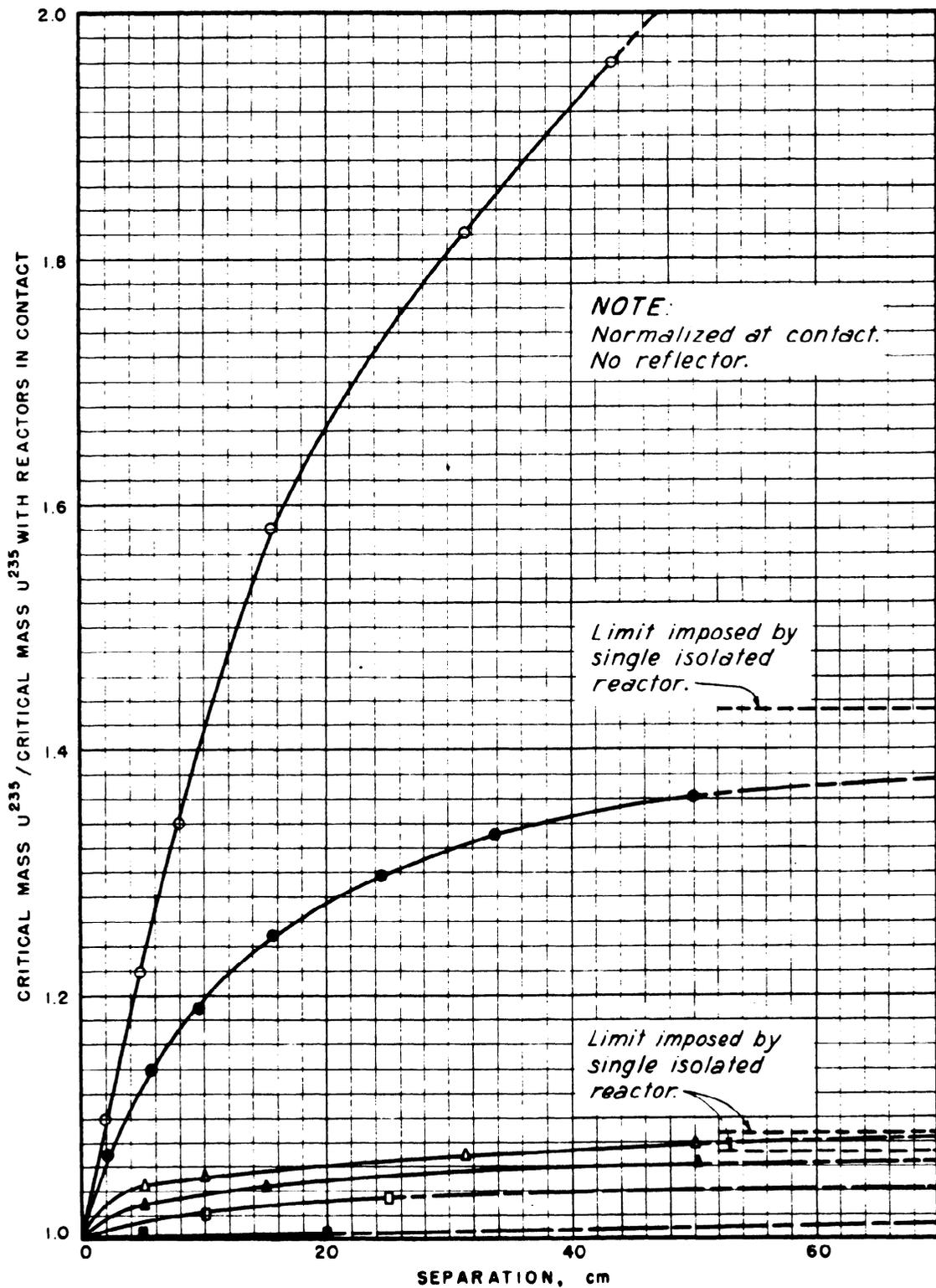


NOTE:

Experiment No. 224
 10" Aluminum reactors
 No reflector
 H:U²³⁵ Atomic ratio = 169

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| LABORATORY DIVISION | |
| CRITICAL MASS PER REACTOR VS REACTOR SEPARATION | |
| K-25 PLANT | |
| CARBIDE & CARBON CHEMICALS CORP | |
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FIGURE 9



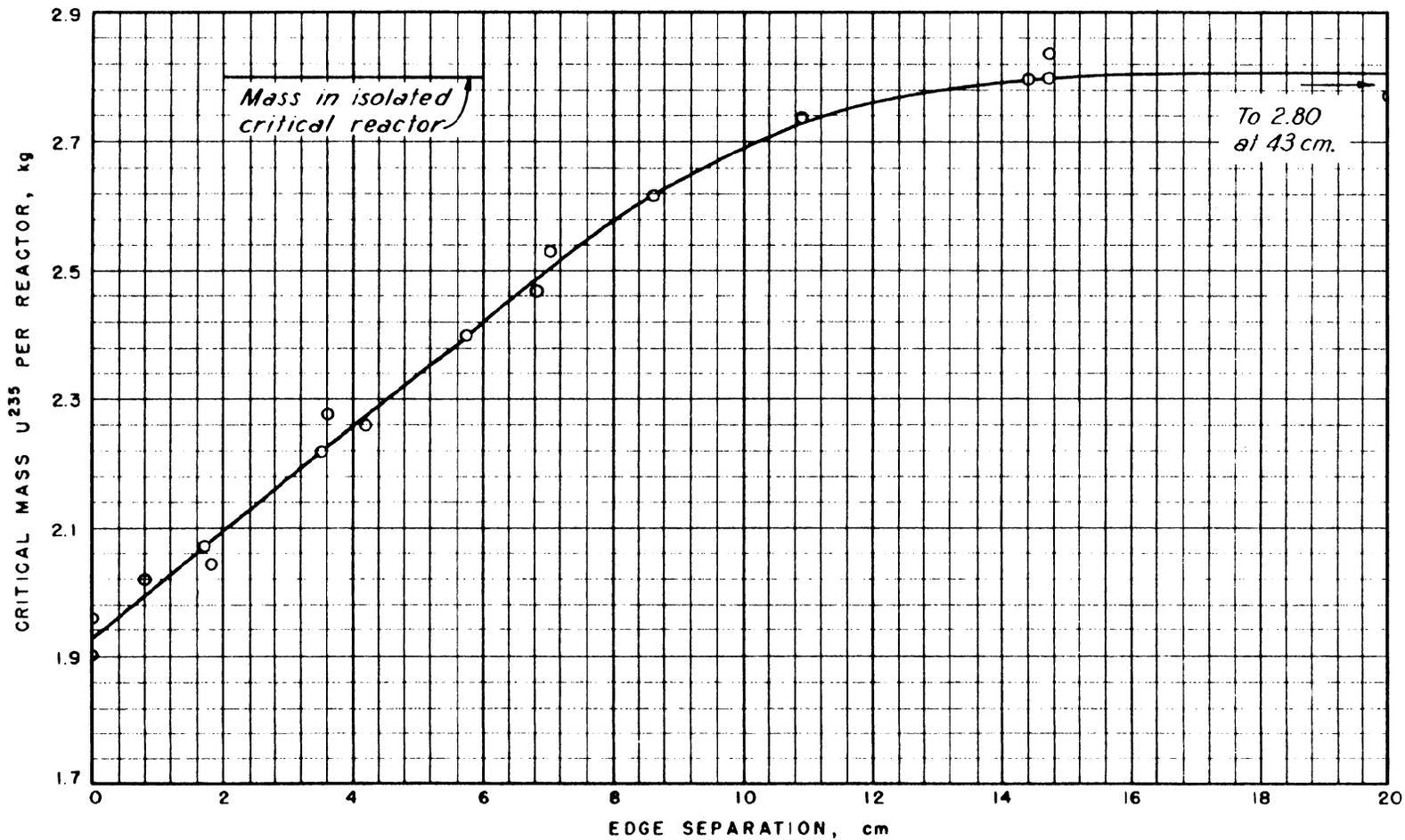
| SYMBOL | REACTOR | H:U ²³⁵ |
|--------|------------|--------------------|
| ○ | Al 10" dia | 329 |
| ● | Al 10" dia | 169 |
| △ | Al 15" dia | 329 |
| ▲ | Al 15" dia | 169 |
| □ | SS 20" dia | 329 |
| ■ | SS 20" dia | 169 |

| | |
|---|-----------------|
| LABORATORY DIVISION | |
| CRITICAL MASS Vs REACTOR SEPARATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
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FIGURE 10

In Fig. 11 is shown a typical curve of critical mass per reactor versus separation of two reactors submerged in water. It will be noted that although the mass per reactor is lower than that of the corresponding unreflected case due to neutron reflection, the reactors become effectively isolated at relatively smaller separations due to neutron absorption by the hydrogen. Since the water acts as both moderator and absorber, when the reactors are separated by a small amount, it might be expected that the moderation of fast neutrons would predominate over the absorption of thermal neutrons, yielding a minimum in the critical mass versus separation curve. Actually no such minimum was found. If it does exist, it must be extremely shallow, of the order of a few millimeters, and it must occur at a separation of 5 mm or less.

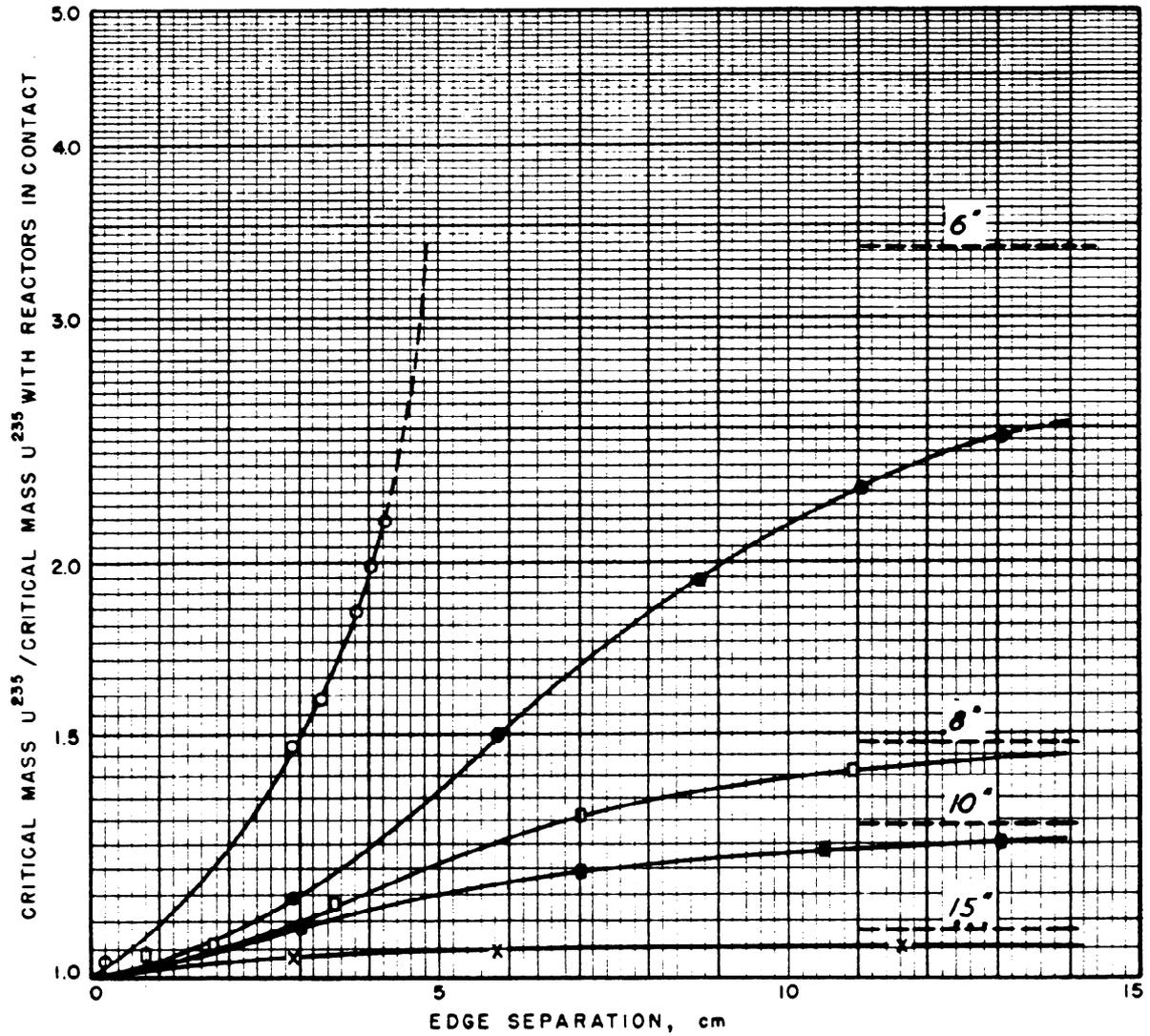
Data from five pairs of interacting water enclosed reactors having diameters of 5 to 15 inches, and with fuel having an H:U-235 ratio of 52.9 are plotted in Fig. 12 as a function of reactor separation. The ordinate is the ratio of the critical mass per reactor at various separations to the critical mass per reactor when the two were in contact. In general the curves are of two types: (a) those for reactors which are singly non-critical at infinite height yielding curves having vertical asymptotes and (b) those for reactors which are singly critical at a finite height which give curves with horizontal asymptotes. The smaller the diameter of each of the pair of reactors, the steeper the corresponding curve, for, when the reactor diameter is sufficiently small, they may remain subcritical in spite of considerable interaction. The positions of the vertical asymptotes cannot be taken, therefore, as distances at which no significant interaction occurs. Conversely, it has already been noted that the large diameter reactors are relatively insensitive to interaction so the behavior of these curves cannot be used to estimate the limiting interaction distance in water. Attention must therefore be confined to those reactors which are critical without interaction and yet are small enough in diameter to be sensitive to exchange neutrons. Unfortunately, such data are extremely limited and a tendency towards over-generalization must be avoided. It can be seen that for reactors having diameters of 8 to 15 inches the interaction is negligible for separations of 15 cm or more. The data for the six inch reactor indicates that significant interaction occurs at greater than 20 cm in this particular case. The behavior of the six inch reactor may be ascribed qualitatively to the geometric properties of those reactors which can be made singly critical at heights large compared to their diameters. In such a system, the contribution to the reactivity of significant quantities of uranium placed at the extremity of the reactor is small, that is, a significant increase in fuel height is required to effect only a small increase in the neutron reproductivity. In this manner, the exchange of a few external neutrons between companion reactors is equivalent in its effect on critical conditions to a relatively large quantity of fuel placed at the extremity of such a reactor.



NOTE:
 Experiments 205 and 220
 8" Aluminum reactors
 H:U²³⁵ Atomic ratio = 52.9
 Water reflector

| | |
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| CRITICAL MASS PER REACTOR VS REACTOR SEPARATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
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FIGURE II



LEGEND:

- ----- 5" Reactor diameter
- ----- 6" Reactor diameter
- ----- 8" Reactor diameter
- ----- 10" Reactor diameter
- x ----- 15" Reactor diameter

Aluminum reactors
 Water reflector
 H:U²³⁵ Atomic ratio = 52.9
 Normalized at contact

----- Limiting value of ratio determined from critical mass in isolated reactor.

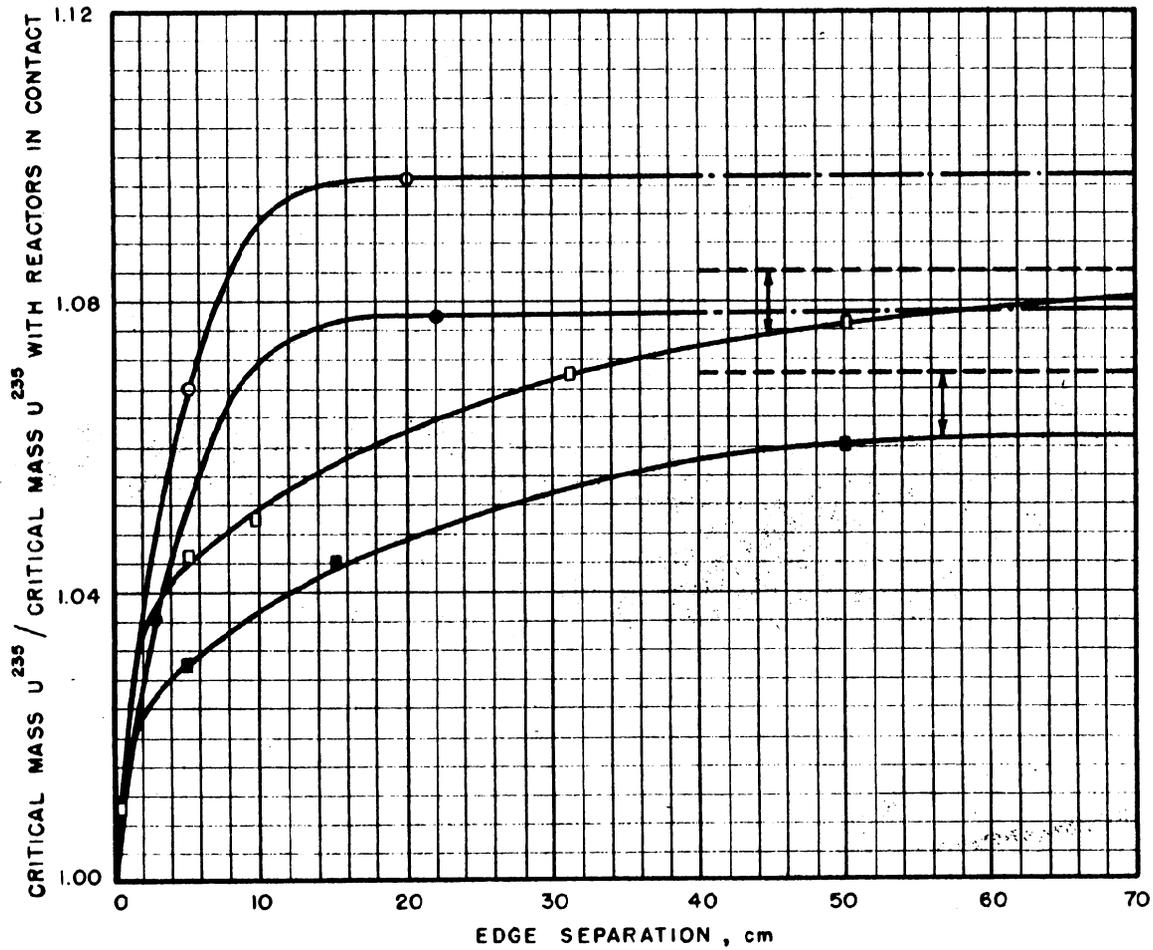
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| LABORATORY DIVISION | |
| CRITICAL MASS RATIO Vs REACTOR SEPARATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
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| DATE: MAY 27, 1949 | DWG NO: LD-B864 |

FIGURE 12

Pairs of reactors having heights several times larger than their diameters will, therefore, apparently display inordinately large interaction even at separations of 20 cm and greater. The apparent interaction is merely the interchange of relatively few neutrons magnified by the geometric conditions of the system. This hypothesis leads to the conclusion that there is no limiting interaction distance in water. However, in practice a limit will be reached in those cases of geometry where the minimum measurable additions of fuel contributes more to the reactivity than do the exchange neutrons. Thus an edge-to-edge separation of 15 centimeters for two reactors 8 inches or more in diameter appears to be large enough to insure that the critical mass per reactor will not differ significantly from that of an isolated one of similar dimensions. There is evidence that, for pairs of reactors between 5-1/2 and 8 inches in diameter, the above value of the separation needs to be increased in order to eliminate measurable interaction between the components. For pairs of reactors having diameters of 5-1/2 inches or less, 15 centimeter separation is again adequate for practical isolation.

Some of the results obtained in these experiments are given in Figs. 13 and 14 in which the ratio of the critical mass per reactor at some separation to the critical mass per reactor with the reactors in contact is plotted against the separation. Fig. 13 refers to reactors 15 inches in diameter at two fuel moderations and Fig. 14 contains data for a 10 inch reactor. Where possible the mass contained in a single isolated reactor is shown as the limiting value. The large increase in the critical mass with separation for the ten inch reactors without reflector is attributed to the geometric effects discussed above for the six inch water moderated system. The height to diameter ratio is sufficiently large to require a considerable increase in uranium content to offset the loss of a few exchange neutrons with increased separation.

The effect of moderation upon the apparent interaction is shown in Fig. 15 where the ratio of the critical mass contained in a single water enclosed isolated reactor to that in each of a pair of reactors in contact is plotted against the H:U-235 atomic ratio. Over the range of conditions under which an isolated reactor can become critical, the interaction is nearly independent of the moderation. Also, the apparent interaction decreases as the reactor diameter increases, being less than 10% for the 15 inch reactor system, an effect equally true for systems of large reactors without water reflector. The lack of further experimental data and the difficulties of treating the problem theoretically preclude the possibility of making more general conclusions at this time. Attempts to describe the observed effects of the interacting neutrons in terms of a simple geometric parameter have likewise led to inconclusive results due to the limited data. Two such parameters will be discussed briefly for general interest.



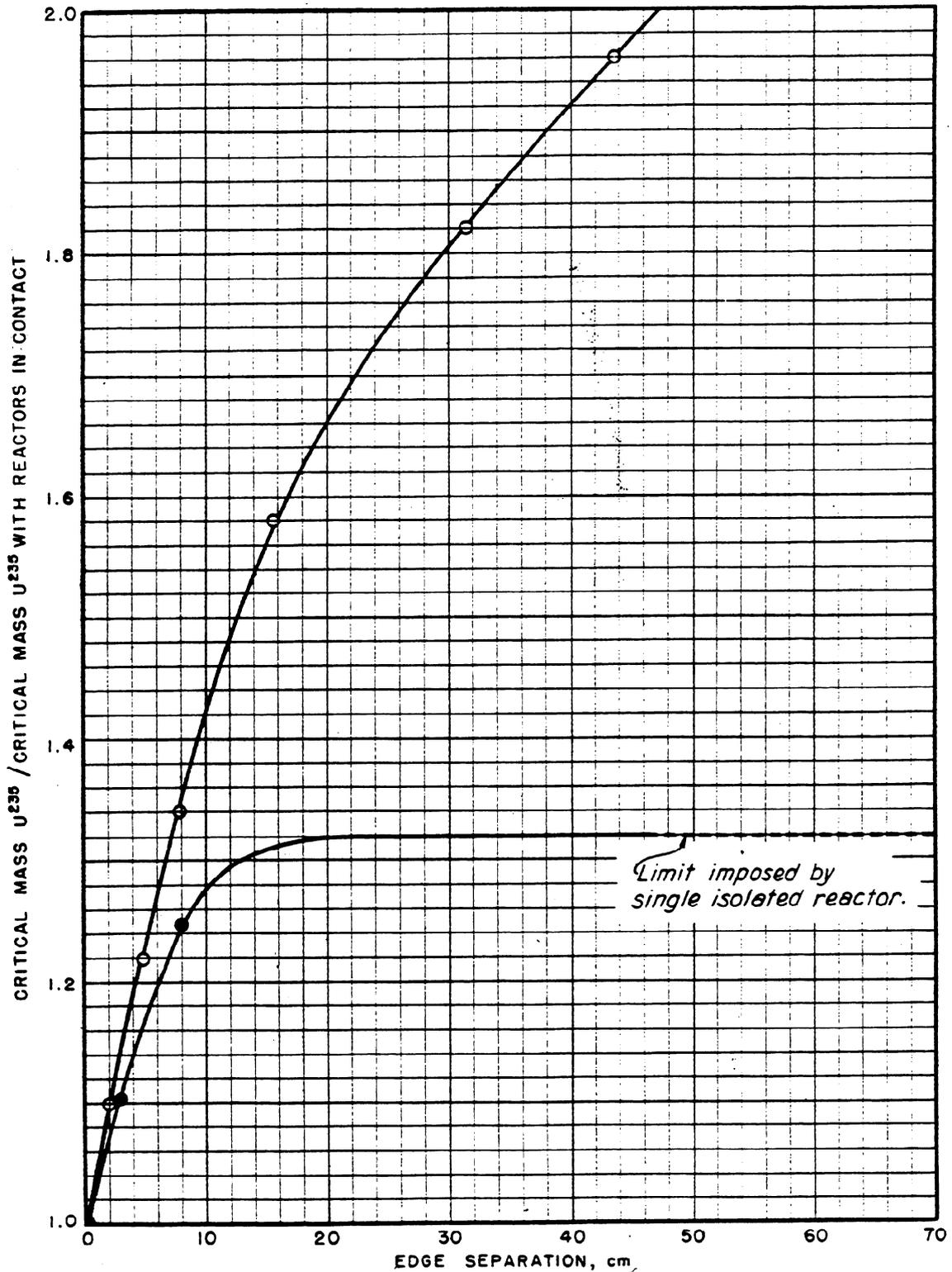
LEGEND:

- Water reflector $H:U^{235}=329$
- Water reflector $H:U^{235}=169$
- No reflector $H:U^{235}=329$
- No reflector $H:U^{235}=169$
- Limit imposed by single isolated reactor.

Normalized at contact
 Reactor diameter = 15"

| | |
|---|-----------------|
| LABORATORY DIVISION | |
| CRITICAL MASS VS REACTOR SEPARATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
| DRWN <i>B. Hammer</i> | APPV'D ----- |
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FIGURE 13



LEGEND:

○ No Reflector

● Water Reflector

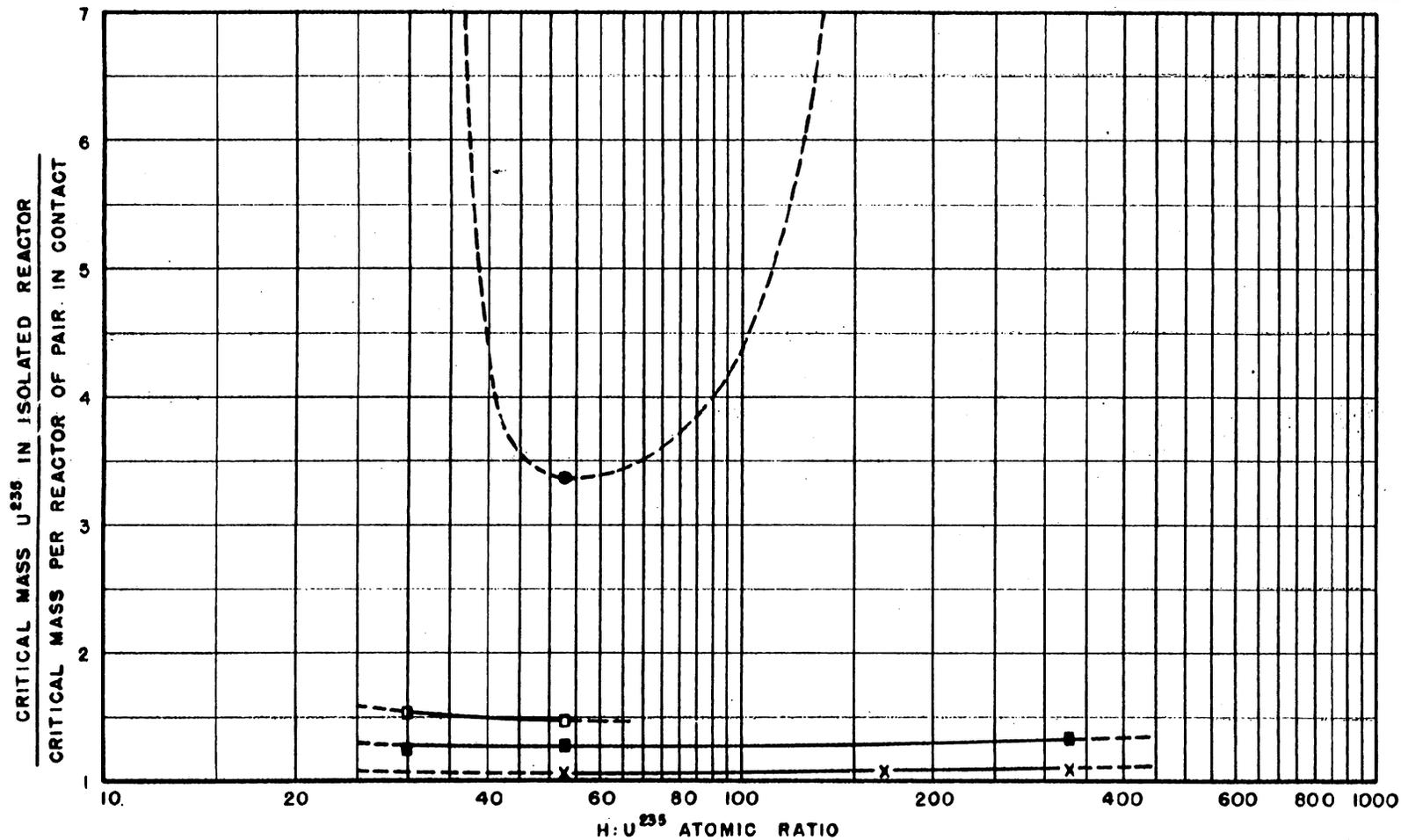
Reactor diameter = 10"

Normalized at contact

H:U²³⁵ Atomic ratio = 329

| | |
|---|-----------------|
| LABORATORY DIVISION | |
| CRITICAL MASS VS REACTOR SEPARATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
| DRWN <u>F.C. Allen</u> | APP'VD _____ |
| DATE: MAY 16, 1949 | DWG NO: LD-B847 |

FIGURE 14



REACTOR DIAMETER:

- 6"
- 8"
- 10"
- x----- 15"

Water enclosed aluminum reactors.

| | |
|---|-----------------|
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| APPARENT INTERACTION VS FUEL MODERATION | |
| K-25 PLANT CARBIDE & CARBON CHEMICALS CORP | |
| DRWN. <i>B. Hammer</i> | APPV'D. _____ |
| DATE: MAY 31, 1949 | DWG NO: LD-8865 |

FIGURE 15

C. Geometric Parameters

In the absence of an interposed reflector, the fractional solid angle between two cylinders has been suggested as a measure of the amount of neutron interaction. As already pointed out, however, in these experiments with cylindrical reactors, the change in critical height with separation of the reactors is not only a function of the exchange neutron flux but is also strongly dependent on the geometry and fuel characteristics of the individual reactors. It is, therefore, not surprising that there appears to be no simple empirical relation between solid angle and the corresponding increase in solution height. In the presence of a water reflector the situation is further complicated by diffusion and absorption effects in the reflecting medium.

A second and possibly more useful concept is that of an equivalent diameter which may be defined in the following manner. Two interacting cylindrical reactors, separated by a fixed distance are found to be critical when filled to the appropriate height with fuel of known moderation. The diameter of a single isolated reactor which is critical at the same height and moderation is defined as the equivalent diameter of the pair. Data on isolated cylindrical reactors are reported in Part III, Figs. 29 and 32 where critical heights are plotted as a function of reactor diameters at constant moderation. Thus the equivalent diameter D_E of the pair can be readily obtained from a knowledge of the critical height and moderation of the reacting pair. It is appropriate to consider now the relation of D_E to the actual diameter, D , of each of two reactors with equal diameter. In the limit of no interaction, D_E is obviously equal to D . As the separation of the two reactors is decreased, the critical height of each of the pair is also decreased due to the exchange of neutrons between them. Hence, a single cylindrical reactor, in order to be critical at this lowered height would require an increased diameter. The equivalent diameter, therefore, increases as the reactors are brought together. In the hypothetical limit of coalescence, the equivalent diameter becomes $\sqrt{2}D$, the diameter of a single vessel which would contain all the fuel present in two reactors each of diameter D . In practice, of course, the limit of approach is contact of the leading edges so the actual limit of D_E is somewhat less than $\sqrt{2}D$ because of the excess leakage from the "figure eight" cross section of two cylinders in contact.

For reactors in contact, the value of D_E remains sensibly constant for each diameter at different values of the H:U-235 atomic ratio of the fuel. The ratios $D_E/D = r$ for reactors in contact at each experimental value of H:U-235 are given in Table 1. As the separation of two water enclosed reactors is increased, the value of D_E changes with the atomic ratio due to

Table 1Equivalent Diameters of Aluminum Reactors in Contact

| <u>Reactor Diameter</u> <u>inches</u> <u>D</u> | <u>H:U-235</u> | <u>Critical</u> <u>Height</u> <u>cm</u> | <u>Equivalent Diameter</u> <u>inches</u> <u>D_E</u> | <u>D_E/D = r</u> |
|--|----------------|---|---|----------------------------|
| <u>Water Enclosed</u> | | | | |
| 5 | 52.9 | 35.4 | 6.7 | 1.34 |
| 5-1/2 | 29.9 | 26.3 | 7.4 | 1.35 |
| 6 | 29.9 | 22.8 | 7.8 | 1.30 |
| | 52.9 | 21.0 | 7.7 | 1.28 |
| | 169. | 28.2 | 8.0 | 1.33 |
| | 329. | 61.0 | 7.7 | 1.28 |
| 8 | 29.9 | 13.4 | 10.1 | 1.26 |
| | 52.9 | 12.8 | 10.1 | 1.26 |
| 10 | 29.9 | 11.0 | 11.6 | 1.16 |
| | 52.9 | 10.2 | 12.0 | 1.20 |
| | 329. | 16.9 | 11.2 | 1.12 |
| <u>Unenclosed</u> | | | | |
| 15 | 169. | 17.2 | 15.9 | 1.06 |
| | 329. | 20.0 | 16.2 | 1.08 |
| 20 Stainless Steel | 329. | 16.7 | 21.6 | 1.08 |

the absorption of exchange neutrons by the water between the reactors. These effects may be noted by reference to Fig. 16 which shows the critical height as a function of H:U-235 atomic ratio of two six inch reactors at several separations. Curves for single 6, 6-1/2 and $6\sqrt{2}$ inch reactors taken from Part III have been included for comparison. It will be noted that the curve for the six inch pair in contact is very nearly parallel to that for the single $6\sqrt{2}$ inch reactor. The curve for the 6-1/2 inch single reactor, on the other hand, crosses both the 6 cm and 8 cm separation curves leading to a noticeable dependence of D_E upon the moderation of the fuel. Such a dependence is not expected in the absence of a water reflector but there is not enough experimental information on the unreflected reactors to support this contention.

D. Application to Processing Specifications

It was shown in Part III that a single water enclosed reactor 5.5 inches in diameter was subcritical under all conditions studied. A pair of reactors having an equivalent diameter of 5.5 inches or less would also be subcritical under the same conditions. It is, therefore, possible to estimate the diameter, D , of the individual reactors having $D_E = 5.5$ inches from the relation

$$\frac{D_E}{D} = r$$

The maximum value of r is not greater than $\sqrt{2}$ and, as shown in Table 1, is not less than 1.34. Thus

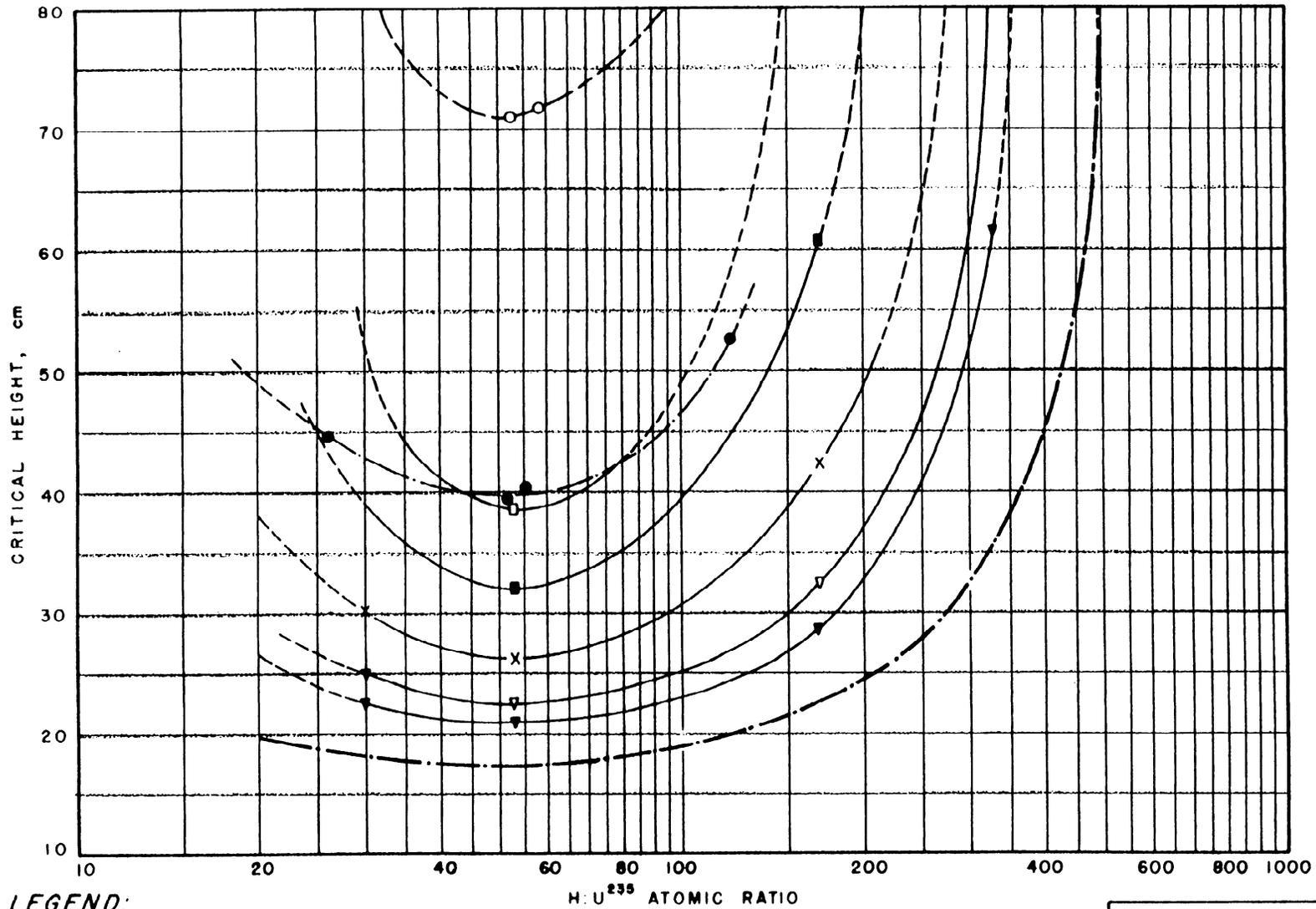
$$\frac{5.5}{\sqrt{2}} \leq D \leq \frac{5.5}{1.34}$$

$$3.88'' \leq D \leq 4.10''$$

These values have not been checked experimentally. The estimation is included here to emphasize the fact that two cylinders 4 inches in diameter in contact and water enclosed may form a critical system.

E. Accuracy and Precision

It is appropriate to consider the accuracy of the results of these experiments and the precision of the measurements. An overall estimate of $\pm 5\%$ L.E. (95% confidence interval) has been made of the accuracy following consideration of height, diameter, solution concentration and separation measurements. Some of the causes of the uncertainties are reviewed below.



LEGEND:

o--- Single 6" reactor
 ●--- Single 6½" reactor
 —•— Envelope - single 6√2" reactor
 Aluminum reactors
 6" diameter - water enclosed

Several reactor separations
 □--- 8 cm separation
 ■--- 6 cm separation
 x--- 4 cm separation
 ▽--- 2 cm separation
 ▼--- 0 cm separation

| | |
|---------------------------------------|-----------------|
| LABORATORY DIVISION | |
| CRITICAL HEIGHT VS FUEL MODERATION | |
| K-25 PLANT | |
| CARBIDE & CARBON CHEMICALS CORP | |
| DRWN <i>B. Hammer</i> | APP'VD _____ |
| DATE: MAY 31, 1949 | DWG NO: LD-B851 |

FIGURE 16

1. Volume of Uranium Solution and Mass of U-235

The height of fuel in a reactor at criticality was reproducible to ± 0.1 cm. The inaccuracies in volume measurement did not exceed $\pm 2\%$ L.E. and were due to variations in the zero level of the height scale and in the reactor diameters resulting from constructional irregularities.

The mass was determined from the volume, the specific gravity, and a gravimetric analysis of the uranium solution. The analytical error, including sampling and specific gravity measurements is estimated at $\pm 2\%$ L.E. The overall error in the mass is thus estimated at $\pm 3\%$ L.E.

2. Separation of Reactors

Errors in measurement of the distances between pairs of reactors were due principally to lack of axial parallelism amounting to a few millimeters over a height of a meter. The separation reported is the distance between adjacent edges at one-half critical height.

3. Reactor Construction

A column of fuel three inches in diameter and about one foot long was attached to the bottom of each stationary reactor because of structural details. The manner in which this appendage contributed to the reactivity of the stationary reactor is discussed in Part III and was considered again in the experiments reported here.

A more direct evaluation of this effect could be obtained with the apparatus of these experiments. A comparison was made between the solution height at criticality in the stationary reactor, filled singly, with the corresponding height in the movable cylinder when it was made critical alone. Data were obtained with reactors of two diameters and are tabulated below:

Table 2

Feed Line Correction

| Reactor Diameter inches | H:U-235 | Critical Height | | Feed Line Correction | |
|-------------------------------|---------|--|--------------------------|--------------------------|----------------|
| | | Stationary Reactor Uncorrected cm | Movable Reactor cm | This Experiment cm | Part III cm |
| 8 | 29.9 | 19.3 | 20.5 | 1.2 | 1.4 |
| 10 | 52.9 | 12.5 | 13.2 | 0.7 | 0.9 |

The agreement between the two independent values of the correction is considered adequate.

Since the bottom surfaces of the reactors, when used together, are coplanar, the solution height in the movable cylinder is measured directly by the sight glass. The critical height of each of a pair of critical reactors recorded in the data is the average of the actual height in the movable cylinder and the corrected height in the stationary cylinder.

A second effect of the three inch appendage is its possible contribution to interaction. In one experiment the movable cylinder was lowered an amount equal to the correction applied to the stationary reactor by placing shims of appropriate thickness between the top flange and the dolly, thus making the effective lower surfaces of the cylinders of fuel coplanar. A comparison was then made between the values of the critical mass at various reactor separations with corresponding values determined with the cylinders in their usual position. No differences exceeding the precision of the measurements were found.

4. Temperature

In the experiments reported here, there was no control of the temperature of the uranyl fluoride solution or of the reflector water and no measure of the temperature coefficient of reactivity. However, the experiments were performed within a period of seven weeks and were not subjected to marked seasonal temperature variations, hence errors introduced by this cause amount to a few tenths of a percent at most.

VI. SUMMARY

Results from the experiments reported here which are of value in the specification of safe conditions for handling enriched uranium solution are summarized in this section.

1. The smallest critical mass observed in these experiments was measured when 680 gm of U-235 were placed in each of two water enclosed aluminum reactors 10 inches in diameter which were in contact with each filled to a height of 16.9 cm. The chemical concentration of the fuel corresponded to H:U-235 = 329. Since the moderation was not further increased and only a limited number of reactors were used, it is probable that the minimum critical mass attainable is significantly less than the above value. The latter is to be compared with 890 gm of U-235 required to become critical in a single reactor 10 inches in diameter at the same moderation.

2. An imposed time limitation made impossible the extension of these measurements to pairs of sub-critical water enclosed reactors. The smallest reactors tested were a pair five inches in diameter which were critical when in contact, containing 2.06 kg U-235 each, and filled to a height of 35.4 cm with a solution of H:U-235 = 52.9. Extrapolation of the data predicts that they would be sub-critical at any height when separated seven centimeters or more.
3. Examination of the data indicates that two reactors four inches in diameter, placed in contact and enclosed by water, might be made critical. This result was not verified by direct experiment.
4. A layer of water approximately 15 cm (6 inches) separating two reactors each 8 inches or more in diameter effectively shields one from the other. Residual neutron exchange was found sufficient to change the critical conditions of two 6 inch reactors when separated by as much as 22 cm of water.
5. Sufficient uranium was not available to reach criticality with two eight inch reactors in contact and without a water reflector. However, at an H:U-235 ratio of 52.9, it was indicated that they would be critical at some finite height greater than 38 cm.
6. Interaction between two reactors without water reflector was observed when they were separated as much as 50 cm. However, the mass in each at that separation was more than 90% of that required to make it critical alone.
7. Apparent interaction effects decrease sharply with increasing reactor diameters for both bare and water reflected pairs. The decrease results from the reduction of side leakage as the reactors approach infinite slabs.

VII. ACKNOWLEDGMENTS

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VIII. APPENDIX

Table 3 Critical Conditions for Two Aluminum Reactors Without Water Reflector

Table 4 Critical Conditions for Two Aluminum Reactors With Water Reflector

TABLE 3
CRITICAL CONDITIONS FOR TWO
ALUMINUM REACTORS WITHOUT WATER REFLECTOR

| REACTOR | | H ² U-235 ATOMIC RATIO | GM U-235 GM SOLUTION | SOLUTION DENSITY GM/CM ³ | CRITICAL HEIGHT CM | CRITICAL MASS PER REACTOR KG U-235 | EQUIVALENT DIAMETER INCHES ^a D _E | RATIO D _E /D = I' |
|-------------------------|------------------|---|-------------------------|---|--------------------------|---|---|------------------------------------|
| DIAMETER INCHES D | SEPARATION CM | | | | | | | |
| 8 | 0.0 | 52.9 | 0.298 | 1.566 | > 38 | > 5.65 | - | - |
| | 0.0 | 169 | 0.127 | 1.187 | | | - | - |
| 10 | 0.0 | 52.9 | 0.293 | 1.566 | > 20 | > 4.65 | - | - |
| | 0.0 | 169 | 0.127 | 1.187 | 28.7 | 2.20 | 11.4 | 1.14 |
| | 2.0 | 169 | 0.127 | 1.187 | 30.7 | 2.35 | 11.1 | 1.11 |
| | 5.8 | 169 | 0.127 | 1.187 | 32.8 | 2.51 | 10.8 | 1.08 |
| | 9.6 | 169 | 0.127 | 1.187 | 34.3 | 2.62 | 10.6 | 1.06 |
| | 15.6 | 169 | 0.127 | 1.187 | 35.8 | 2.74 | 10.4 | 1.04 |
| | 24.6 | 169 | 0.127 | 1.187 | 37.2 | 2.85 | 10.3 | 1.03 |
| | 33.9 | 169 | 0.127 | 1.187 | 38.2 | 2.92 | 10.2 | 1.02 |
| | 50.0 | 169 | 0.127 | 1.187 | 39.1 | 2.99 | 10.2 | 1.02 |
| | 0.0 | 329 | 0.0715 | 1.101 | 40.8 | 1.63 | - | - |
| | 1.9 | 329 | 0.0715 | 1.101 | 44.9 | 1.79 | - | - |
| | 4.8 | 329 | 0.0715 | 1.101 | 50.0 | 1.99 | - | - |
| | 8.0 | 329 | 0.0715 | 1.101 | 54.8 | 2.18 | - | - |
| | 16.6 | 329 | 0.0715 | 1.101 | 64.7 | 2.58 | - | - |
| | 31.3 | 329 | 0.0715 | 1.101 | 74.4 | 2.97 | - | - |
| 43.3 | 329 | 0.0715 | 1.101 | 80.1 | 3.19 | - | - | |
| 15 | 0.3 | 169 | 0.127 | 1.187 | 17.3 | 2.98 | 15.9 | 1.06 |
| | 5.0 | 169 | 0.127 | 1.187 | 17.8 | 3.06 | 15.5 | 1.03 |
| | 15.0 | 169 | 0.127 | 1.187 | 18.0 | 3.10 | 15.3 | 1.02 |
| | 50.0 | 169 | 0.127 | 1.187 | 18.3 | 3.15 | 15.1 | 1.01 |
| | 0.2 | 329 | 0.0715 | 1.101 | 20.1 | 1.80 | 16.2 | 1.08 |
| | 5.0 | 329 | 0.0715 | 1.101 | 20.8 | 1.87 | 15.5 | 1.03 |
| | 9.7 | 329 | 0.0715 | 1.101 | 21.0 | 1.88 | 15.4 | 1.03 |
| | 31.3 | 329 | 0.0715 | 1.101 | 21.3 | 1.91 | 15.2 | 1.01 |
| 50.0 | 329 | 0.0715 | 1.101 | 21.5 | 1.93 | 15.1 | 1.01 | |
| 20** | 0.0 | 169 | 0.127 | 1.187 | 14.7 | 4.50 | - | - |
| | 5.0 | 169 | 0.127 | 1.187 | 14.8 | 4.53 | - | - |
| | 20.0 | 169 | 0.127 | 1.187 | 14.8 | 4.53 | - | - |
| | 0.0 | 329 | 0.0715 | 1.101 | 16.7 | 2.66 | 21.6 | 1.08 |
| | 10.0 | 329 | 0.0715 | 1.101 | 17.0 | 2.71 | 20.8 | 1.04 |
| | 25.0 | 329 | 0.0715 | 1.101 | 17.3 | 2.76 | 20.2 | 1.01 |

^a DIAMETER OF A SINGLE CYLINDER CRITICAL AT EQUAL HEIGHT OF SOLUTION.(FROM PART III)

** THE REACTOR 20 INCHES IN DIAMETER WAS CONSTRUCTED OF STAINLESS STEEL.

TABLE 4
 CRITICAL CONDITIONS FOR TWO
 ALUMINUM REACTORS WITH WATER REFLECTOR

| REACTOR DIAMETER INCHES D | SEPARATION CM | H:U-235 ATOMIC RATIO | GM U-235 GM SOLUTION | SOLUTION DENSITY GM/CM ³ | CRITICAL HEIGHT CM | CRITICAL MASS PER REACTOR KG U-235 | EQUIVALENT DIAMETER INCHES ^a D _E | RATIO D _E /D = Σ |
|------------------------------------|------------------|----------------------------|-------------------------|---|--------------------------|---|---|-----------------------------------|
| 5 | 0.2 | 52.9 | 0.293 | 1.566 | 36.4 | 2.12 | 6.6 | 1.32 |
| | 2.9 | 52.9 | 0.293 | 1.566 | 51.9 | 3.02 | 6.2 | 1.24 |
| | 3.3 | 52.9 | 0.293 | 1.566 | 56.2 | 3.26 | 6.1 | 1.22 |
| | 3.8 | 52.9 | 0.293 | 1.566 | 65.3 | 3.80 | 6.0 | 1.20 |
| | 4.0 | 52.9 | 0.293 | 1.566 | 70.2 | 4.08 | - | - |
| | 4.2 | 52.9 | 0.293 | 1.566 | 76. | 4.4 | - | - |
| 5½ | 0.5 | 29.9 | 0.394 | 1.926 | 27.1 | 3.15 | 7.4 | 1.34 |
| | 2.9 | 29.9 | 0.394 | 1.926 | 31.0 | 3.60 | 7.1 | 1.29 |
| 6 | 0.1 | 29.9 | 0.394 | 1.926 | 22.8 | 3.16 | 7.8 | 1.30 |
| | 2.9 | 29.9 | 0.394 | 1.926 | 26.9 | 3.73 | 7.4 | 1.23 |
| | 0.0 | 52.9 | 0.293 | 1.566 | 21.0 | 1.76 | 7.7 | 1.28 |
| | 2.9 | 52.9 | 0.293 | 1.566 | 24.0 | 2.01 | 7.3 | 1.22 |
| | 5.8 | 52.9 | 0.293 | 1.566 | 31.5 | 2.64 | 6.8 | 1.13 |
| | 8.7 | 52.9 | 0.293 | 1.566 | 40.7 | 3.41 | 6.4 | 1.07 |
| | 11.0 | 52.9 | 0.293 | 1.566 | 47.6 | 3.98 | 6.3 | 1.05 |
| | 13.0 | 52.9 | 0.293 | 1.566 | 52. | 4.3 | 6.2 | 1.03 |
| | 0.4 | 169 | 0.127 | 1.187 | 28.9 | 0.796 | 7.9 | 1.32 |
| | 2.0 | 169 | 0.127 | 1.187 | 32.5 | 0.895 | 7.6 | 1.27 |
| | 4.0 | 169 | 0.127 | 1.187 | 42.2 | 1.16 | 7.1 | 1.18 |
| | 6.0 | 169 | 0.127 | 1.187 | 60.8 | 1.65 | 6.7 | 1.12 |
| | 8.0 | 169 | 0.127 | 1.187 | | | - | - |
| | 0.2 | 329 | 0.0715 | 1.101 | 63.1 | 0.905 | 7.6 | 1.27 |
| | 0.5 | 329 | 0.0715 | 1.101 | 66.2 | 0.950 | 7.6 | 1.27 |
| 0.8 | 329 | 0.0715 | 1.101 | 69.8 | 1.00 | 7.5 | 1.25 | |
| 8 | 0.0 | 29.9 | 0.394 | 1.926 | 13.4 | 3.29 | 10.1 | 1.26 |
| | 0.0 | 29.9 | 0.394 | 1.926 | 13.4 | 3.29 | 10.1 | 1.26 |
| | 0.4 | 29.9 | 0.394 | 1.926 | 13.5 | 3.32 | 9.8 | 1.23 |
| | 1.5 | 29.9 | 0.394 | 1.926 | 14.0 | 3.43 | 9.7 | 1.21 |
| | 0.0 | 52.9 | 0.293 | 1.566 | 12.8 | 1.90 | 10.1 | 1.26 |
| | 0.0 | 52.9 | 0.293 | 1.566 | 13.2 | 1.96 | 9.9 | 1.24 |
| | 0.8 | 52.9 | 0.293 | 1.566 | 13.6 | 2.02 | 9.8 | 1.23 |
| | 1.7 | 52.9 | 0.293 | 1.566 | 13.9 | 2.07 | 9.7 | 1.21 |
| | 1.8 | 52.9 | 0.293 | 1.566 | 13.7 | 2.04 | 9.8 | 1.23 |
| | 3.5 | 52.9 | 0.293 | 1.566 | 14.9 | 2.22 | 9.4 | 1.18 |
| | 3.6 | 52.9 | 0.293 | 1.566 | 15.3 | 2.28 | 9.0 | 1.12 |
| | 4.2 | 52.9 | 0.293 | 1.566 | 15.2 | 2.26 | 9.0 | 1.12 |
| | 4.2 | 52.9 | 0.293 | 1.566 | 15.2 | 2.26 | 9.0 | 1.12 |
| | 5.7 | 52.9 | 0.293 | 1.566 | 16.1 | 2.40 | 8.7 | 1.09 |
| | 6.8 | 52.9 | 0.293 | 1.566 | 15.6 | 2.47 | 8.6 | 1.07 |

TABLE 4 (Cont.)

| REACTOR | | H:U-235 ATOMIC RATIO | GM U-235 GM SOLUTION | SOLUTION DENSITY GM/CM ³ | CRITICAL HEIGHT CM | CRITICAL MASS PER REACTOR KG U-235 | EQUIVALENT DIAMETER INCHES ^a D _E | RATIO D _E /D = r |
|-------------------------|------------------|----------------------------|-------------------------|---|--------------------------|---|---|-----------------------------------|
| DIAMETER INCHES D | SEPARATION CM | | | | | | | |
| 8 | 7.0 | 52.9 | 0.293 | 1.566 | 17.0 | 2.53 | 8.5 | 1.06 |
| | 8.6 | 52.9 | 0.293 | 1.566 | 17.6 | 2.62 | 8.3 | 1.04 |
| | 10.9 | 52.9 | 0.293 | 1.566 | 18.4 | 2.74 | 8.1 | 1.01 |
| | 14.4 | 52.9 | 0.293 | 1.566 | 18.8 | 2.80 | 8.0 | 1.00 |
| | 14.7 | 52.9 | 0.293 | 1.566 | 18.8 | 2.80 | 8.0 | 1.00 |
| | 14.7 | 52.9 | 0.293 | 1.566 | 19.1 | 2.84 | 8.0 | 1.00 |
| | 20.1 | 52.9 | 0.293 | 1.566 | 18.6 | 2.77 | 8.0 | 1.00 |
| | 43.0 | 52.9 | 0.293 | 1.566 | 18.8 | 2.80 | 8.0 | 1.00 |
| 10 | 0.0 | 29.9 | 0.394 | 1.926 | 11.0 | 4.23 | 11.6 | 1.16 |
| | 0.2 | 52.9 | 0.293 | 1.566 | 10.3 | 2.40 | 12.1 | 1.21 |
| | 3.0 | 52.9 | 0.293 | 1.566 | 11.2 | 2.60 | 11.4 | 1.14 |
| | 7.0 | 52.9 | 0.293 | 1.566 | 12.2 | 2.84 | 10.8 | 1.08 |
| | 10.5 | 52.9 | 0.293 | 1.566 | 12.7 | 2.95 | 10.5 | 1.05 |
| | 13.0 | 52.9 | 0.293 | 1.566 | 12.9 | 3.00 | 10.1 | 1.01 |
| | 20.0 | 52.9 | 0.293 | 1.566 | 13.0 | 3.02 | 10.0 | 1.00 |
| | 0.0 | 329 | 0.0715 | 1.101 | 16.9 | 0.674 | 11.3 | 1.13 |
| | 3.0 | 329 | 0.0715 | 1.101 | 18.7 | 0.745 | 10.7 | 1.07 |
| | 8.0 | 329 | 0.0715 | 1.101 | 21.1 | 0.841 | 10.2 | 1.02 |
| 15 | 0.0 | 52.9 | 0.293 | 1.566 | 7.3 | 3.82 | - | - |
| | 2.9 | 52.9 | 0.293 | 1.566 | 7.6 | 3.97 | - | - |
| | 5.8 | 52.9 | 0.293 | 1.566 | 7.65 | 4.00 | - | - |
| | 11.6 | 52.9 | 0.293 | 1.566 | 7.7 | 4.03 | - | - |
| | 0.0 | 169 | 0.127 | 1.187 | 9.0 | 1.55 | - | - |
| | 3.0 | 169 | 0.127 | 1.187 | 9.3 | 1.60 | - | - |
| | 22.0 | 169 | 0.127 | 1.187 | 9.7 | 1.67 | 15.0 | 1.00 |
| | 0.0 | 329 | 0.0715 | 1.101 | 11.5 | 1.03 | - | - |
| | 5.0 | 329 | 0.0715 | 1.101 | 12.3 | 1.10 | - | - |
| | 20.0 | 329 | 0.0715 | 1.101 | 12.6 | 1.13 | 15.0 | 1.00 |

^a DIAMETER OF A SINGLE CYLINDER CRITICAL AT EQUAL HEIGHT OF SOLUTION. (FROM PART III).